Analytical perturbative theories of motion in highly inhomogeneous gravitational fields

Executive Summary

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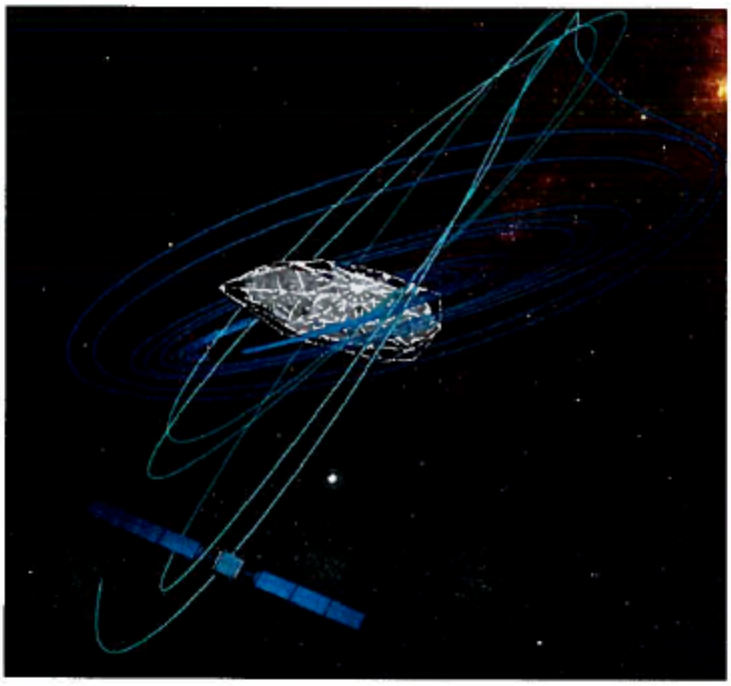
Picture:



Motivation:

This study addresses the problem of generating analytic approximations for spacecraft orbits in highly inhomogeneous gravitational fields. The project aims to develop an analytical tool that can be implemented algorithmically in standard symbolic software tools, such as Mathematica, or specialised algebraic tools, such as Piranha. The aim is to develop a general perturbative theory implementable in technical software that can rapidly generate orbits in highly inhomogeneous gravitational fields, for example, frozen orbits around asteroids for mission analysis.

Methodology:

The methodology enables the rapid generation of frozen orbits in highly inhomogeneous fields under certain assumptions. These frozen orbits could potentially provide initial guesses for frozen orbits in high fidelity asteroid simulations. Although an understanding of the methodology is not required to produce these initial guesses it is highlighted in the following. The methodology is based on a general perturbative theory, which considers all the terms of the gravitational potential. The solar radiation pressure could also be incorporated into this framework. The generalized perturbative method exploits the analytical methods of the relegation of the argument of node by Deprit (2001) and a Delauny normalisation of arbitrary order. These analytical methods are extremely powerful but up to now only first order approximations have been undertaken due to the high computational requirement. However, using standard technical computing software such as Mathematica or, better still, specialist algebraic manipulation software tools such as Piranha (an algebraic software tool developed by ACT) the analytical theory is realised as a useful tool in the analysis of the dymnamics in an inhomogeneous gravitational field. Therefore, the proposed theory presents a method to derive more accurate descriptions of the dynamics and moreover provide a method to find initial conditions to yield frozen orbits around asteroids. These frozen orbits can then be used as reference trajectories in missions that require close inspection of such rigid bodies. Alternatively, the initial conditions that yield frozen orbits in these systems can be used to provide accurate initial guesses for frozen orbits in high fidelity simulations.

Results:

The most significant results obtained during the study are:

- Two exact formulations of the gravitational potential of an inhomogeous body in polar-nodal coordinates to apply the perturbative theory. This corrects two of the expressions stated in the literature San-Jaun et. al. (2004), Segerman (2000). The mistake went unrecognised in the literature as the expressions were correct to low order. However, to develop the method to higher order required this correction.
- A generalized perturbative theory is presented. It considers all the terms of the gravitational potential and thus generalizes previous methods in the literature.
- An application of Deprit (2001) to relegate the argument of node is presented. This is used to reduce the degrees of freedom to simplify the analysis. The novelty here was that the procedure was implemented to arbitrary order rather than the first order undertaken in the literature San-Jaun (2002). This means that the analytic solution is arbitrarily more accurate than previously stated.
- The main result is that the initial problem is reduced from finding initial conditions for frozen orbits in a complex dynamical system of ordinary differential equations to the problem of solving an algebraic equation.
- A software script was developed in Mathematica that can rapidly generate frozen orbits. These can be used to provide reference orbits or an initial guess in a numerical optimizer to compute highly accurate frozen orbit in high fidelity simulation.

Publications:

Two conference papers and a Journal paper:

- Ceccaroni, M., Biggs, J., Biscani, F., “An analytical perturbative theory of motion in highly inhomogeneous gravitational fields”, Celestial Mechanics and Dynamical Astronomy, 2012.
- Ceccaroni, M., Biggs, J., Biscani, F., “A general perturbative theory of motion in highly inhomogeneous gravitational fields”, Analytical methods of Celestial mechanics,Euler International Mathematical Institute, St Petersburg, Russia, 2012.
- Ceccaroni, M., Biggs, J., Biscani, F., “A second order analytic solution for the motion of spacecraft about Hektor 624”, Proceedings of the International Astronautical Conference, Napoli, 2012.

Highlights:

The main result is that the perturbative theory allows a high-order approximation of frozen orbits to be rapidly generated using the Mathematica symbolic software. This can be used to generate reference orbits or provide an initial guess for frozen orbits in high fidelity simulations of spacecraft about arbitrary asteroids. The general methodology could be extended to include solar radiation pressure if required.