BODIES UNDER CONNECTED ELASTIC DYNAMICS: A THEORETICALLY GROUNDED ASSESSMENT OF ELASTIC TETHER DYNAMICS FOR ACTIVE DEBRIS REMOVAL MISSIONS

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ABSTRACT

The BOUNCED project has assessed the behaviour of elastic tethers within flexible link Active Debris Removal (ADR) systems. This has been achieved by solving a multi-dimensional set of linear equations, through the separation of dynamic modes; and the construction of analytic and numeric models of the resulting system.

These models can be used to simulate the behaviour of both single and multi material tethers across the full lifetime of a tether mission, from pretension, through re-entry burn and free body motion post burn, to re-entry.

The models have been validated and used to explore the behaviour of the system, and as a result have highlighted the following benefits of using highly elastic tethers:

- Reduced shock loading to the target and chaser
- Improved control due to slower oscillatory motion during the burn phase.
- The opportunity to retain control authority, by selecting the burn stop time at a point of low tension in the tether.

However, these benefits come at the cost of:

- Large amplitude lateral and rotational motion of the target.
- Potential interaction with the sloshing resonance of the target and chaser.
- More energy being stored in the tether, leading to the target and chaser being pulled together more quickly when the burn ends.
- Very low tether damping levels required to achieve the benefits identified above.

Importantly, the study has also demonstrated that, through the selection of appropriate tether characteristics, it is possible to design systems that avoid a collision between the target and chaser after the burn, without requiring retention of control authority, or a manoeuvre.

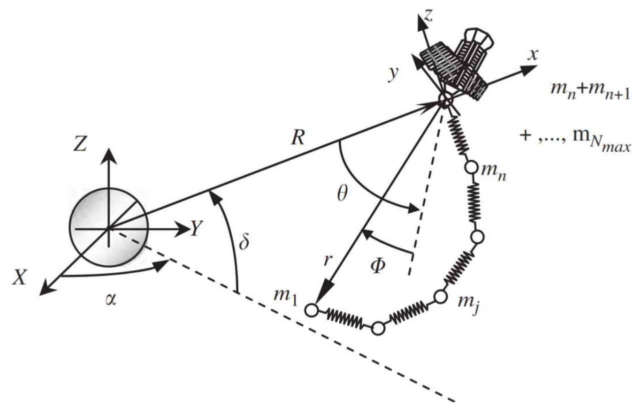

Figure 1: Bead model in Hills reference frame

1. OBJECTIVES

The objectives of the BOUNCED study were:

- To provide a grounded theoretical basis for the modelling of an elastic tethered ADR system.
- To perform a parametric study to determine potential advantages and disadvantages of elastic tethers.
- To assess potential resonances of the elastic system with spacecraft dynamics.
- To assess the potential collision risk between the target and chaser.
- To assess the early interaction with the atmosphere.

2. MODELS DEVELOPED

A discrete bead model representation is used for both the analytic model and a numerical model as shown in Fig.1.

2.1. Theoretical Model

The equations of the system can be formulated such that the resulting matrices are symmetric tridiagonal. This allows the eigenvalues and eigenvectors to be found using standard linear algebraic techniques, enabling the system to be solved for an arbitrary number of beads and, with some constraints, an arbitrary number of materials.

This 1D model is applicable to the main burn, unloading of residual tension in the tether at burn stop, and the subsequent re-tension events which will occur if control of the system is lost. It has been coupled with a 2D solution of Hill's equations to cover orbital motion during periods when the tether is slack.

2.2. Numerical Model

The analytic capability of the study is extended using a 2D on-orbit numerical model of the same bead system.

This allows assessment of effects of the tether becoming slack, orbit eccentricity, propellant consumption, mass change, and the final equilibrium state of the system in the case control is lost.

2.3 Target Rotation Model

A simple model for the rotation of the target has been devised, based on the assumption that the motion of the target body does not have a significant effect on the tether modes.

3. MODEL RESULTS

3.1 Bead model fidelity

The dependence of the calculated eigenfrequencies on the number of beads modelled is shown in Tab. 1. The frequencies are consistently underpredicted if the discretisation is too coarse, with a minimum of 100 beads required to predict the first five modes accurately.

Table 1: Eigenfrequency dependence on resolution

	Frequencies (Hz) for first five modes				
Bead Number	1	2	3	4	5
1	0.0251				
2	0.0251	0.792			
5	0.0251	0.866	1.645	2.264	2.662
10	0.0251	0.876	1.730	2.542	3.290
100	0.0251	0.880	1.759	2.637	3.515
1000	0.0251	0.880	1.759	2.638	3.517
2000	0.0251	0.880	1.759	2.638	3.517

3.2 Elastic Behaviour

Key diagnostics of the elastic behaviour such as the frequencies of the motion, the tether length and tension; as well as the rotational motion of the target can be predicted by the analytical model.

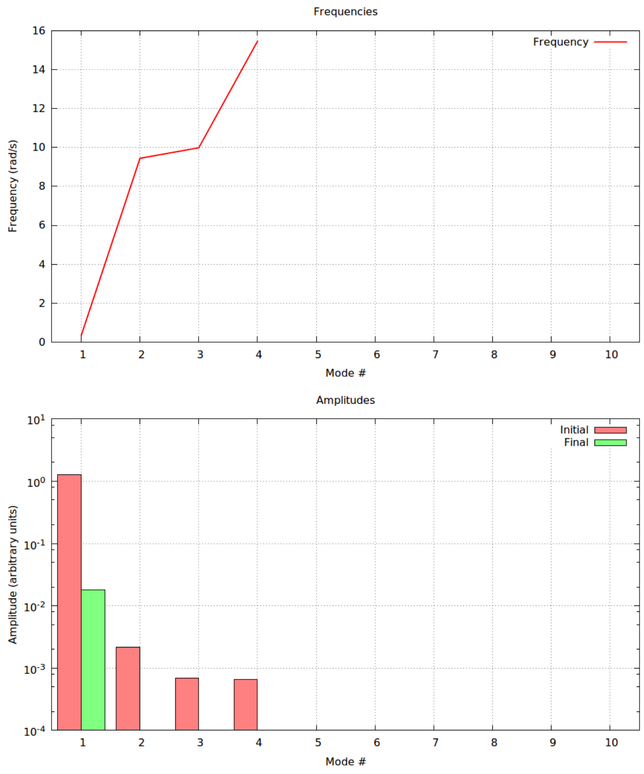

Figure 4.Modal response

Fig. 4. illustrates that only the lowest modes are underdamped, which reduces the likelihood of modes being excited by vibrations from spacecraft components, such as engines. This limits the range of frequencies where potential resonant coupling requires mitigation.

Additionally, by the end of the burn the amplitude of the motion is significantly reduced and only the fundamental mode remains excited. This is confirmed by the tether length prediction, shown in Fig. 5.

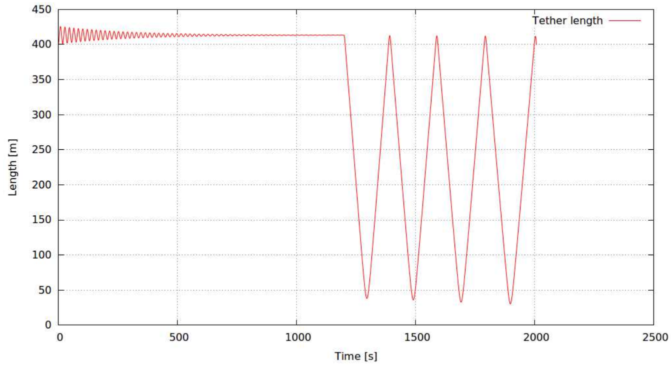

Figure 5. Tether length in test case 2

Once the main burn stops, the remaining tension in the tether unloads, and the bodies are accelerated until the tether becomes slack. This does not necessarily imply that the end masses collide; indeed the minimum distance between them is a little below 40m in Fig. 5.

When the tether's unstretched length is again reached, it retensions and again unloads. A cycle of retension and unloading events can then be seen, which is dependent upon the interaction of the tether and orbit dynamics, as seen in Fig. 6.

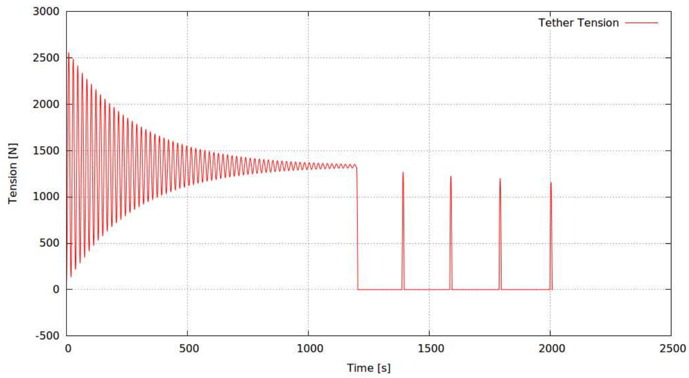

Figure 6. Tether tension

The peak tension reached in each of the events falls slightly due to damping in the tether during each snatch tension and relaxation event.

Eventually, damping is such that the tension forces become small, and the gravity gradient force dominates high altitudes. This is demonstrated by numerical model results for the whole entry, which are shown in Fig. 7.

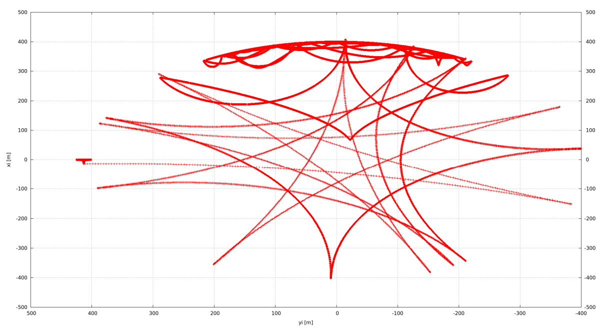

Figure 7. Transition to gravity-gradient libration

The relative magnitude of the accelerations suggests that Coriolis is dominant. If the target misses the chaser on the first pass, it will continue to miss the chaser after each subsequent retension event. Eventually, the motion evolves to a gravity gradient stabilised oscillating state.

3.3 Parametric Study

The effects of tether properties such as stiffness, density and damping, as well as system properties such as the target and chaser masses and thrust level are assessed.

As expected, the frequency of the motion becomes higher as the tether stiffens; and an increase in damping results in the motion being attenuated more quickly.

Decreasing the chaser mass, relative to that of the target, results in a higher tension in the tether for a given main burn thrust. This is due to a higher proportion of the force being required to slow the target.

Pretension of a few Newtons is also required to ensure that the tether remains in tension for the whole of the main burn. This ensures that non-uniform tension, due to modal excitation, is not sufficient to cause slackness in the tether which can result in snatch loading.

An assessment of independence between the parameters affecting tether dynamics has also been made. Independent parameter variation is seen to provide sufficient insight into expected elastic tether behaviour. This is a useful result as it simplifies the understanding of the system behaviour for tether design activities.

3.4 Target Rotation Dynamics

By selecting a tether stiffness such that the fundamental frequency is similar to the pendulum frequency of the target, the tension force can vary in phase with the pendulum motion and thus resonances are possible, as seen in Fig. 8.

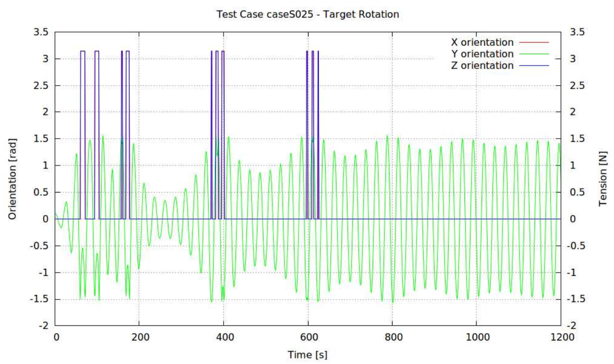

Figure 8. Resonant oscillations of target

If the tether force is not acting along a principal axis of the target, then coupling between the inertias results in there being no point at which zero rotation is observed as shown in Fig. 9. As a consequence there will always be residual rotational motion of the target post burn.

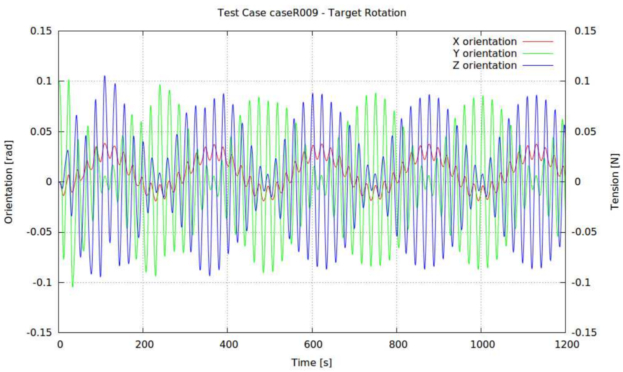

Figure 9. Coupled rotational motion of target

3.5 Post Burn Dynamics

Tether stiffness is important in post burn dynamics as is shown in Fig. 10. A more elastic tether (green) results in larger relative velocities between the end masses once the higher elastic energy stored in the tether has accelerated the chaser and target towards each other. This is reflected in the distance of the closest approach, which is much smaller for the more elastic tether.

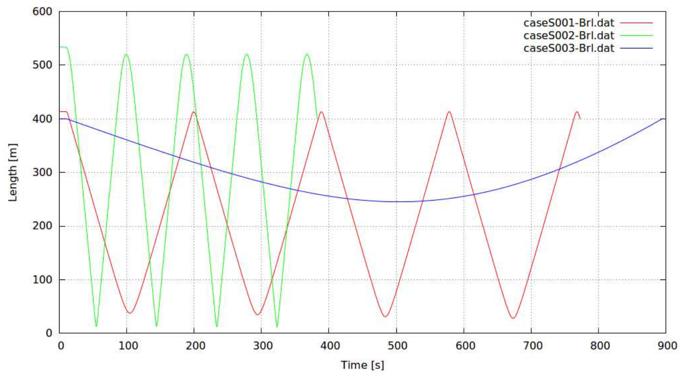

Figure 10. Effect of elasticity of post burn dynamics

The timing of the burn stop also has a large effect on the closest pass of the end masses, due to the different tension in the tether. However, the required damping levels in order to be able to benefit from this at burn stop are very low. Instead, it is likely that the tension will be approaching an equilibrium level at burn stop, and that the post burn dynamics will be elasticity driven.

For a given velocity, snatch loads are higher for a stiffer tether due to less potential energy being converted into elastic energy.

3.6 Collision Risk

The minimum distance between the target and chaser is driven by the relative velocity between the end masses at the point that the tether becomes slack. Analysis of the orbit dynamics demonstrates that the minimum distance between target and chaser scales with the tether length as demonstrated in Fig. 12.

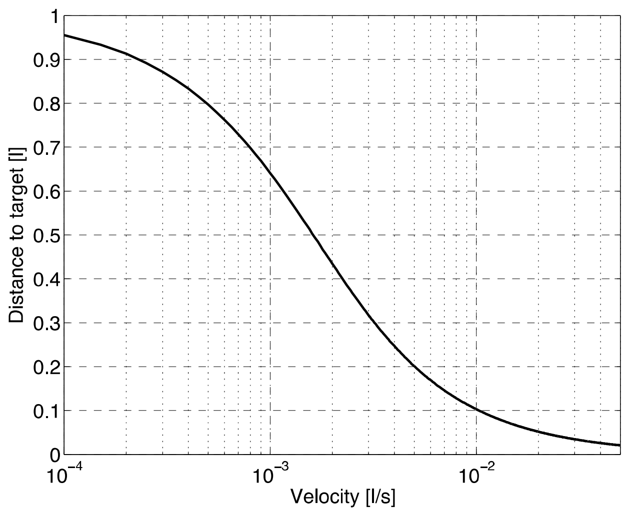

Figure 12: Minimum post burn target-chaser separation

For a given tether length, the minimum pass distance as a function of the tether stiffness can be found. Fig. 13. shows that a separation distance of 20m is only violated by a very elastic tether.

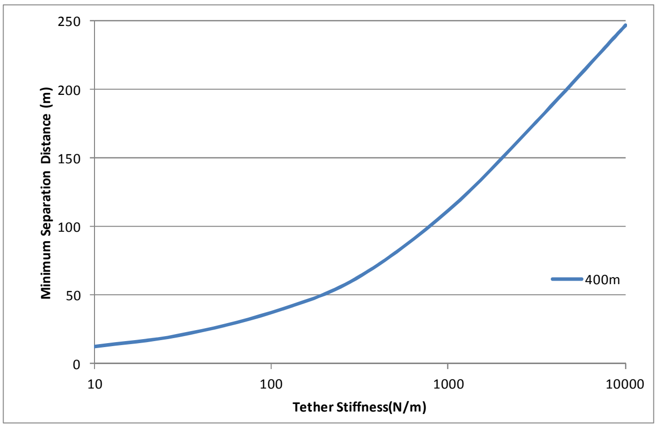

Figure 13: Minimum separation distance for 400m tether

Inverting this analysis, Fig 14. shows the tether lengths required to avoid collision for 10m and 20m separation distances, values which are relevant for the Envisat mission.

This demonstrates that collision risk can be designed out of the system through appropriate selection of tether characteristics.

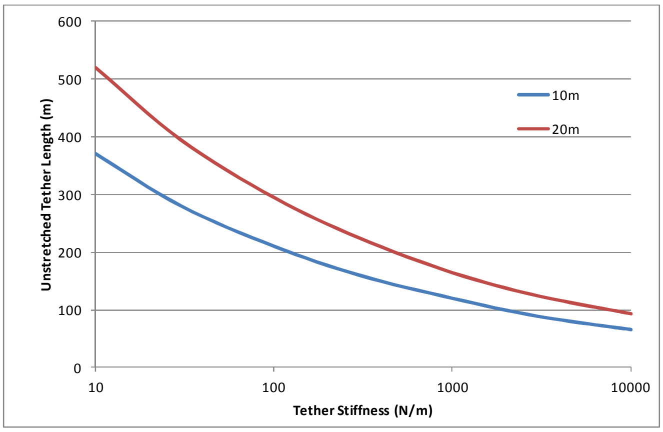

Figure 14: Required tether lengths to avoid collision

3.7 Lateral Motion

Lateral oscillations can be induced into the tether motion by the tension force and the orbit path not being perfectly aligned. Simulations using the numerical model allow the assessment of the resultant pendulum motion of the target about the orbit path, as shown in Fig. 15.

This undesirable motion can be avoided by use of a sufficiently stiff tether.

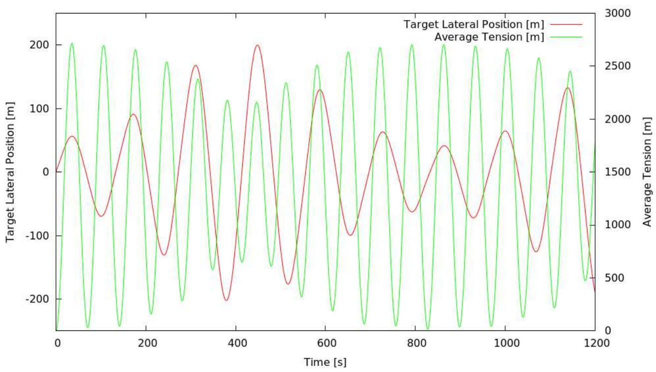

Figure 15. Lateral oscillations of target

3.8 Multi-Material Tethers

The analytic model is also able to assess the affect of including a short stiff section to resist the heat from the thruster plume into an elastic tether.

As expected, the resultant motion, shown in Fig. 16. is dominated by the longer elastic section of the tether. The influence of the stiffer section in reducing the equilibrium tether length and increasing the frequency of the dynamic motion is also visible.

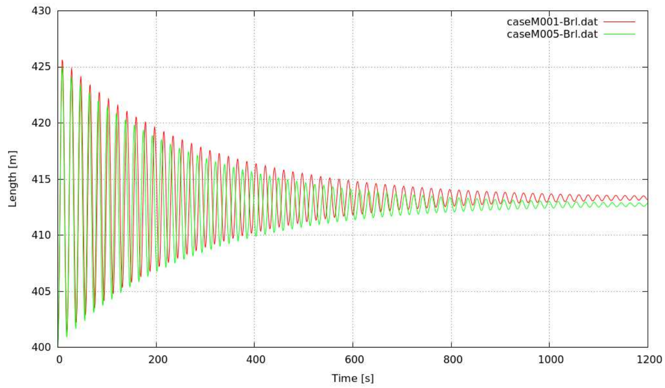

Figure 16. Multi-material tether response

4. ATMOSPHERIC INTERACTION

At altitudes below 300km, drag becomes important, modifying the post burn retension dynamics. Here, two phenomena compete for dominance, the drag serving to pull the target and chaser apart due to ballistic coefficient differences; and the tension in the tether due to the snatch loads at retension. The effect of the relative importance of these phenomena is shown in Fig. 17.

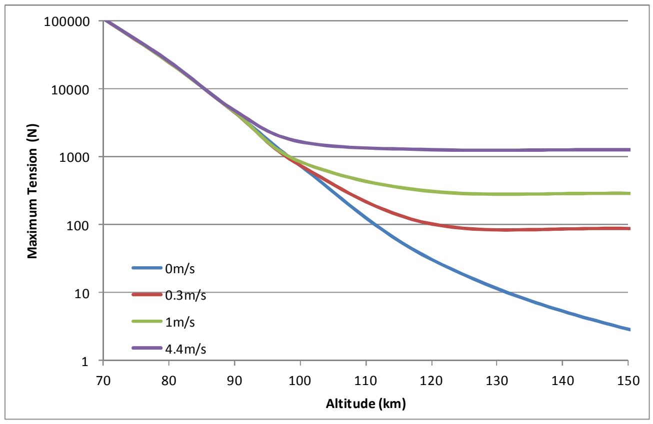

Figure 17. Transition to drag dominated motion

The maximum tension in the tether is used as the diagnostic to indicate the behavioural regime. The blue curve shows the maximum tension in the tether with no initial relative velocity, modelling the situation where all tension is due to the atmosphere. Where the other curves with higher initial relative velocity overlay this baseline, atmosphere drag is the dominant phenomenon.

As expected, little effect of drag is seen above 120km, and drag is not dominant until an altitude of approximately 95km. The transition between regimes depends on the relative ballistic coefficients of the target and chaser, and on the tether stiffness.

The tether is also subjected to aerothermal heating during the entry, which has been assessed using the SAM tool. Tab. 3. demonstrates that the tether would be expected to melt relatively early during re-entry, even if composed of steel.

Table 3. Tether melt altitudes

Material	Steel	Dyneema	Technora
1mm radius	84.5km	106.0km	98.6km
2mm radius	83.0km	102.6km	95.5km
4mm radius	80.5km	99.4km	92.2km

5. SYSTEM ASPECTS

5.1 Resonant Modes

The majority of spacecraft frequencies are found to be above those of interest, due to launch requirements. However, there is a potential issue with deployable structures introducing additional low frequency modes.

However, of greater concern are tank sloshing frequencies, typically in the range of a few Hz or lower, which is similar to the fundamental frequency of the elastic oscillations of a tether. As a consequence there is a risk that the tether and tank sloshing will resonate.

5.2 Space Debris Impact

NASA's Debris Assessment Software has been used to produce an initial set of debris impact probabilities. Approximately 1 impact from a 0.1mm object can be expected per mission, and a potential mission threatening 1mm object would be expected to impact the tether every 30-100 missions.

This analysis suggests that debris impact is in fact a higher risk than the collision of target and chaser.

6. TETHER RUPTURE

Through assessment of the different types of tether rupture, created by effects such as slow stretching, sudden stretching and cutting by impact; a set of maximum recoil velocities have been produced for an elastic rupture of various materials, as shown in Tab. 5. These show good agreement with predictions produced by the analytic model.

Table 5. Tether rupture recoil velocities

	Nylon	Kevlar	Steel Alloy	Dyneema	Technora
Youngs Modulus (Gpa)	3.9	124	210	100	73
Strength (Mpa)	616	3930	1330	2000	3400
Density (kg/m^3)	1150	1440	7800	970	1390
c0 (m/s)	1841	9279	5188	10153	7246
Post rupture velocity (m/s)	145.4	147.1	16.4	101.5	168.8
Theoretical model velocity (m/s)	139.1	152.0	16.3	105.2	165.0

7. CONCLUSIONS

A grounded theoretical basis of tether behaviour has been formulated, and an analytic and a numerical elastic tether model have been constructed, cross validated, and used to explore the behaviour of such systems.

From this investigation the following points have been observed:

- Significant resonances can occur between the rotational motion of the target and the elastic tether motion. These may be mitigated by use of a sufficiently stiff tether.
- Resonances can also occur between the elastic motion and lateral target oscillations, again these can be mitigated by use of a sufficiently stiff tether.
- It is not possible to guarantee zero rotational motion of the target at the end of the main burn, and therefore, wrapping of the tether around the target is a concern.
- Post burn collision risk is dependent upon the residual energy in the tether. Further, this risk can be designed out through the use of appropriate tether characteristics.
- The collision risk is smaller for stiff tethers, given the same level of damping.
- The level of damping required in an elastic tether, to match the collision risk of a stiff tether, is sufficiently low as to make elastic tethers unfavourable.
- Stiffer tethers dissipate less energy per retension cycle and induce higher snatch loads increasing the risk of rupture.
- A single burn strategy is preferred due to the residual rotational motion of the target and the control needs offsetting the propellant mass gain.
- Resonances may be possible with spacecraft deployables, and with tank sloshing modes.
- Any tether is most likely to melt during re-entry before the forces are sufficient to cause a purely mechanical break.

8. ACKNOWLEDGEMENTS

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