Sinergy between electromagnetic and gravitational fluid dynamics
ESA Contract No. 4000115042/15/NL/KML/fg

P. Amodio[*], C.M. Coppola[†], F. Iavernaro[‡],
L. Iorio[§], S. Longo[¶], F. Mazzia[‖]

Summary

In several applications it is relevant to quantify the energy exchanges for a system embedded in an electromagnetic field, and to solve the typical equations of electromagnetic fluid dynamics. In order to model such systems, several approximations are usually adopted because of the difficulty (in some cases, impossibility) to solve, even numerically, the electromagnetic fluid dynamics equations.

It is then worth to move to a similar problem that can be handled and that can, in principle, allow to export the developed methodology into the electromagnetic case. In this context, gravitational fluid dynamics represents an interesting and useful test case, due to the possibility to express the basic equations in exactly the same formalism as the Maxwell equations, which describe the evolution in time and space of the electric and magnetic fields. Indeed, also in the case of the gravitational field, Maxwell-like equations can be derived. These equations allow to define two fields that describe the gravitational field and its evolution in time and space. The additional advantage of such an approach is the handy definition of the numerical constants that appear in the Maxwell-like equations and that are naturally provided by writing the gravitational field dynamics equations.

Consistently with the goals of the present project, the solution of these equations have been performed for the case study of a perfect gas enclosed in a spherical box placed in the interstellar space where all external forces are negligible. Thus the gas is subject to only its own self-induced gravitational field, other than to thermal and viscous dissipation effects.

---

[*] Department of Mathematics, University of Bari, Italy, `pierluigi.amodio@uniba.it`

[†] Department of Chemistry, University of Bari, Italy, `carla.coppola@uniba.it`

[‡] Department of Mathematics, University of Bari, Italy, `felice.iavernaro@uniba.it`

[§] MIUR-Istruzione, Bari (BA), Italy, `lorenzo.iorio@libero.it`

[¶] Department of Chemistry, University of Bari, Italy, `savino.longo@uniba.it`

[‖] Department of Computer Science, University of Bari, Italy, `francesca.mazzia@uniba.it`

The gas is initially assumed in a steady-state equilibrium at a given uniform temperature T_i for $t \leq 0$. In a subsequent finite time interval $t \in [0, t^*]$, the temperature is continuously changed on the inner interface of the shell from the initial value T_i to a final value T_f, according to a given law and, consequently, the gradient of the temperature will set the gas in motion. The question is then to detect the configuration of the system for $t \geq 0$. The numerical solution of the set of equations describing the dynamics of the gas is challenging due to the difference in orders of magnitude between the speeds of sound and lights which makes the problem stiff and due to the long integration times needed to ascertain the asymptotic behavior of the gas. To overcome these numerical difficulties, a particular attention must be paid to the choice of reliable discretization techniques both in space and in time. Furthermore, to validate the obtained results, a comparison among different numerical methods would be in order.

During the study and the numerical simulation of the problem, interesting elements have already emerged in the case where some simplifying assumptions have been initially considered. These aspects concern both the stationary configurations and the long-time behavior of the solutions and, to the best of our knowledge are new and thus deserve a particular attention. More specifically, due to the symmetry of the problem, when the gas is at rest, all physical quantities describing the state of the gas at a given point are radially distributed around the center of the sphere. We have initially assumed a radial distribution of all variables also during the motion of the gas, which may be easily achieved by imposing that the temperature on the boundary of the sphere is changed with the same law at all points. If, from one hand, this simplifying assumption have made the mathematical description and numerical treatment of the evolution equations easier, from the other hand, the solutions of the resulting problem have shown new and interesting physical properties which have been the object of a separate discussion from the more involved general non-radial case.

With this premise, the work structure of the project has deployed according the following three tasks. Task 1 is theoretical in character and focusses on the linearization of general relativity equations in the weak-field approximation for a continuous medium, leading to a set of Maxwell-like equations for the gravito-electromagnetic fields. Task 2 is devoted to the formulation of the boundary-value problem yielding the stationary states of the gas. The obtained results have been then exploited during the research activity of Task 3 to set up the initial and boundary data needed by the dynamical system governing the motion of the gas. Several numerical experiments have been carried out to show the different asymptotic properties of the solution and, in particular, to understand the stability nature of the stationary states of the gas.