
TUMBLING MOTION ASSESSMENT FOR SPACE DEBRIS OBJECTS

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LIST OF ABBREVIATIONS

AIUB	Astronomical Institute of the University of Bern
ADR	Active Debris Removal
DEC	Declination
ESA	European Space Agency
GEO	Geostationary Earth Orbit
GTO	Geostationary Transfer Orbit
IOTA	In-Orbit Tumbling Analysis simulator
LEO	Low Earth Orbit
MEO	Medium Earth Orbit
RA	Right Ascension
R/B	Rocket Body
SGF	Savitzky-Golay Filter
SRP	Solar Radiation Pressure
WP	Work Package

1 INTRODUCTION

One of the methods to limit the space debris population growth and to stabilize the space debris environment in LEO is Active Debris Removal (ADR). Most of the proposed ADR concepts need to grab or catch the objects to be removed by some means or require attaching a device on the target object. In all these cases precise knowledge of the attitude state of the target object will be absolutely critical. To understand the attitude state and evolution of a target, one has to understand the physical mechanisms which are affecting it. These are mainly eddy currents, atmospheric torques, gravity gradients, outgassing, solar radiation pressure, collisions with small-size objects, or the motion of internal components which lead to damping or acceleration of the spin period and to changes of the spin axis orientation. By knowing the physical parameters of the target, one should be able to model its attitude. However, current models to predict the evolution of the attitude cannot fit accurately the observed behavior for many objects. The goal of the current project was to develop methods for the determination of the attitude motion of debris objects and its evolution. Existing methods and models developed in previous studies were improved to be able to better predict and determine the attitude state of debris objects. Specifically the existing In-Orbit Tumbling Analysis simulator (IOTA) was further developed to be able to simulate more and more real spacecraft configurations and real physical effects.

2 ATTITUDE EVOLUTION

The attitude dynamics of space objects is dependent from the characteristics of the object itself but also from its orbit and the initial attitude state. Different orbits are subject to different environmental forces which modify the attitude. In a simplified approach we can distinguish two categories of objects: satellites and rocket bodies. For the same purpose of categorizing the acting forces we distinguish two main orbital regions: LEO and GEO. The same perturbing force can influence the attitude at different orbital regimes but to a different extent, being predominant in one and negligible in another one. Therefore we propose idealized categories, where rocket bodies and satellites in LEO are mostly concerned by gravity gradient and eddy currents torques, while in GEO especially satellites through solar panels are sensitive to solar radiation pressure (SRP).

2.1 Rocket bodies in LEO

2.1.1 Evolution due to eddy current torque

Simulations in [RD-1] show that the spin rate follows three distinct phases. In the first phase the angular velocity and the angular momentum shift towards the axis of maximum inertia (to flat spin), following the dissipation of the eddy current torque when the object rotates around the non-principal axis. In the second phase the angular velocity decays exponentially under the effect of the geomagnetic field, before it approaches a stationary value in the third phase. In the presence of a uniform magnetic field the angular momentum will tend to align to it under the eddy current torque. In reality the geomagnetic field is not uniform, and if a dipole approximation is used, the object will encounter a different magnetic field direction and intensity along the orbit, depending on the inclination and altitude. In this case the angular momentum will

initially try to align to an average magnetic field direction (averaged over the encountered magnetic regions). Finally, when object's and relative geomagnetic field's angular velocity are in the same order of magnitude, the angular momentum will reach a stationary point dependent from the distribution of the geomagnetic field.

2.1.2 Evolution due to eddy current and gravity gradient torques

If in addition to the eddy current torque also the effect of the gravity gradient is considered, the angular velocity is only affected in the last phase: since the angular velocity is small, the dissipating effect of the eddy current torque almost vanishes and the gravity gradient torque shows its contribution. The angular momentum under the effect of the gravity gradient will not tend to align to the magnetic field, as in the eddy current only case, it will show instead a precession around the normal to the orbital plane. After this phase the angular velocity becomes small and is not enough to maintain a precession dynamics. The attitude motion will enter the stabilization phase. In the stabilization phase transitions between the enclosed regions can take place and the evolution is characterized by chaotic motion. At the end of the chaotic phase the final gravitational stabilization (gravitational capture) occurs, where the object rotates synchronously to the orbital period. The convergence to the stable gravity-gradient orientation is a consequence of the significant elongation of the considered rocket body. For less elongated objects like short rocket bodies or satellites, other final motion regimes are possible.

2.2 Satellites in LEO

Simulations in polar LEO orbits show that in addition to the gravitational stabilization, there is an alternative final regime related to the eddy current torque. For low spinning rates the model needs to be extended with additional terms related to the orbital motion of the object and the Earth's rotation. In fact in this regime the latter start to be the dominant terms. A stable motion can be reached at the resonance condition close to $\omega/\omega_0 = 2$ (angular velocity ω and orbital angular velocity ω_0), in which an object on a polar orbit has its induced north pole in correspondence of the Earth's south pole and vice versa, half revolution later. Satellites differ from rocket bodies not only in the moments of inertia but also in the presence of solar panels. The influence due to solar radiation pressure (SRP) can be an additional effect that needs to be considered. The torque acting on the solar panels can increase or decrease the spin rate and change the angular momentum vector. The resulting effect is close to the one experienced by objects in GEO.

2.3 Rocket bodies and satellites in GEO

At this altitude the main torque contribution is due to SRP, while gravity gradient and eddy currents effects decrease with the distance from Earth. As seen previously eddy currents have a dissipating effect and together with the gravity gradient there is a tendency to stabilization of the attitude motion. In absence of these forces and of dissipation there is no damping effect and the object keeps its rotation status indefinitely. If forces like e.g. SRP are present, then the angular velocity can even increase. As discussed in [RD-2] the rotation axis can move from major towards minor axis of inertia through phases of torque free precession (the reverse order of the case with dissipation). Simulations show that a monotone increase or decrease of the angular velocity can be achieved by rotating the solar panels asymmetrically or by introducing an offset in the center of mass location so that it is not aligned with the center of figure. The

canted solar panels are subject to a mechanism similar to the one of a wind wheel or fan. Simulations were performed to investigate the spin rate evolution for a box wing model with canted solar panels. An existing angular momentum stabilizes the attitude of the satellite, keeping the orientation of the “wind wheel” constant during the year. The SRP torque contributes essentially to the increase or decrease of the spin rate depending on the orientation of the wheel. The maximal torque occurs with the wheel pointing to the Sun. After 6 months the maximal effect is obtained in the opposite sense of rotation. Maximum and minimum of the function are reached during the year when the axis is perpendicular to the Sun direction. The yearly variation of the Sun position causes a periodic pattern in the angular velocity. In addition to obtain a periodic dependency with a secular increasing or decreasing trend the torque should be smaller/bigger in magnitude during part of the year, e.g. decreasing the efficiency of the back side of the “wind wheel” keeping unchanged the front side.

3 METHODS FOR ATTITUDE DETERMINATION

3.1 Amplitude method

The amplitude method was chosen as one of the methods to be implemented in the current project. Originally the approach was proposed by Williams in [RD-3] and it could later be successfully applied in similar investigations [RD-5]. In the original idea the method applies to cylindrical elongated objects where we assume that a stable attitude state is reached, with rotation around the major axis of inertia, perpendicular to the cylinder axis. The Sun light is diffusely reflected by the lateral surface of the cylinder without considering top and bottom. The existence of maximal and minimal values depending on the observation configuration can be exploited to retrieve information about the orientation of the cylinder axis. The **brightness ratio** between maximal and minimal intensity can be related to a set of possible rotation axes. More than one brightness ratio, obtained observing the same object at different epochs or from different stations, is necessary to reduce the solution set to a unique rotation axis.

3.1.1 Method of Yanagisawa & Kurosaki

An improvement to the original amplitude method is the one proposed by Yanagisawa & Kurosaki [RD-4]. It can be applied to a general tri-axial ellipsoidal object, although the authors also suggest a formulation for a cylindrical model. The same assumptions about diffuse reflection and the rotation about the major axis of inertia apply as in the general approach.

In this approach the brightness ratio is considered independent from the position of the Sun up to a scaling factor, which is a function of the phase angle and a constant related to the composition of the target. This simplifies the formulation of the problem and the brightness amplitude is only a function of the angle θ , between the rotation axis and the observer, and the phase angle. The procedure involves a least squares optimization to estimate the rotation axis, given the measured and modeled brightness ratios. This approach has the advantage that additional parameters to estimate can be easily included in the optimization, as well as all the available observations. However, this procedure as well is subject to solution ambiguities in terms of multiple local minima in the loss function. Since the method bases on relative brightness values, only the ratio of parameters related to shape and reflecting properties can be estimated, as e.g. the albedo. A general problem is that in the current algorithm only the observer LOS through the angle θ is taken into account and not the Sun direction. The latter is implicitly present in the phase angle α but is not uniquely defined. The same phase angle and LOS with a different

illumination direction clearly results in different reflected intensities. The absence of information about the Sun direction in general prevents the model from accurately describing the illumination conditions of the object, e.g. for faces in the shadow. In general the model of Yanagisawa & Kurosaki is less accurate and can be only used as an approximation for θ around 90° , i.e. for situations with large brightness variations.

3.2 Epoch method

The photometric observations can be collected at different sampling intervals or exposure times depending on the technology used, and both parameters can vary anywhere from a fraction of millisecond (photon counters) to several seconds or minutes (CCD, CMOS). In case of the Graz detection system, the single-photon resolution of the collected optical flux allows recording a high level of signal details of sunlit objects from all orbital regimes. The applied 10 ms binning interval enables for an accurate attitude parameters determination through the epoch analysis of collected time series data.

The epoch-based light curve post-processing pipeline consists of the following steps:

- a) **Pattern normalization with time derivative.** The Savitzky-Golay (SG) filtering [RD-6] is used as a method for denoising the raw, observational data. The SG filter is based on local least-squares fitting of the data by low-degree polynomials for smoothing data and calculation of the light-curve time derivative.
- b) **Frequency signal extraction with spectral analysis.** The spectral content of the photometric pattern is extracted by processing the light curve time derivative with Lomb-Scargle Periodogram [RD-5] which estimates a frequency spectrum of unequally spaced data based on the least squares fit of sinusoids to the data samples.
- c) **Detrending.** The light curve detrending process is based on the Chebyshev polynomials (Gram polynomials) which is a type of discrete orthogonal polynomials used in approximation theory. The degree of the polynomial fit is decided upon the frequency output of the spectral analysis and the pass duration.
- d) **Inertial spin rate and orientation determination with Phase Dispersion Minimization.** The light curve pattern periodicity analysis can be performed with the Phase Dispersion Minimization (PDM). The photometric samples are folded into 1-degree rotational phase bins and the variation (RMS) of the binned residuals is calculated as a measure of dispersion. Testing range of possible spin period values allows to identify the pattern periodicity by minimizing the dispersion coefficient.

A statistical averaging of the attitude results obtained with the amplitude and epoch methods allows for data fusion on the result level, but it may be more beneficial to perform data fusion on the observational level by combining the SLR depth dimension with the solar phase angles. Such a combination could allow for more complete attitude determination with a lower set of the initial assumptions regarding the satellite shape and an approximate tumbling state. The spinning satellite body causes rotation of the retroreflector array (RRA) resulting in periodic oscillation of the observer – reflector slant range. The spin-related oscillation of SLR data, at the periodicity shorter than the pass duration, can be determined with respect to the best-fit trend function that can be constructed as a low-degree polynomial fit to the SLR data.

4 IOTA AND ATTITUDE MODELS

4.1 IOTA improvements

IOTA is a highly modular software tool to perform short- (days), medium- (months) and long-term (years) propagation of the orbit and attitude motion (6-DoF) of spacecraft in Earth orbit. IOTA capabilities have been extended in the current project, including

- Modelling
 - Earth radiation model (Albedo and thermal radiation)
 - Earth shadow penumbra region
 - Generic attitude damping model
 - Material dependent surface reflectivity (considered for radiation pressure models and synthetic light curves)
 - Replacement of the eddy current damping model with a magnetic tensor based model
 - Thermosphere wind model, i.e. HWM14
 - Variable solar activity as input for the atmosphere model implemented (NRLMSISE-00)
- Performance
 - Geometry handling optimizations
 - Pre-computed coefficients for aerodynamics and radiation effects
- Usability
 - Additional simulation output extension: Sun direction, along-track and radial vectors
 - Extension of the GUI, to support setup, management and execution of simulations, with queuing and (parallel) execution of simulations
 - TLE/OMM import

Besides the many advantages, owing to the new modelling capabilities it was possible to validate the evolution models due to eddy currents, gravity gradient and solar radiation pressure seen above.

4.2 Observation campaign

For the observation campaign, different objects have been selected as a representative sample of the tumbling population up to the geostationary altitudes. The list covers:

- 4 R/B and 3 satellites at altitudes up to 1500 km where all optical methods, including the noncooperative satellite laser ranging, should perform well.
- 2 satellites at altitudes 1500-3000 km.
- 3 R/B and 3 satellites in Medium Earth Orbit (MEO), circular and elliptical.
- 3 R/B in Transfer Orbit and Geostationary Transfer Orbit (GTO).
- 3 satellites in Geostationary Orbit (GEO).
- Special Interest Objects: ERS-2, ADEOS, Envisat.

The orbital configuration of the selected objects assures frequent availability during the terminator period (dusk and dawn) for the photometric observations from Graz and Zimmerwald. It is also required that the closest approach of OOI is frequently above 30° of topocentric elevation to give opportunity for longer passes at higher SNR. The types of targets – Rocket Bodies and defunct Payloads – represent group of large radar cross-section objects for efficient optical observation and analysis of the tumbling motion. The constraints of the attitude methods require multiple rotations per pass (suggested >10) and some basic information about the R/B technical and optical properties. In total more than 2000 measurements (light curves and SLR data), performed during this and previous observation campaigns, were provided. The amplitude method was applied to four different R/Bs while the epoch method to almost twenty different space objects. Most of the measurements were used to determine the spin rate evolution of more than twenty different objects over a period of one or more years.

In addition to the regular campaign, a joint tracking campaign with TIRA, Graz SLR station, and the Zimmerwald Observatory was performed. The purpose of the 3-systems tracking campaign was the collection of the photometric, laser and radar data for attitude determination of selected objects-of-interest. The following objects have been selected for the joint tracking campaign with Zimmerwald, Graz and TIRA systems: Jason-2 (COSPAR 08032A, NORAD 33105), ENVISAT (02009A, 27386), SL-16 R/B (98045B, 25407), CZ-2D R/B (19063B, 44548), Globalstar M060 (00008C, 26083).

4.3 Amplitude method

In the list of observed objects in the planned campaign only rocket bodies (R/B) are a suitable choice for the amplitude method. All measured light curves of R/Bs were examined and only a smaller subset of objects was appropriate for the intended analysis. Some of the light curves did not show periodicity, while for certain objects interval between the observations was too large to guarantee the condition of having a constant rotation axis. Complementary observations from Graz were not used for this method, since photon counters do not proportionally react to detected photons, so the data are not suitable for a brightness ratio analysis. For four objects it was possible to apply the method and find the spin axis solution with accuracies varying from few degrees to tens of degrees. Although solutions could be found there is no absolute guarantee that the provided results corresponds to reality. The assumption of having a constant rotation axis throughout the period of the observations cannot always be ensured. The consequence of not having a constant rotation axis may cause much larger errors than the pure formal error computed for the brightness ratio. This error component is mostly present, even for observations within short time, but can be reduced to acceptable levels choosing appropriate time intervals according to the orbit of target.

4.4 Epoch method

The analysis of several objects observed during the campaign showed that, in general, for the spin axis orientation determination a significant apparent motion must be present. In the case of GEO objects, the magnitude of apparent rotation is too small to act as a marker of satellite orientation. This is not a problem for LEOs, where the apparent rotation can exceed 100° per pass thus allowing for the inertial spin axis orientation analysis. However the accuracy in the latter case turned out not to be enough for cross validation with e.g. the amplitude method.

In addition to the limitations due to the orbital regime, the detrending of light curves is a further challenge in the post-processing as the cause of the trend cannot be predetermined. We have

utilized a Savitzky-Golay filter (SGF) that works on a local data sample (rather than full pass) and produces detrended time derivative series that can be post-processed with the epoch methods to determine the satellite tumbling motion parameters. A derivative time series is naturally detrended and distributed about zero level which makes it a perfect input for the epoch method analysis. Moreover, the use of SGF allows for the derivative determination from small data samples, which makes this method suitable for near real-time operation without the need for a full-pass detrending.

4.5 Comparison with simulated light curves

4.5.1 Rocket body CZ-3B

The results obtained using the amplitude method were compared with the output of the IOTA simulator. The spin axis orientation of one of the observed rocket body was used as input, as well as the state vector at the three epochs previously computed. The objective was to evaluate if the simulated brightness ratio is similar to the one obtained from the measured light curves. In this case the implementation of the Williams model in the amplitude method was validated against the diffuse reflection model incorporated in IOTA. The comparison showed similarities in the brightness ratios, although the formal error in the brightness ratio alone and the consequent error in the determined spin axis direction was not sufficient to justify the difference obtained in the simulations. Other not considered sources of error might be responsible for this, e.g. the fact that the adopted cylinder model has not a perfect symmetric surface and is approximated by a fine-structured prismatic shape.

4.5.2 Envisat satellite

We decided to simulate light curves of an object with known spin axis orientation and we considered the measurements of Envisat performed in the joint CMOS-SLR-Radar campaign of the night 2016-09-21 in the frame of the project ESA/ESOC 4000112447/14/D/SR [RD-7]. The model of Envisat provided in the IOTA release was considered with varying reflection coefficients for the different components, like solar panel, radar antenna, polymer composite, etc. We were able to refine the coefficients to match the dominant peaks in the measured light curves (see e.g. Figure 4-1).

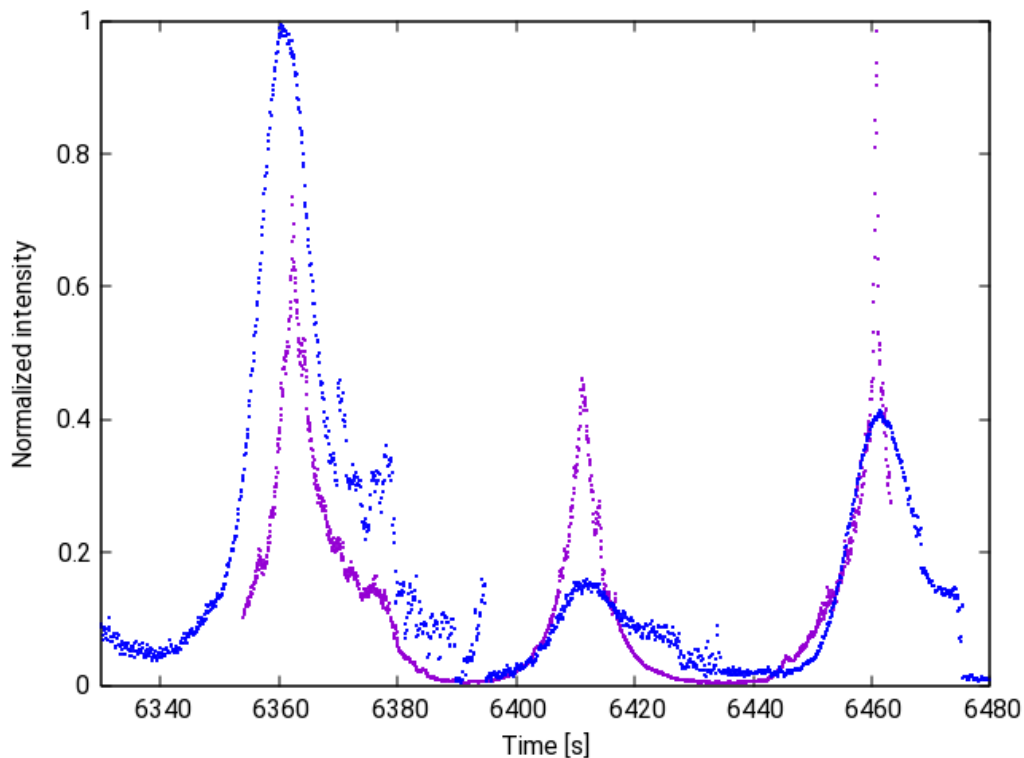


Figure 4-1: Observed (violet) and simulated light curves with modified reflection coefficients (blue). Time in seconds since 18:53:21 UTC.

5 SPIN RATE EVOLUTION

The observation campaign performed in the frame of this project covers a period longer than one year. From the observations the change in the spin rate of several objects could be determined. An example is shown in Figure 5-1, where spin rate measurements of a R/B from the Graz station, the Zimmerwald observatory, and the TIRA radar system are indicated. For the considered objects we can easily assume that it is in the decay phase, since usually the flat spin transition phase takes few months, while the stabilization phase show periods of one hour or more, exceeding the length of our measurements.

We were able to reproduce the decay rate visible in the plot with IOTA simulations using refined values for the moments of inertia and the magnetic tensor.

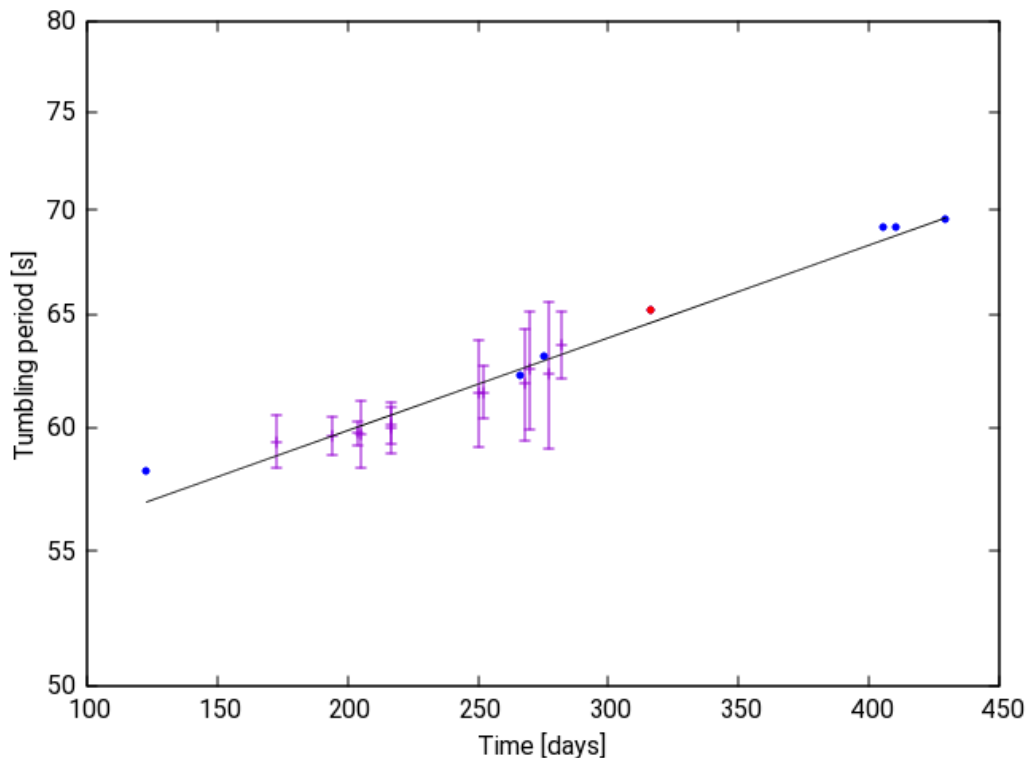


Figure 5-1: Trend of spin period of CZ-3B R/B (2019-090D). Time in number of days from 1.1.2021. Violet points with error bars are from Graz station, while blue and red dots are from Zimmerwald observatory and TIRA, respectively.

6 PROCESS AND PRODUCT DATA FORMATS

One of the tasks in this project was about suggesting possible data formats for information exchange about intermediate processing and attitude state results. The format for the following points were proposed:

- **Power spectrum.** The output of the FFT or Lomb frequency analysis
- **Phase folded timeseries.** The phase folded data time series with phase angle and observed photometric property (e.g. visual magnitude, photon flux).
- **Apparent rotation angle.** The accumulated apparent rotation data series with azimuth and elevation angles of a spin vector in given reference frame, and the predicted accumulated apparent rotation angle for the pass.
- **Brightness ratio.** Ratio between the average maximal and minimal brightness expressed in magnitudes and its standard deviation.
- **Attitude parameters.** Spin axis orientation and rate of rotation
- **Data fusion.** Data fusion process on the result level performed with a weighted average of the contributing results and their errors.

7 CONCLUSIONS

Within this project different aspects of the attitude motion of space objects were treated, methods for attitude determination were developed and attitude evolution models were studied. The existing IOTA software was improved and was used to validate models and to predict the attitude evolution.

Applications of the amplitude method on several R/Bs were shown. It was possible to find the orientation of the spin axis, and different accuracies, varying from case to case, from few degrees to tens of degrees, were achieved. However, the mentioned formal accuracy does not always correspond to reality. If the rotation axis is not constant during subsequent measurements or the assumed reflection model is not good enough, larger errors in the rotation axis can be expected. Ideally, observations separated by short intervals of at most few days should be performed.

Regarding the epoch method, the analysis of objects with different characteristics and orbits helped find out the limitations and possible improvements on the method itself but also on the preprocessing to determine the rotation period, which needs to be very accurate. In the detrending of the light curves a Savitzky-Golay filter (SGF) working on local data samples (rather than full passes) and producing detrended time derivative series was successfully used. For the spin axis orientation determination the additional requirement of significant apparent motion was the limiting factor. In the cases of GEO objects, the magnitude of apparent rotation is too small to act as a marker of satellite orientation. This is less of a problem for LEOs, where the apparent rotation can exceed 100° per pass, thus in general allowing for a better inertial spin axis orientation analysis. However the accuracy of the latter turned out not to be enough for cross validation of the two attitude determination methods. More research should be dedicated to the utilization of time derivatives for satellite spin determination. The use of SGF allows for the derivative determination from small data samples, which makes this method suitable for near real-time operation without the need for a full-pass detrending.

The IOTA simulator was used to reproduce the light curves of a R/B and of the Envisat satellite. A comparison of simulated and measured light curves shows only slight differences that can be due to non-realistic reflection models or reflection coefficients. The case of Envisat indicates that to a certain extent it is possible to reproduce the light curve measurements of complex objects with different materials, having an accurate knowledge of their attitude and characteristics.

The developed attitude evolution models and the ones referenced in the literature were compared with the output of IOTA and checked for consistency. In particular the evolution due to the effect of the eddy currents torque, with a characteristic exponential decay, and the behaviour in presence of gravity gradient torque was reproduced.

The spin rate trend of different objects was studied, on one hand to check the consistency of the different type of measurements (Radar, SLR, CCD, Photon counter), and on the other hand, to predict the evolution based on the studied models. The spin rate of a R/B in GTO orbit could easily be explained with the damping effect of the eddy currents showing a typical exponential decay.

A short analysis on possible data formats in the domain of attitude characterization was performed. Different types and suggested distribution formats of the intermediate processing outputs and the attitude state results were proposed.

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