

AO/1-9934/19/D/SR
Tumbling motion assessment for space debris
objects

Final Presentation

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Outlook

- Introduction
- Attitude models
- IOTA software
- Observation campaign
- Radar observations
- Amplitude method
- Epoch method
- Attitude evolution
- Conclusions

Introduction

- Space debris population growth
- Possible solution: Active Debris Removal
- Knowledge of attitude is necessary
- Current project: development of methods for attitude determination and prediction:
 - Attitude models
 - Attitude determination (amplitude and epoch method)
 - Software for attitude prediction and observation simulations
 - Observations (CCD, photon counter, SLR, radar)
 - Process and product formats and standards

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Attitude models

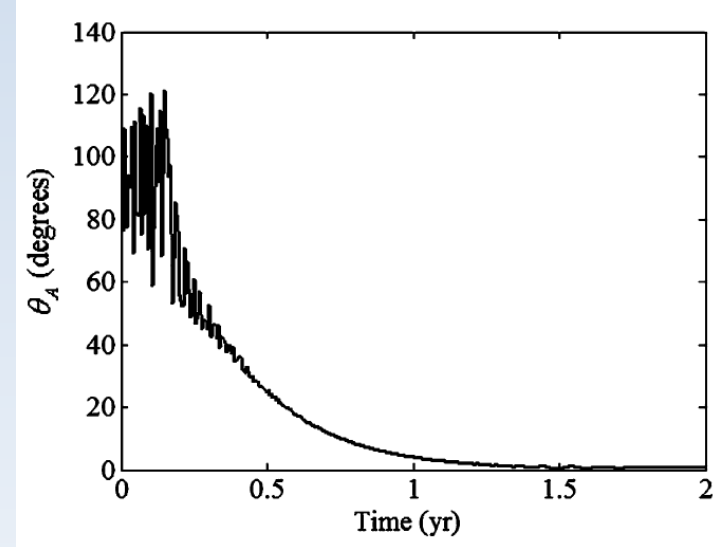
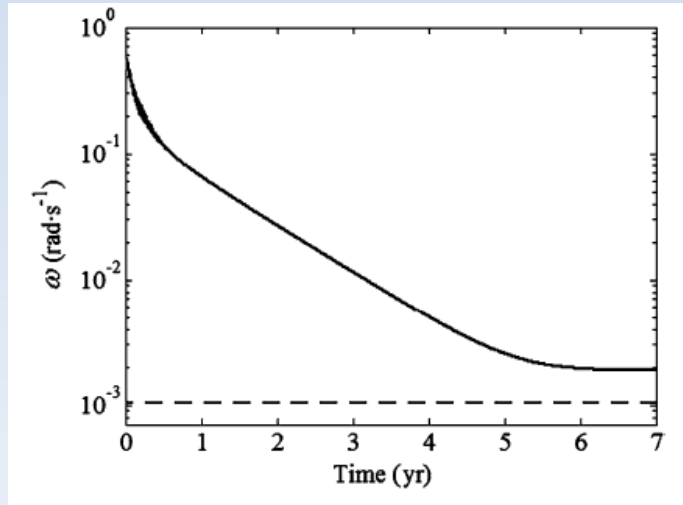
Orbit	Object	Grav. grad.	Eddy curr.	SRP	Dissip.	Long term behav.
LEO	R/B	✓	✓		✓	Stable
	Satell.	✓	✓	✓	✓	Almost stable
GEO	R/B				✓	Rotating
	Satell.			✓	✓	Rotating

- Idealized categories
- LEO: stronger influence of gravity gradient and eddy currents
- Eddy currents is stabilizing perturbation
- Satellites have solar panels and SRP is not stabilizing
- MEO/GEO: stronger influence of SRP

Eddy currents

- Eddy currents torque $\mathbf{M}_{EC} = \left[\mathbb{S} \left(\boldsymbol{\omega} \times \mathbf{B} - (\mathbf{v}, \nabla) \mathbf{B} - \dot{\mathbf{B}} \right) \right] \times \mathbf{B}.$
- Dissipative term (Eddy current): $\boldsymbol{\omega} \times \mathbf{B}.$
- Related to orbital motion: $(\mathbf{v}, \nabla) \mathbf{B}$ and $\dot{\mathbf{B}}$
- Dissipative terms dominates at first (proportional angular velocity)
- Orbital terms dominate after spin velocity has diminished (in the phase of gravitational capture)
- Orbital terms tend to align momentum vector to orbit normal and stabilize to a momentum depending on the orbit
- Magnetic tensor M: $\vec{T} = (M\vec{\Omega}) \times \vec{B}.$
- E.g. spherical shell $M = \frac{2\pi}{3} \sigma R^4 e \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- For flat surfaces computation similar to moment of inertia
- Distribution of conductive material σ

Eddy currents



3 phases:

- Dissipation -> flat spin
- Exponential decay
- Stabilization

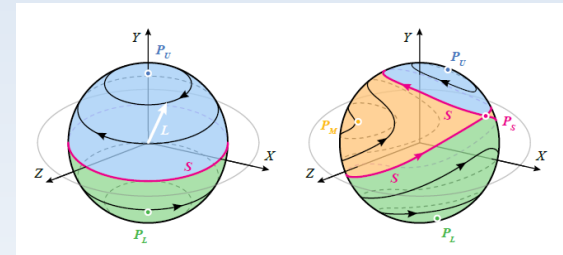
θ_A angle between angular velocity and major axis of inertia (dissipation phase)

Gravity gradient

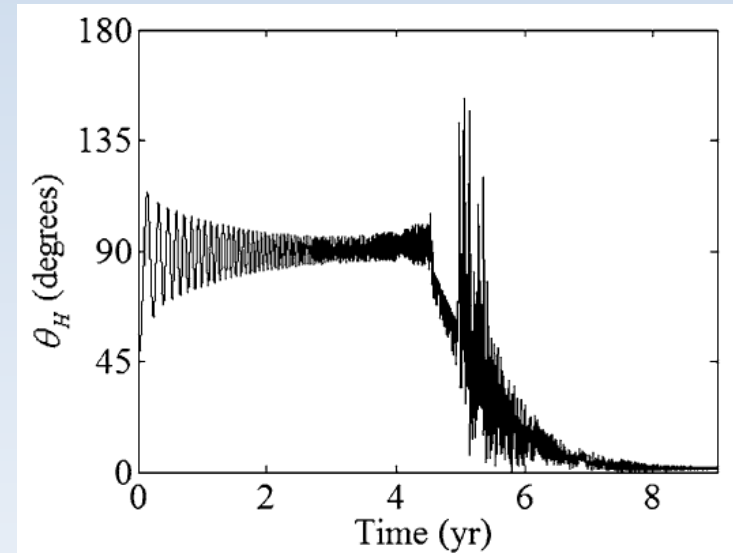
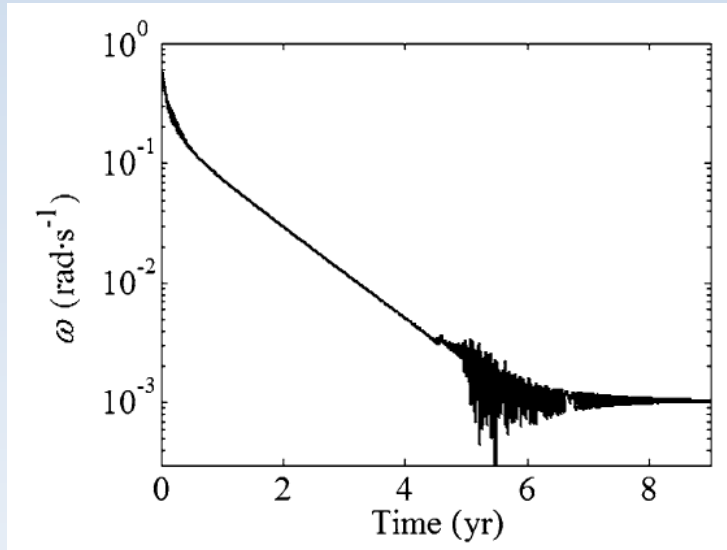
- No contribution to energy loss (conservative)
- Big contribution to change of momentum direction
- Ideally, precession of momentum around orbit normal (decay phase)
- E.g. rocket body:

$$P_g = \frac{2\pi^2 I_0 r^3}{GM_e \Delta I P_{rot} \cos \theta}$$

- Not ideal precession due to orbit precession:
- With low spin rate (gravitational capture phase):
=> alignment to orbit normal
- Oscillations during capture, motion like a pendulum



Eddy current + grav. grad.



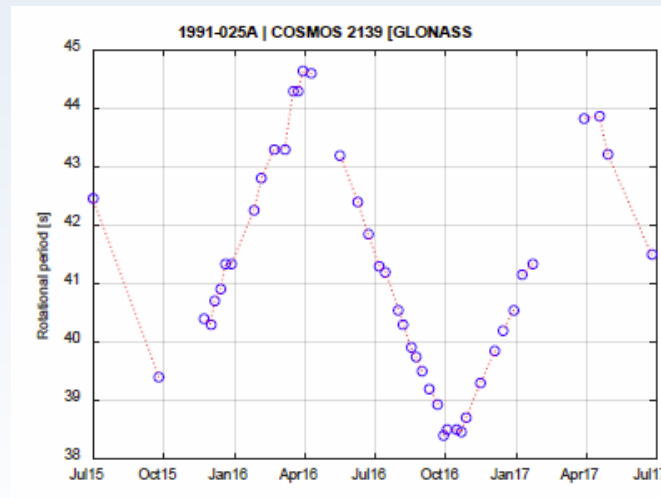
θ_H angle between angular momentum and normal to orbital plane

Gravitational capture:

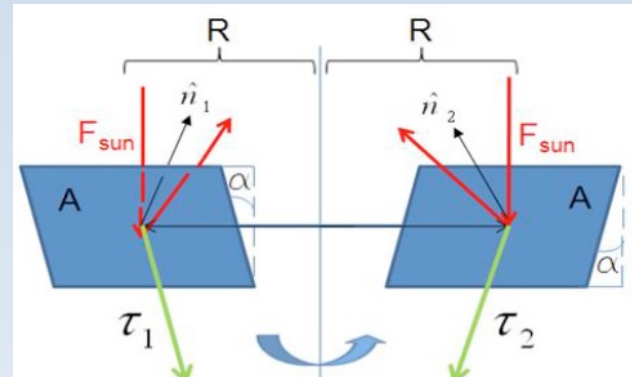
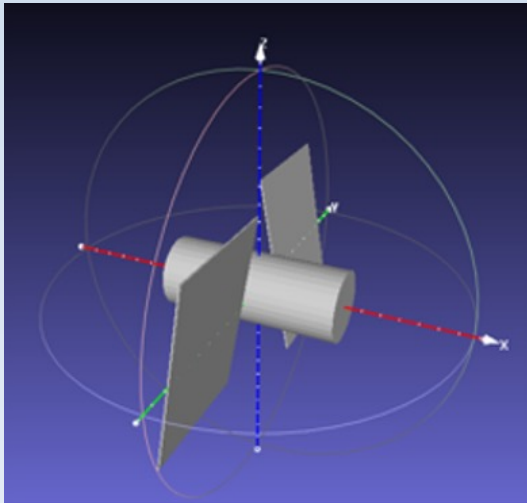
- Angular rotation low
 - Gravity gradient bigger than eddy currents
- => gravity gradient aligns momentum vector to orbit normal
=> Spin-orbit resonance (mostly 1:1, but depends on orbit)

Solar Radiation Pressure

- No magnetic dissipation effect
- Spin rate constant or changes due to SRP
- Mostly S/C with large illuminated surfaces
- Increase/decrease spin rate:
 - Asymmetric solar panels
 - Offset of center of mass w.r.t. figure
- Cyclic and secular trends

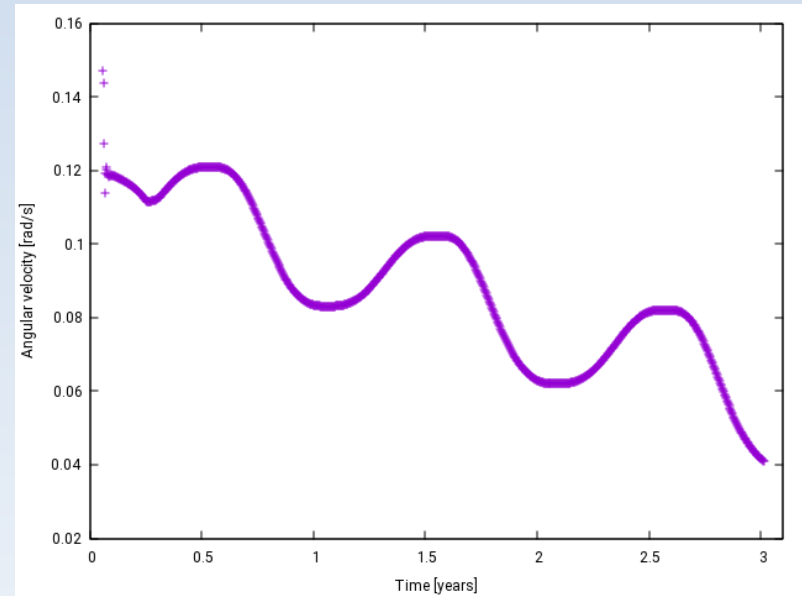
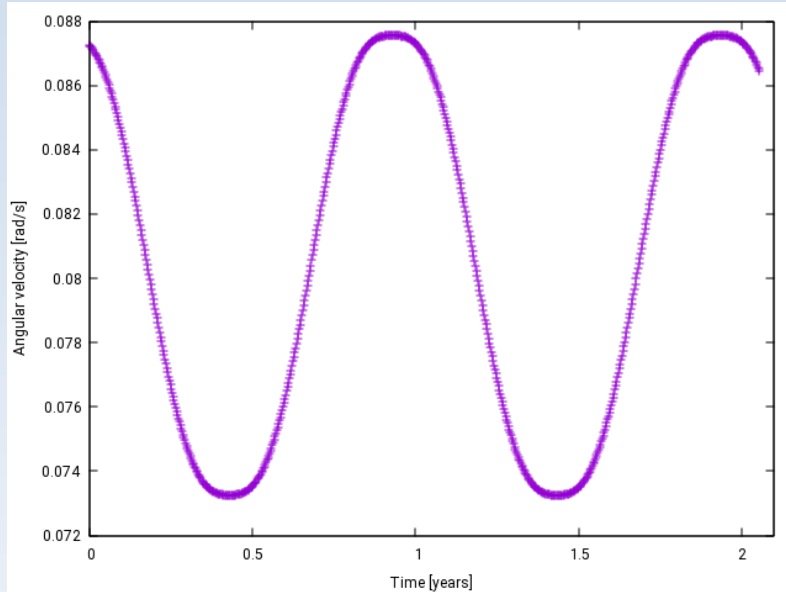


Simulation: wind wheel



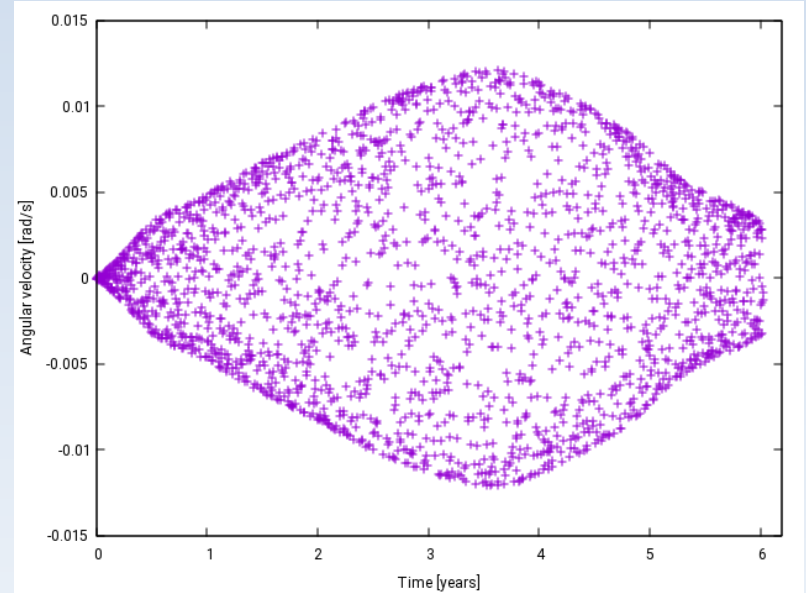
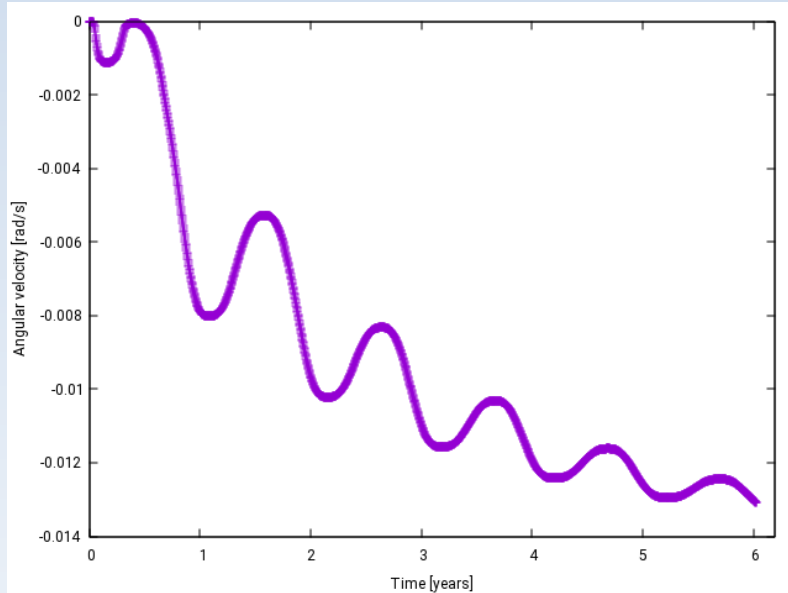
- Wind wheel, rotation around x axis
- Sun illumination parallel to x \Rightarrow torque along x
- Sun illumination in x-y plane \Rightarrow partial torque along x, no torque with Sun parallel to y
- Simulations:
 - Polar orbit, 36'000 km altitude
 - Moments of inertia are 3×10^3 , 3×10^3 , 1×10^3 kg m²
 - Box (2 x 2 x 2 m), wings (1 x 2 m)
 - 10° canting angle

Periodic pattern



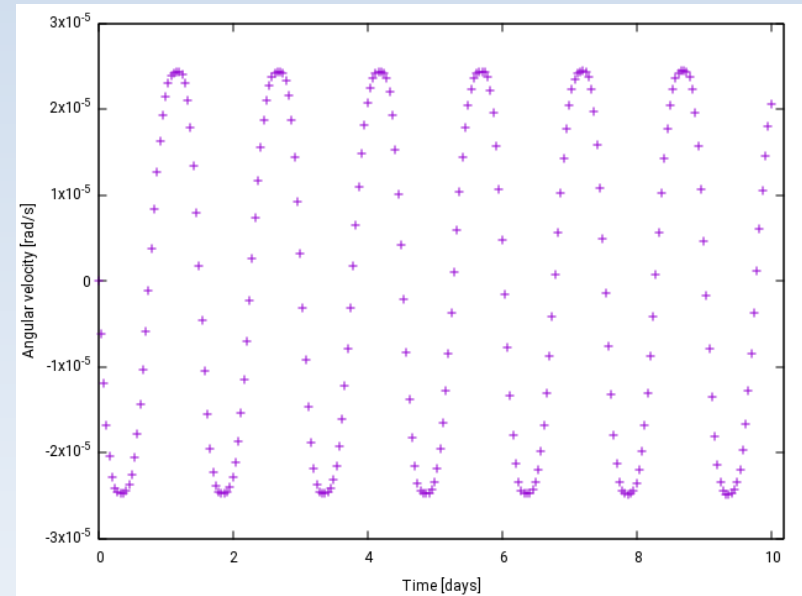
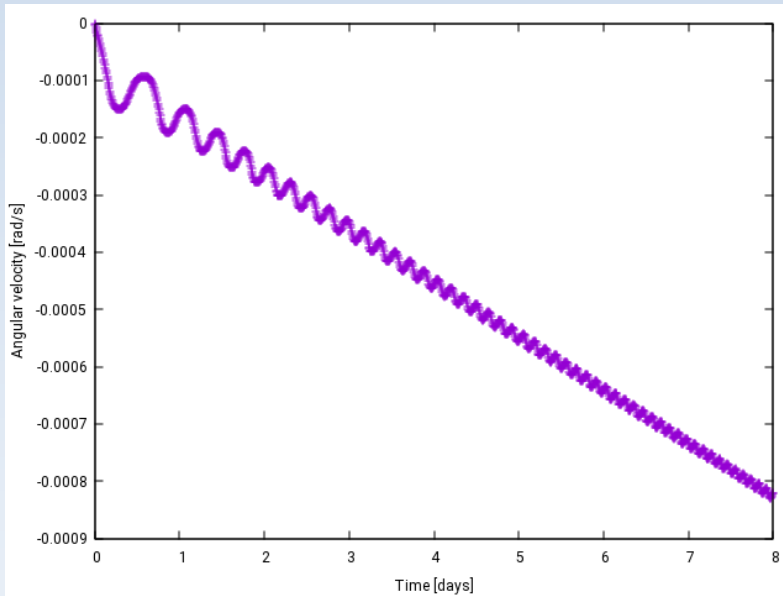
- Initial rotation
 - => stabilize the direction of angular velocity
 - => throughout the year Sun illuminates front/back of wheel
 - => increase/decrease: yearly period
- Different front/back reflection coefficient
 - => increase/decrease stronger in the first half of year
 - => secular slope

Pseudo periodicity

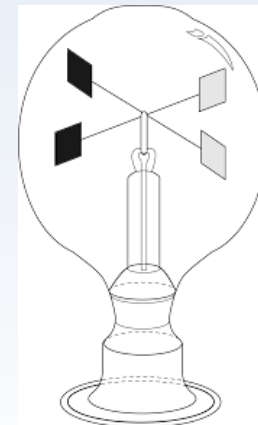


- No initial velocity + different front/back coeff.
=> secular + periodic trend
=> pseudo periodic (chaotic)
- Same results with only 1 wing

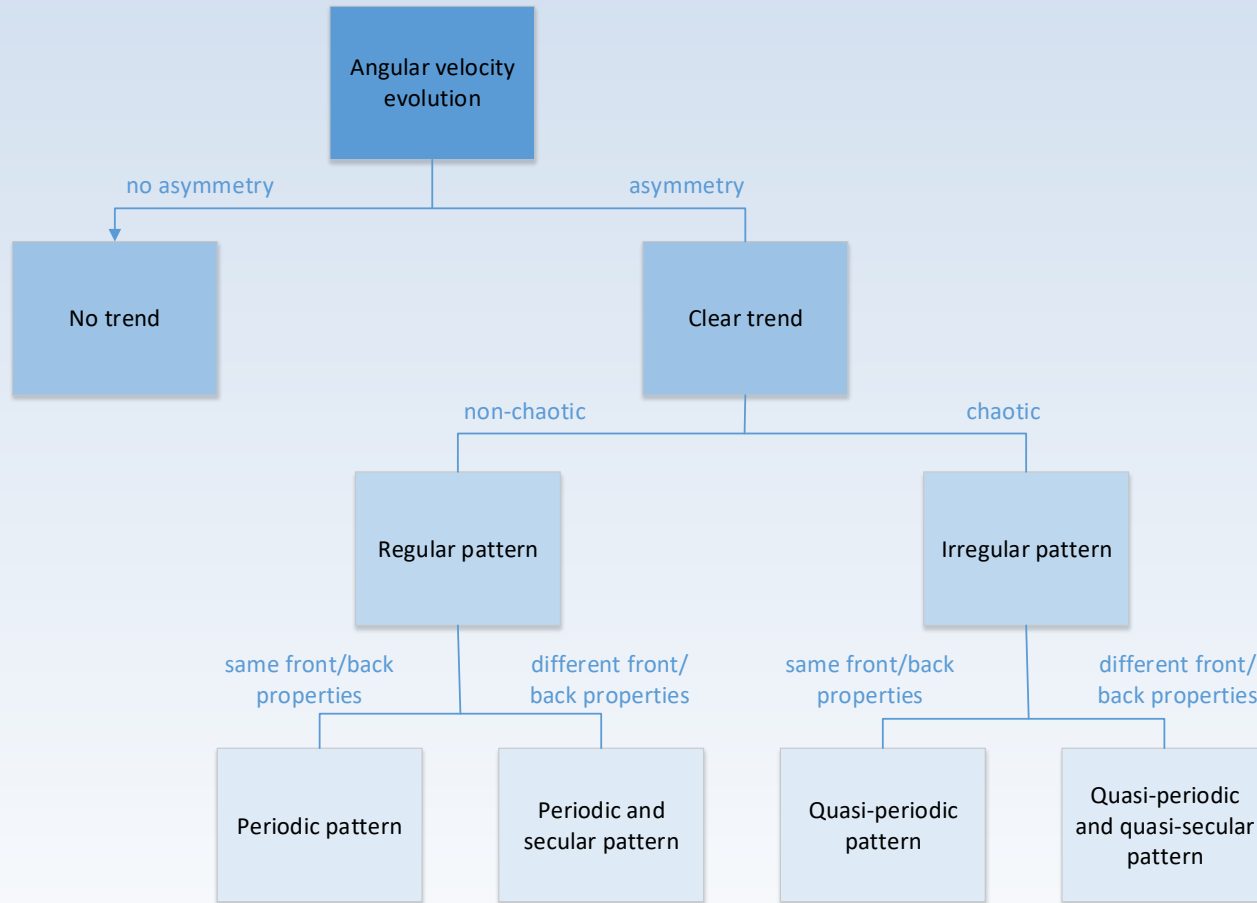
Monotonic trend



- 1 wing with no canting
=> monotonic trend (radiometer)
=> or almost static attitude (wind vane)



Different behaviours



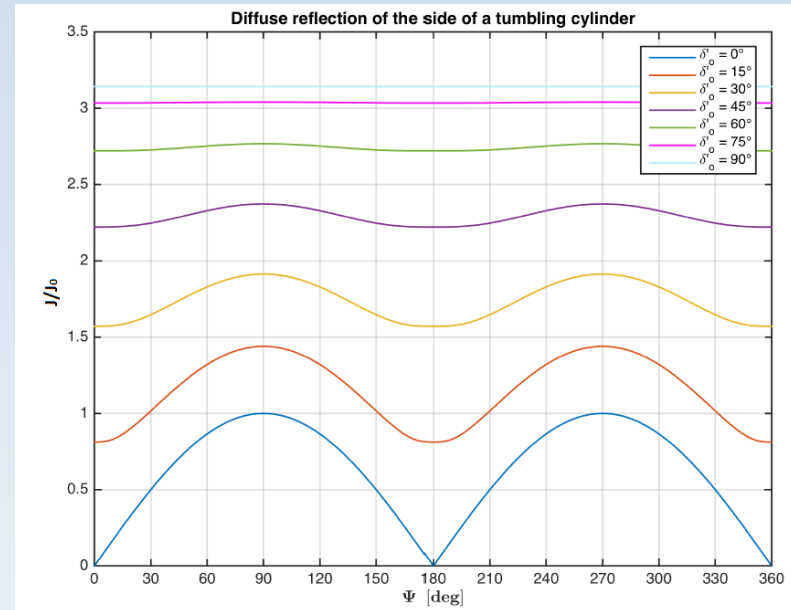
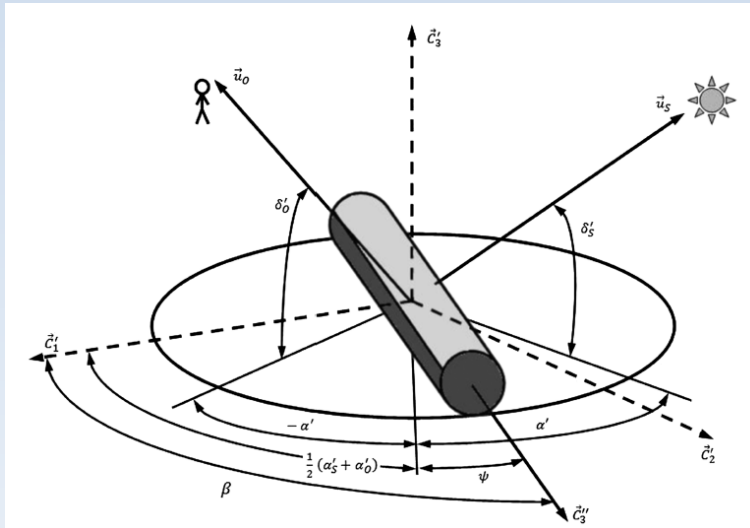
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Amplitude method

- Method proposed by V. Williams (1979)
- Determination of the spin axis orientation
- Cylindrical elongated objects: R/B
- Stable attitude state (flat spin)
- Diffuse reflection of Sun light
- No reflection on top/bottom of cylinder
- Constant spin axis during one observation passage
- Constant spin axis over time period of several observations

Amplitude method



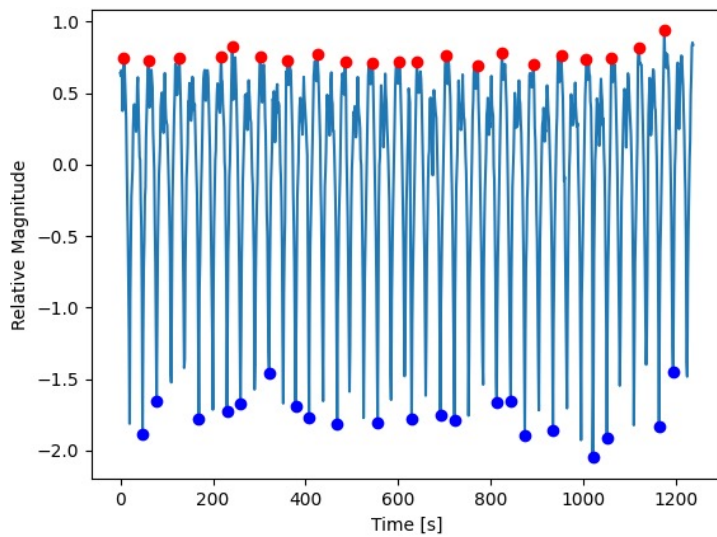
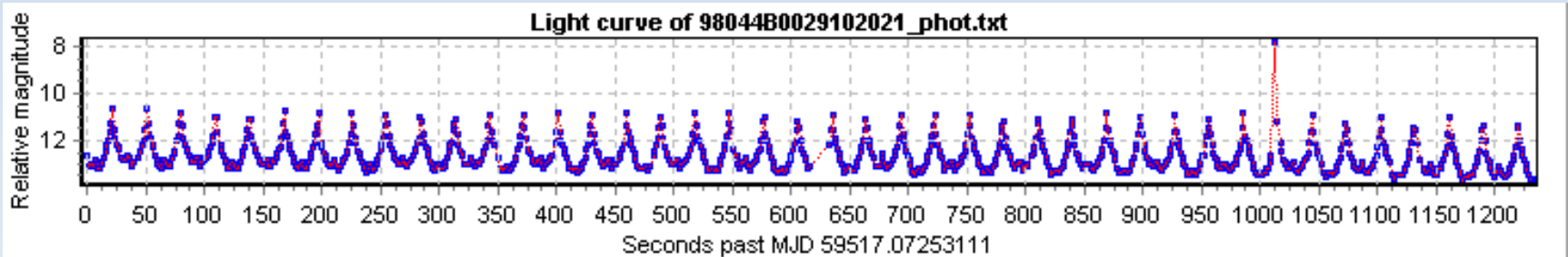
$$\alpha' = 0^\circ, \delta'_S = 90^\circ$$

- Brightness ratio (BR): $M_{max} - M_{min} = -2.5 \log_{10} \frac{J_{max}}{J_{min}}$
- Ratio depends on direction of spin axis
- For given Sun/observer direction and brightness ratio
=> there is a set of solutions
- More ratios are necessary for a unique solution

Amplitude method

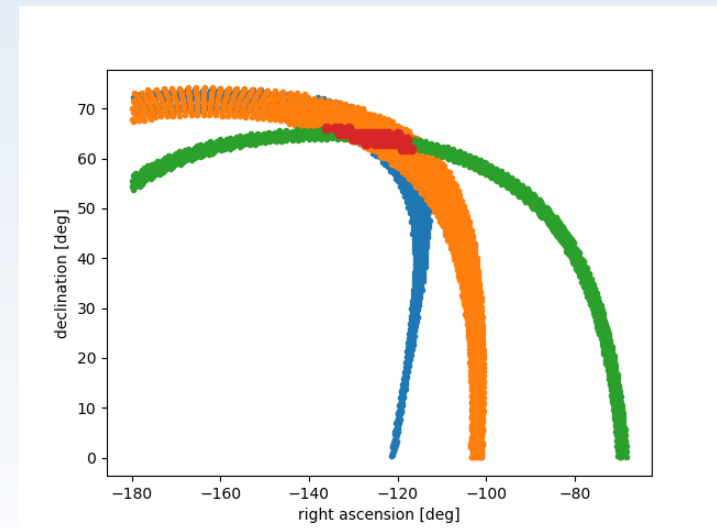
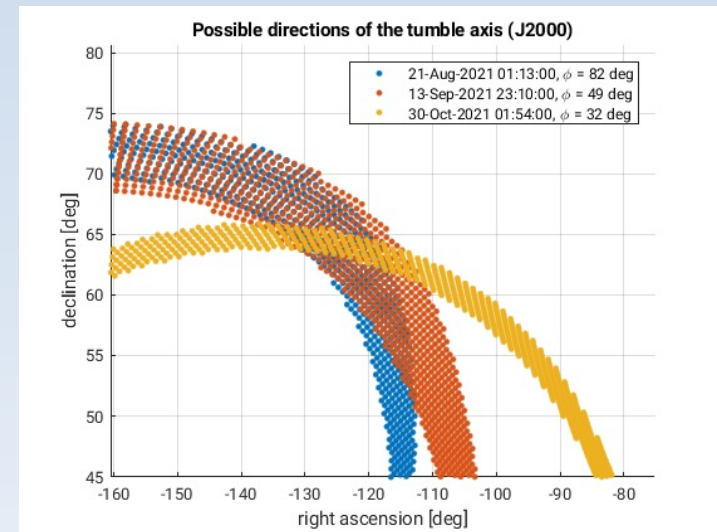
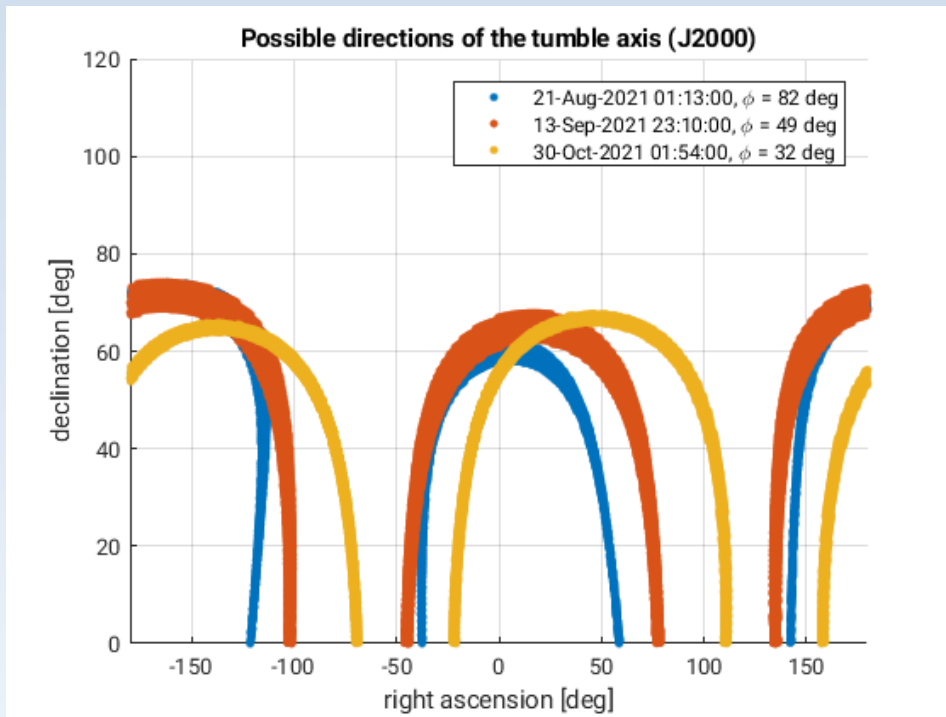
- Measurement of brightness ratio from light curve.
- Determination of the celestial coordinates of satellite, Sun, observer.
- Calculation of the phase angle.
- Construction of the satellite coordinate system
- Numerical search of tumble axis directions comparing measured and simulated brightness ratios.
- Conversion of tumble axis directions in celestial coordinates.
- Iteration of the previous steps for at least three different observation geometries of the same object.

Amplitude method: CZ-3B R/B



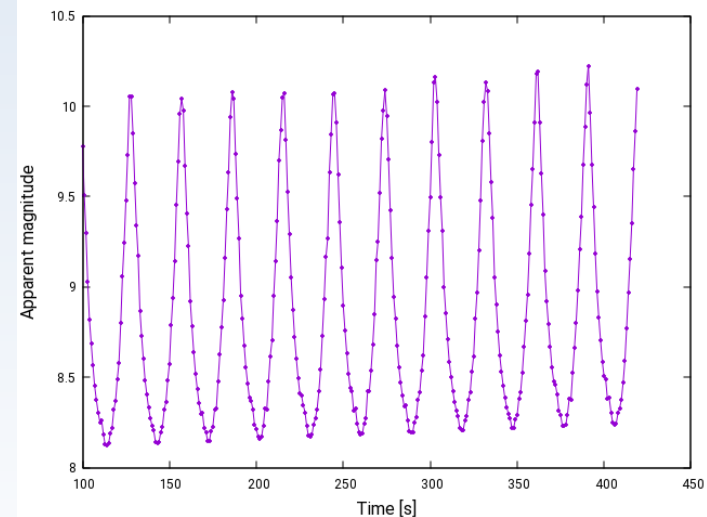
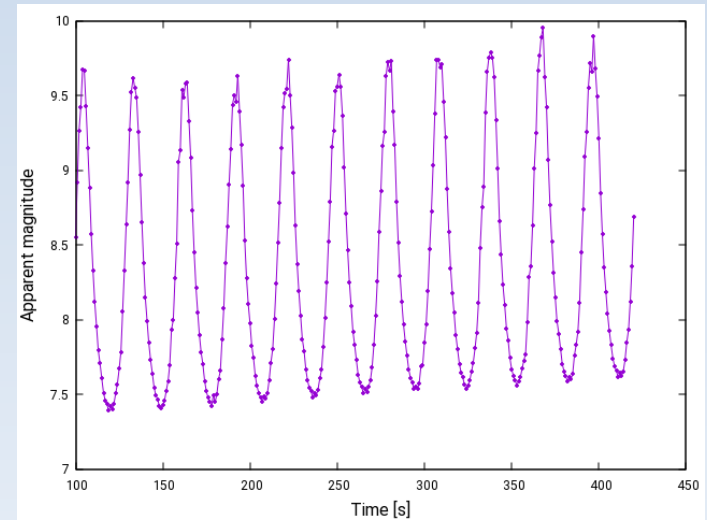
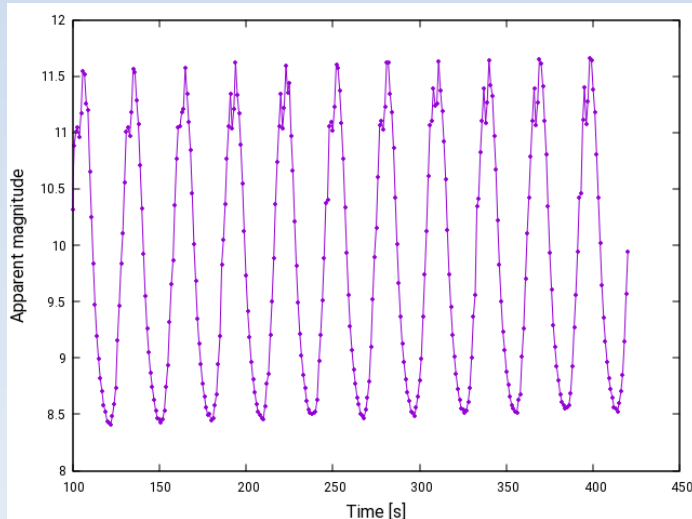
Epoch	Phase angle [deg]	Brightness ratio	Std. deviation	Threshold
21-Aug-2021 01:13:00	82	-3.14	0.30	$\pm 1\sigma$
13-Sep-2021 23:10:00	49	-2.81	0.38	$\pm 1\sigma$
30-Oct-2021 01:54:00	32	-2.38	0.14	$\pm 1\sigma$

Amplitude method: CZ-3B R/B



- Solution regions for 3 light curves
- Intersection shows spin axis direction
- Script identifies region of overlapping

Simulated light curves: CZ-3B R/B



- IOTA simulations at 3 obs. epochs
- Top/bottom absorption coeff. = 1
- Rotation axis (RA 235.2°, DEC 64.1°)
- Brightness ratios from obs.:
-3.1, -2.8, -2.4
- Simulated brightness ratios:
-3.1, -2.3, -2.0
- Possible divergence due to shape/reflection model

Solution evaluation

- Graphical intersection shows that in principle there is a solution
- Although unlikely, it can be that intersection shows not the real spin axis
- If assumption not fulfilled, error exceeds the formal value given by measurement noise
- If spin axis not constant, solution is kind of an average orientation

- Simulations for the above case CZ-3B R/B on GTO orbit:
 - Time interval of 3 observations is about 2 months
 - Gravity gradient torque causes precession of spin axis
 - Spin axis might change orientation of 30° within 2 months
- Ideally for low orbits: observation series within few days

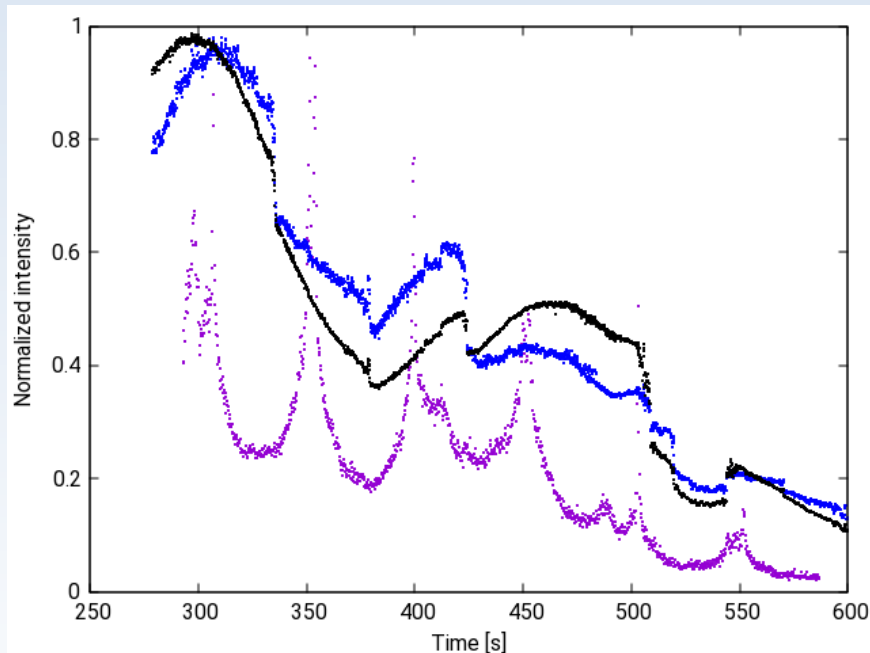
Simulated light curves: Envisat

- Validation of IOTA simulated light curves
- Availability of quite accurate Envisat attitude state in the past
- Joint CMOS-SLR-Radar campaign 21.09.2016 (previous ESA Attitude project)

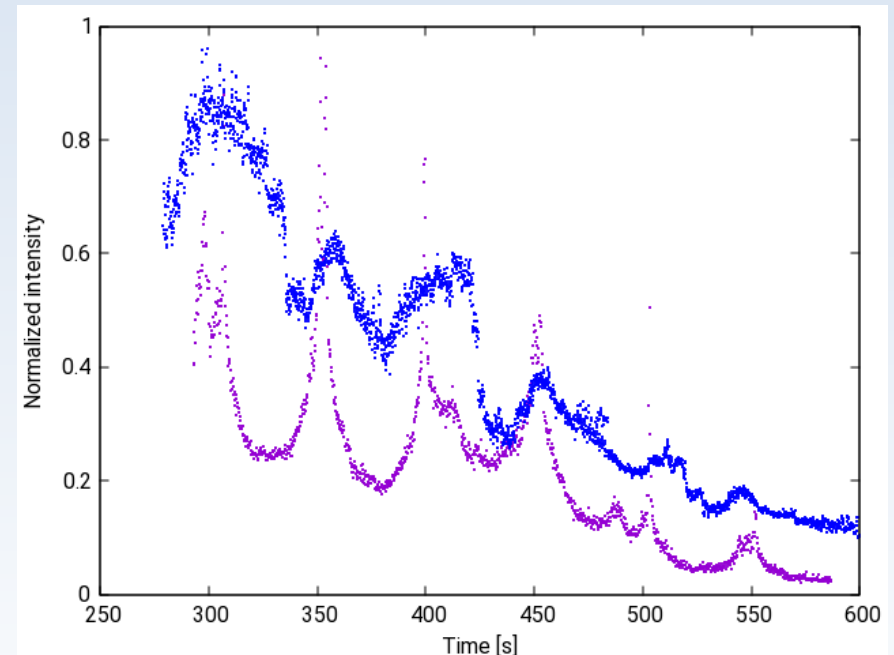
Solution ID	Passage	Source	roll, pitch, yaw [deg]	angular velocity vector: W_x, W_y, W_z [deg/s]
1.1	ENVISAT 2016-09-21 18:53 UTC	ISAR, refined	147, -76, -102	0.04, 0.025, -1.659
3.1	ENVISAT 2016-09-21 20:31:18.6 UTC	SLR, time corrected	117.0, -46.3, -69.8	0.0, 0.0, -1.6774

Simulated light curves: Envisat

	a286 (stainless steel)	aa7075 (aluminum alloy)	cfpr (polymer composite)	cfpr-new (polymer composite)	envi-comp1	s1-sarmat (radar antenna)	sol-ar-swarm (solar panel)
nominal	0.5 / 0.47	0.5 / 0.16	1 / 0	1 / 0	0.5 / 0.16	0.5 / 0.5	1 / 0.9
modified	0.1 / 0.9	0.1 / 0.1	0.9 / 0.1	0.9 / 0.1	0.9 / 0.1	0.1 / 0.1	0.2 / 0.98

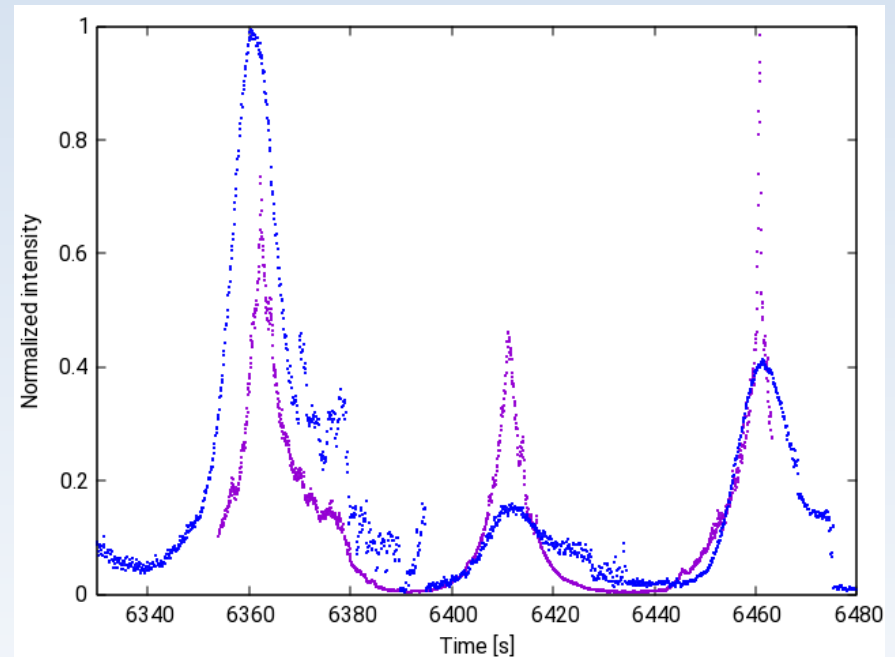
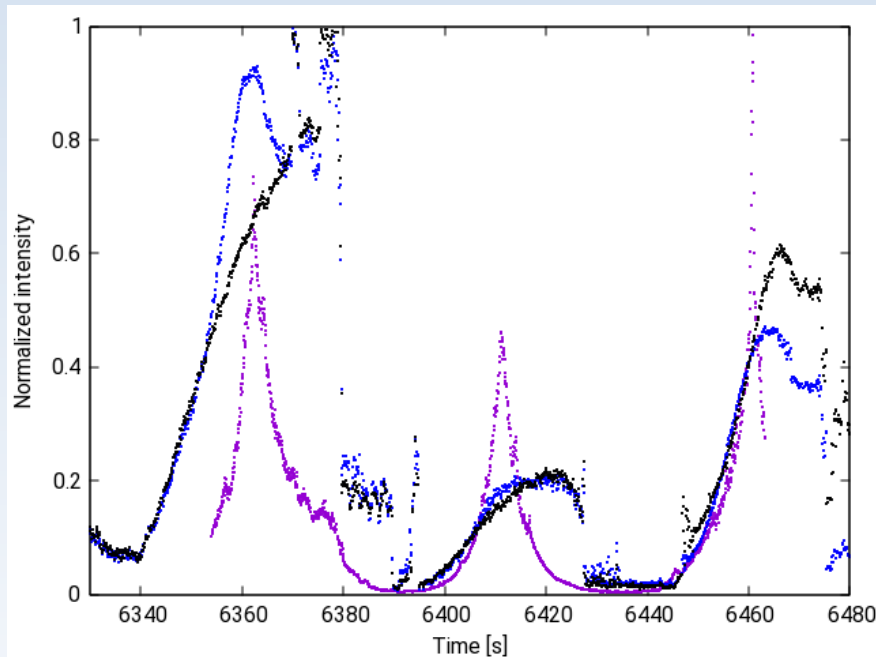


- Violet: observed light curve
- Black: one material
- Blue: nominal material reflection coeff.



- Violet: observed light curve
- Blue: modified material reflection coeff.

Simulated light curves: Envisat



- **Violet:** observed light curve
- **Black:** one material
- **Blue:** nominal material reflection coeff.

- **Violet:** observed light curve
- **Blue:** modified material reflection coeff.

Method of Yanagisawa et al.

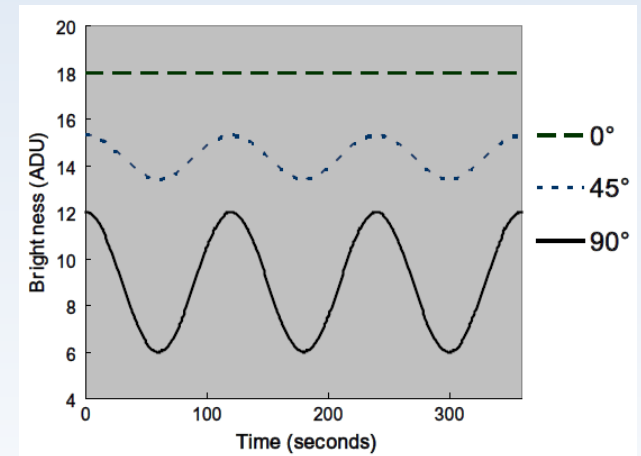
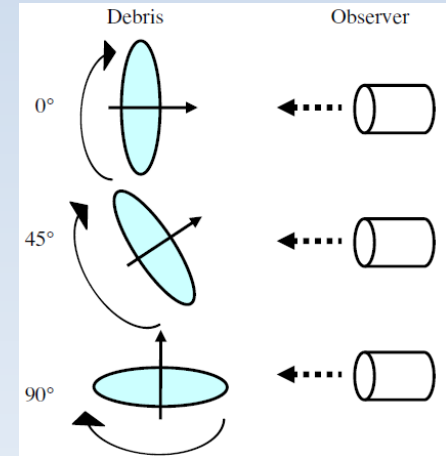
- Simple formulation for ellipsoid, function of θ
- α phase angle, M optic coeff.

$$A = (1 + M\alpha)A_0 \quad A_0 = \frac{a\sqrt{b^2 \cos^2 \theta + c^2 \sin^2 \theta}}{b\sqrt{a^2 \cos^2 \theta + c^2 \sin^2 \theta}}$$

- For cylinder:

$$A_0 = 1 / \left(\cos \theta + \frac{\pi}{4} R d \sin \theta \right)$$

- d diameter divided by length
- R albedo of bottom/top divided by side albedo
- Solution as least squares problem
- At least as many observations as parameters
- Precession parameters can be estimated



Precession

$$A_0 = 1 / \left(\cos \theta + \frac{\pi}{4} R d \sin \theta \right)$$

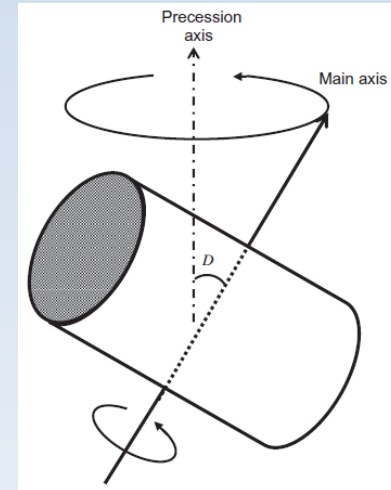
$$R.A. = RA_0 + D \sin(2\pi t/T + \beta) / \cos(Dec.)$$

$$Dec. = Dec_0 + D \cos(2\pi t/T + \beta)$$

$$l = \cos(R.A.) \cos(Dec.)$$

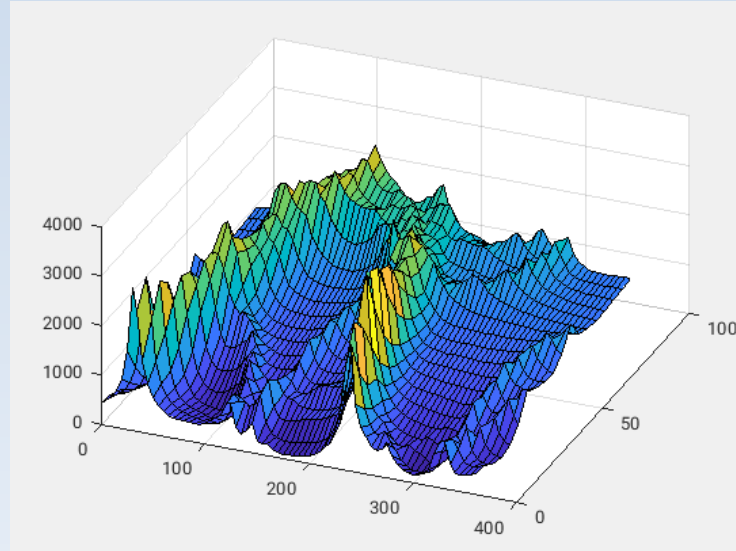
$$m = \sin(R.A.) \cos(Dec.)$$

$$n = \sin(Dec.)$$



- RA_0 Dec_0 (precession axis)
- RA , Dec (rot. axis)
- Approximation for small D
- For larger D it is more complicated (e.g. Rodrigues rotation formula)
- Additional estimated parameters:
 - RA_0 DEC_0
 - D (precession angle), T (precession period), β (phase)

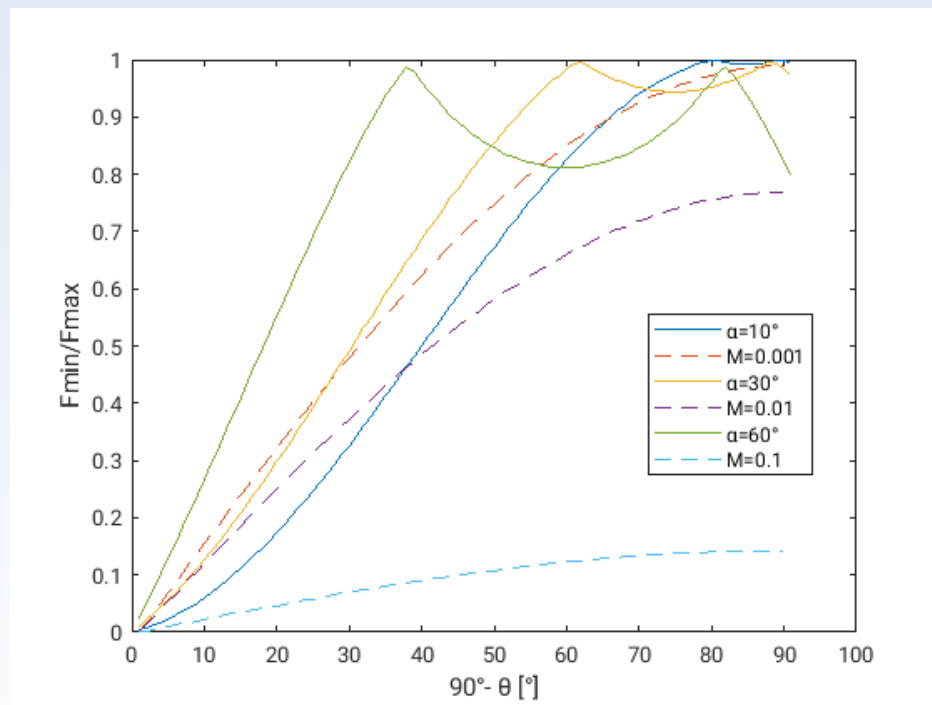
LSQ convergence



- Loss function vs RA/DEC of spin axis
- Tests with all precession parameters show low reliability
- Use several starting points (multiple local minima)
- Tests w/o precession parameters show reproducible results (different LSQ start points with same result).
- However, high residuals
- Problem of Yanagisawa model?

Comparison with Williams

- Yanagisawa depends on **phase angle** through an empirical formula: $(1 + M\alpha)$
 - But it does **not** depend on the **absolute direction of Sun** and of the **spin axis**
 - Compare with Williams, no top/bottom reflection ($R=0$) $\Rightarrow \frac{F_{min}}{F_{max}} = \frac{|\cos \theta|}{1+M\alpha}$
- \Rightarrow Yanagisawa model: approximation for θ close to 90°



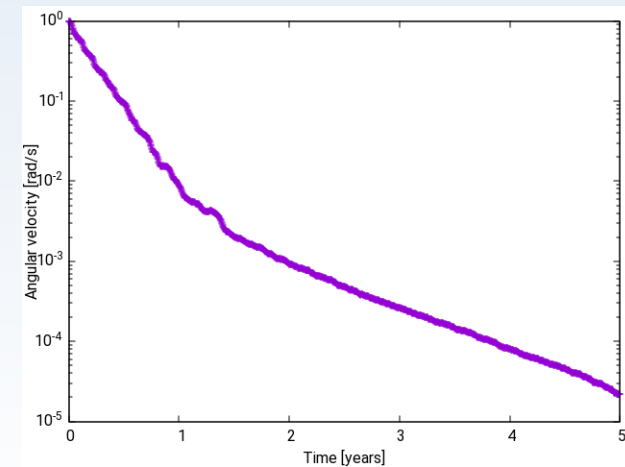
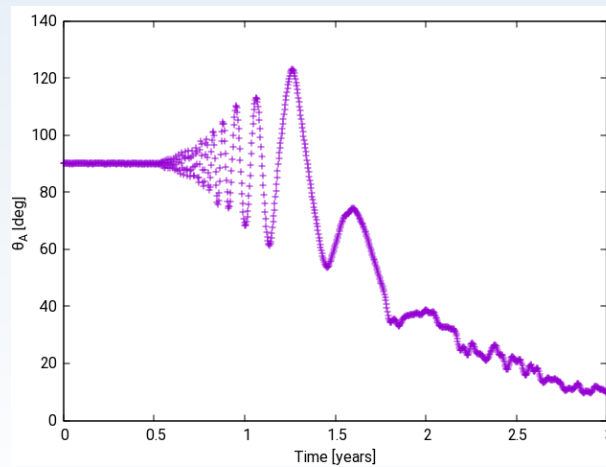
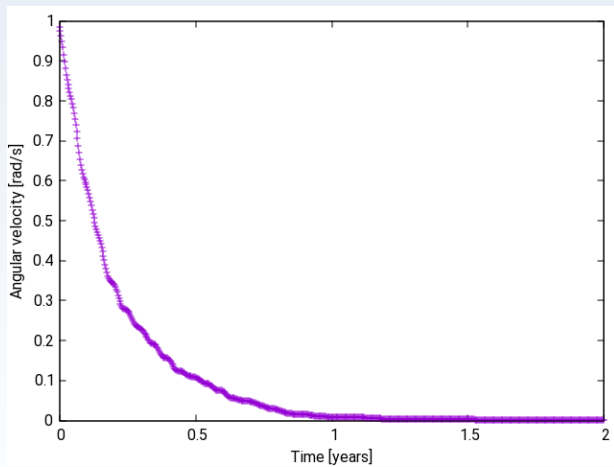
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Attitude evolution

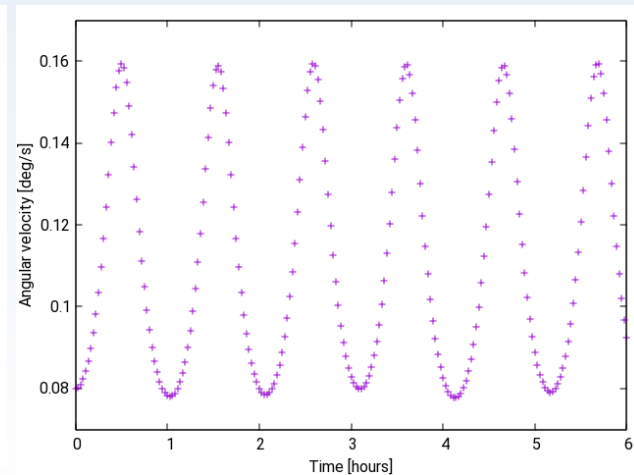
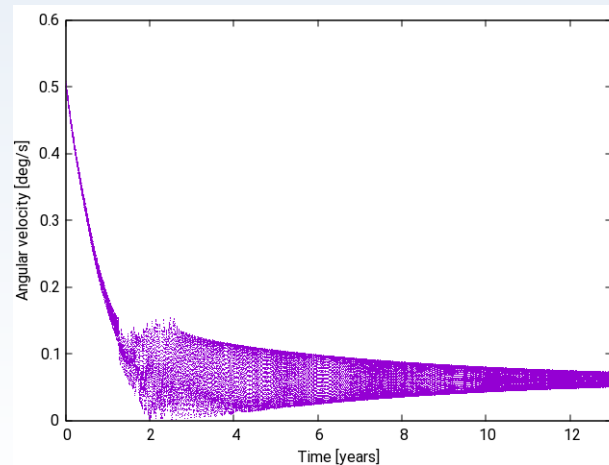
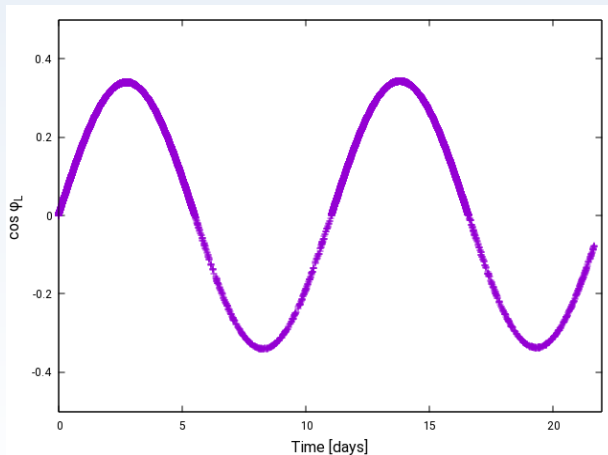
- Compare IOTA simulations with literature results
- Orbit: 800 km, inclination 70°, angular velocity 1 rad/s
- Only effect of Eddy currents
- Exponential damping and stabilization
- Angle θ_A betw. angular velocity and axis of maximum inertia => flat spin
- Eddy currents: transition phase (typical bend) after 1 year
- $I = 5 \times 10^3 \text{ kg m}^2$, $M = 2 \times 10^5 \text{ S m}^4$, $B = 40'000 \text{ nT}$

=> Decay rate $\tau \approx \frac{I}{M B_{\perp}^2} \approx 2.2 \text{ yr}^{-1}$



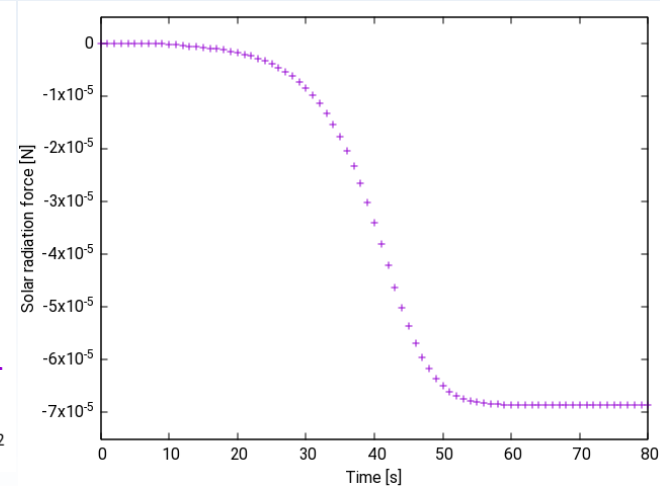
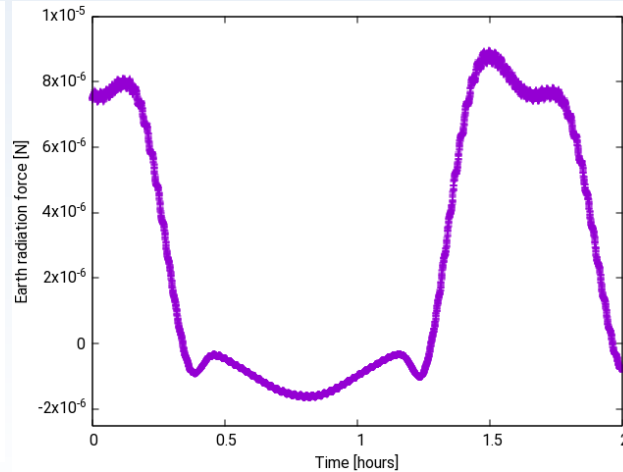
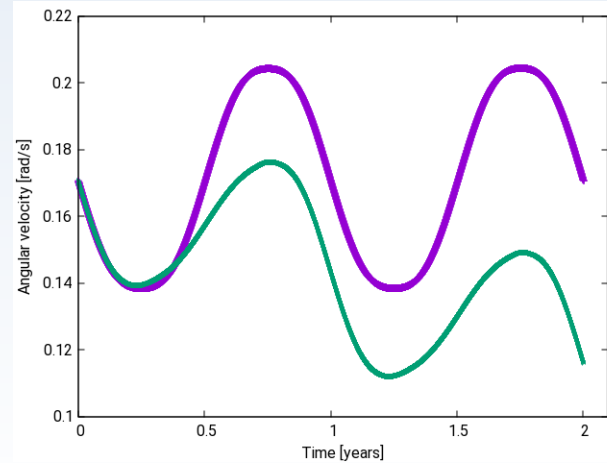
Attitude evolution

- Combined effect of eddy currents and gravity gradient
- Initial angular velocity $0.5^\circ/\text{s}$
- Precession of angular momentum, phase φ_L relates to projection angular momentum onto the orbital plane
- $P_{rot} = 60 \text{ s}$, orbit at 600 km altitude, ratio $\frac{I_0}{I_0 - I_3} \approx 1.28$, and $\theta = 70^\circ$
- $P_g = \frac{2\pi^2 I_0 r^3}{\mu(I_0 - I_3) P_{rot} \cos\theta} \approx 2 \text{ weeks}$
- Stabilization with gravity gradient torque
- Orbital resonance 1:1, angular velocity $0.06^\circ/\text{s}$
- Oscillation of angular velocity (capture phase) $\omega_g = \sqrt{\frac{2\mu(I_0 - I_3)}{I_0 r^3}} \approx 1.3 \text{ h}$



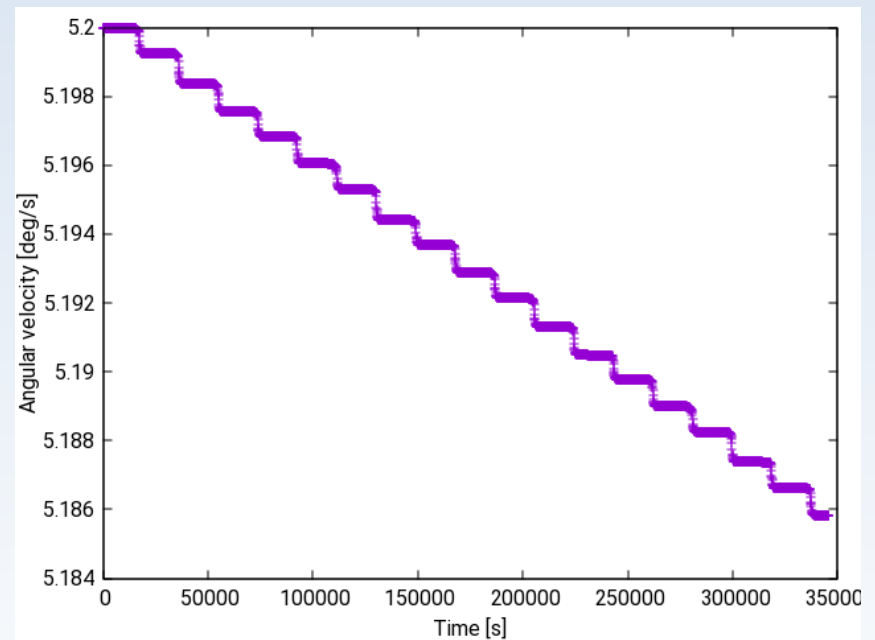
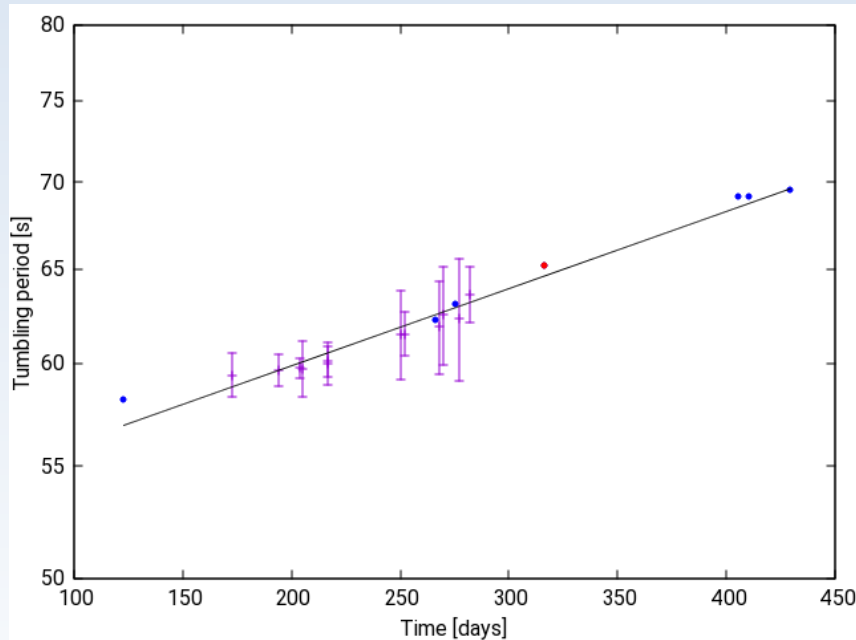
Attitude evolution

- IOTA with only Solar Radiation Pressure considered
- Box (2 x 2 x 2 m), wings (1 x 2 m), 10° canting angle, polar orbit
- Simulation with different front/back panel properties
- Annual variation with secular slope of 0.02 rad/s
- Earth radiation force and penumbra with LEO at 800 km
- Surface of 8 m² => 10⁻⁵ N expected
- Penumbra time in LEO about 10s



Spin rate evolution: CZ-3B R/B

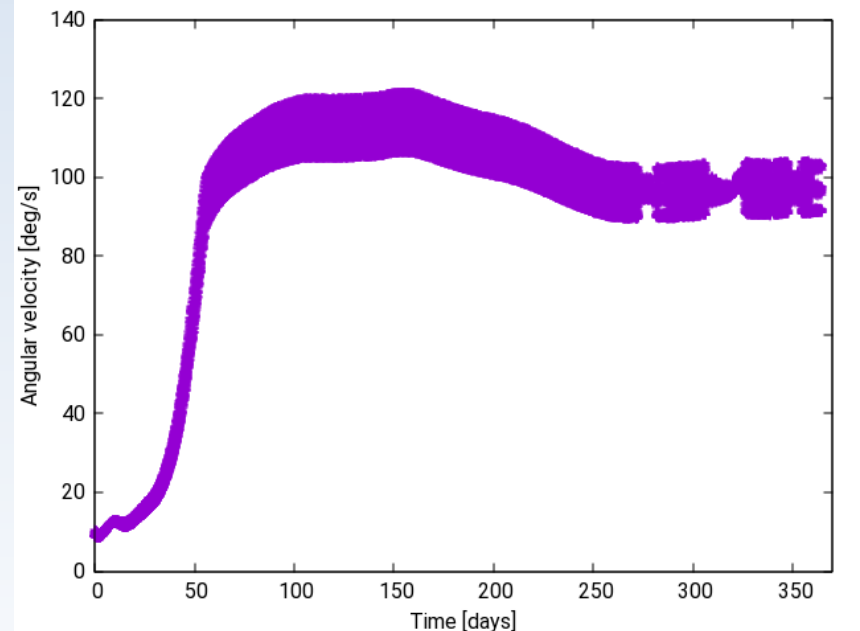
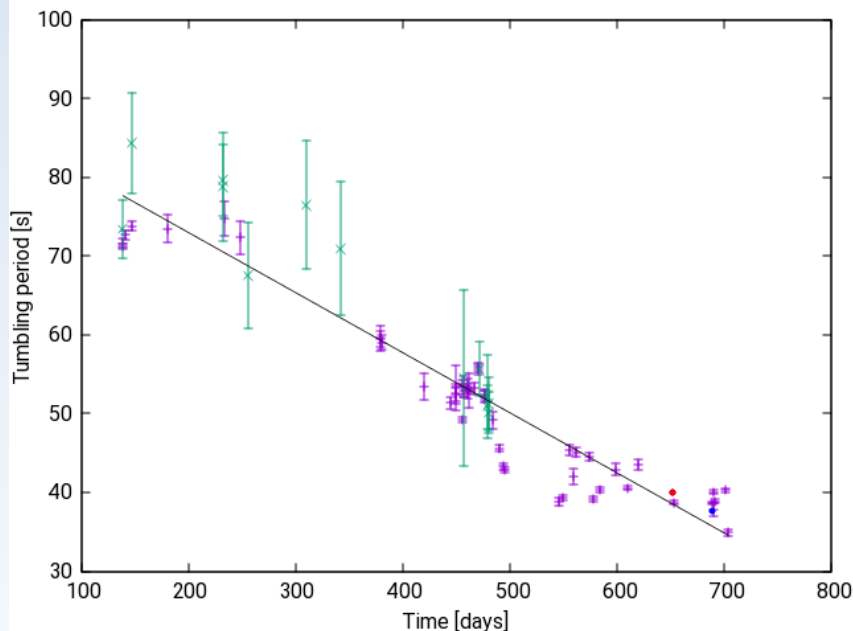
- Combination of observations from Graz, Zimmerwald, TIRA
- Evolution of spin rate determined for several objects
- Check the consistency of the extracted periods
- Example: CZ-3B rocket body (2019-090D) => $\tau \approx \frac{I}{M B_{\perp}^2}$
- IOTA simulation: consider real distribution of Earth magnetic field



- Violet: Graz photon counter
- Red: TIRA, Blue: Zimmerwald
- Spin rate from fit: 0.24 yr^{-1}
- Consider: CZ-3B ~ Ariane 4 H10
- Refined values: $35'000 \text{ km m}^2$, $4 \times 10^6 \text{ S m}^4$
- Simulation gives: $\sim 0.2 \text{ yr}^{-1}$

Spin rate evolution: Jason-2

- Low effect of damping eddy currents, 1300 km altitude
- Solar panels => effect of SRP
- From SLR, radar: spin axis approx. perpendicular to orbit plane
- From radar: solar panels almost orthogonal
- Different reflection coeff. for front/back side of panels



- Violet: Graz photon counter
- Green: Graz SLR
- Red: TIRA, Blue: Zimmerwald

- Spin rate increase, not regular pattern
- Low angular velocity does not keep attitude stable
- Wind wheel model not applicable

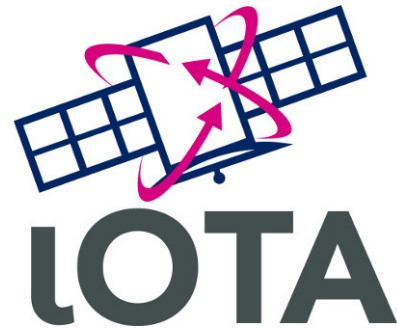
Outlook

- Introduction
- Attitude models
- IOTA software
- Observation campaign
- Radar observations
- Amplitude method
- Epoch method
- Attitude evolution
- Conclusions

Conclusions

- Development of attitude models, attitude determination methods, simulation s/w
- Attitude determination: amplitude method
 - Method of Williams: spin axis orientation of R/B can be determined with 3 or more observations
 - Accuracy from few to tens of degrees
 - Some assumption, e.g. constant spin axis during observations interval, strongly influence the accuracy
 - Method of Yanagisawa uses approximate model and can be used only in a limited range of observation geometries
- Attitude determination: epoch method
 - Characterization of the limits and required spin period accuracy
 - => Improvement of preprocessing for period extraction. Savitzky-Golay on local data samples for successful detrended time derivative series
 - Apparent motion is a limiting factor: in GEO too small
 - In LEO enough apparent motion, but accuracy limits a significant prediction of spin orientation
- Attitude simulation software and evolution models
 - Simulation of light curves for objects with different materials (Envisat, R/B)
 - Simulation of attitude evolution under the effect of eddy currents, gravity gradient, solar radiation
 - Prediction of spin rate for R/B
 - Large amount of observations, more than 20 objects, over 2 and more years, spin periods and trends
- Definition of new process and product formats and standards

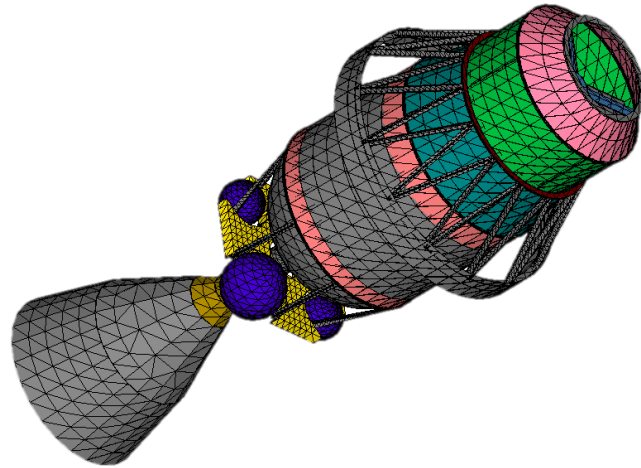
Tumbling motion assessment for space debris objects

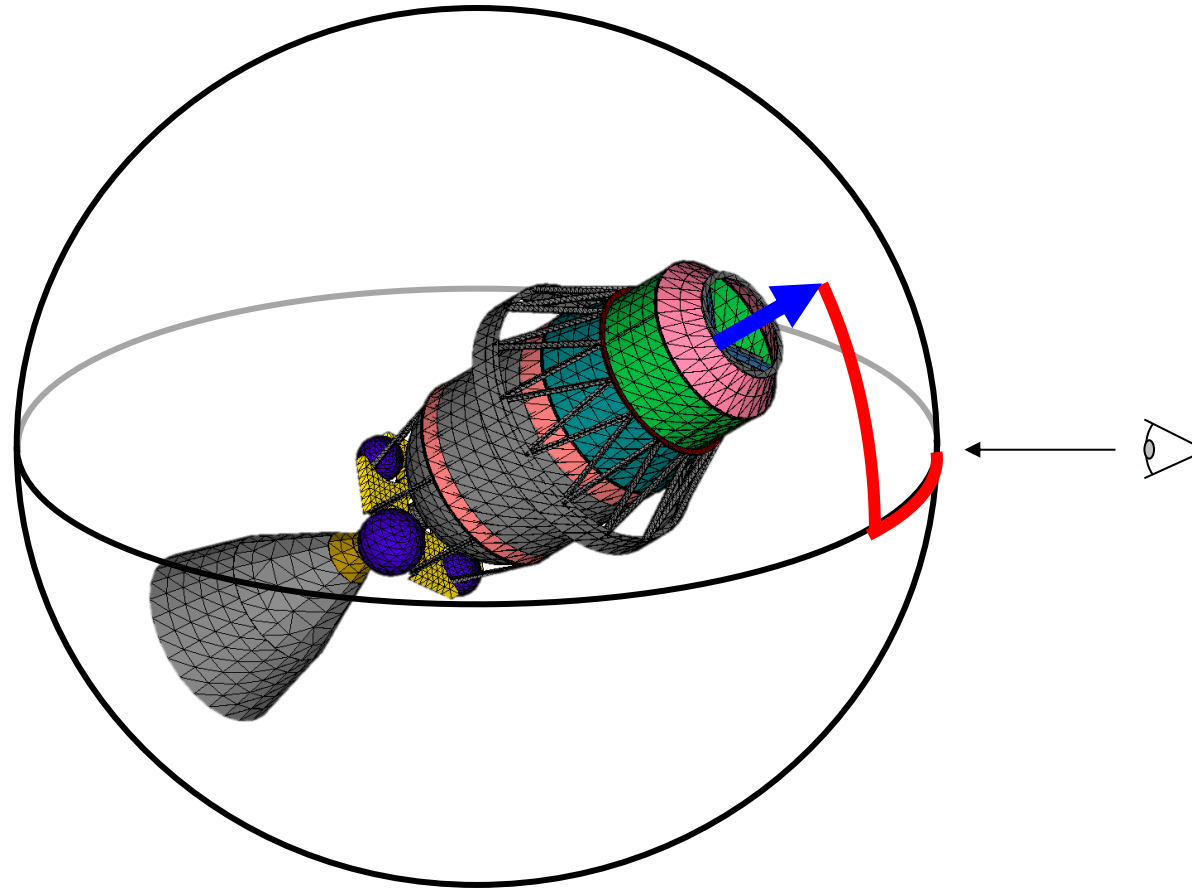


Final presentation

September 9, 2022

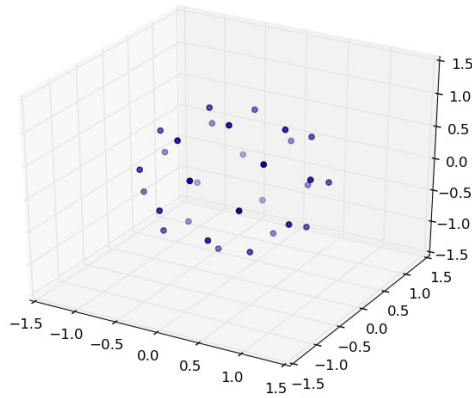
- Major tasks
 - Pre-computed coefficients
 - Magnetic tensor model
 - Earth radiation model
 - IOTA GUI simulation manager and simulation setup
- Minor tasks
 - S/C geometry surface reflectivity
 - Dynamic solar activity for the aerodynamics model
 - Geometry handling optimization
 - TLE import via CSTATE
 - Earth shadow penumbra region
 - Generic attitude damping function
 - Space-based observer
 - Thermosphere wind model (HWM14)
 - Output extension: Sun direction, along-track, and radial vectors [and torque vectors as debug output]



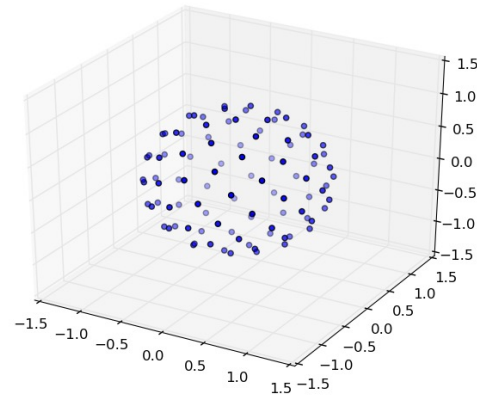


Pre-computed Aerodynamic and Radiation pressure coefficients

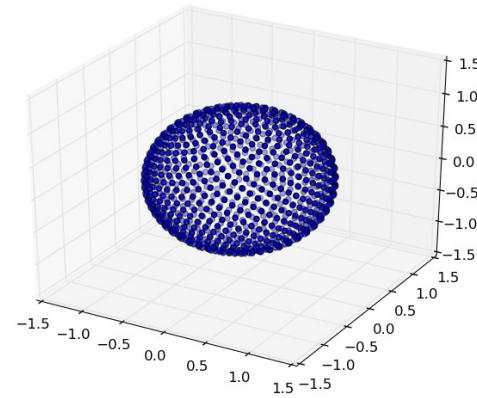
Sphere of view direction
Number of points=30



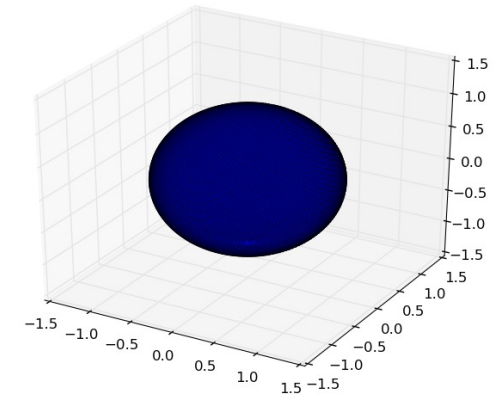
Sphere of view direction
Number of points=100

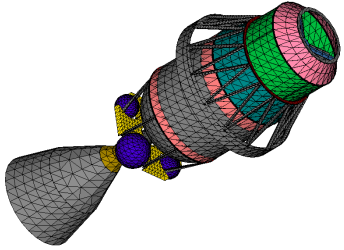


Sphere of view direction
Number of points=1000

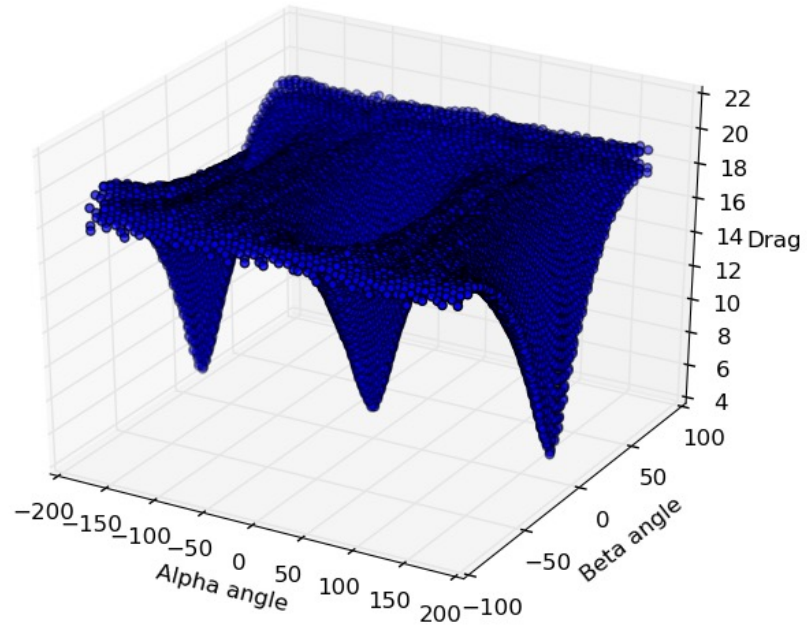


Sphere of view direction
Number of points=10000

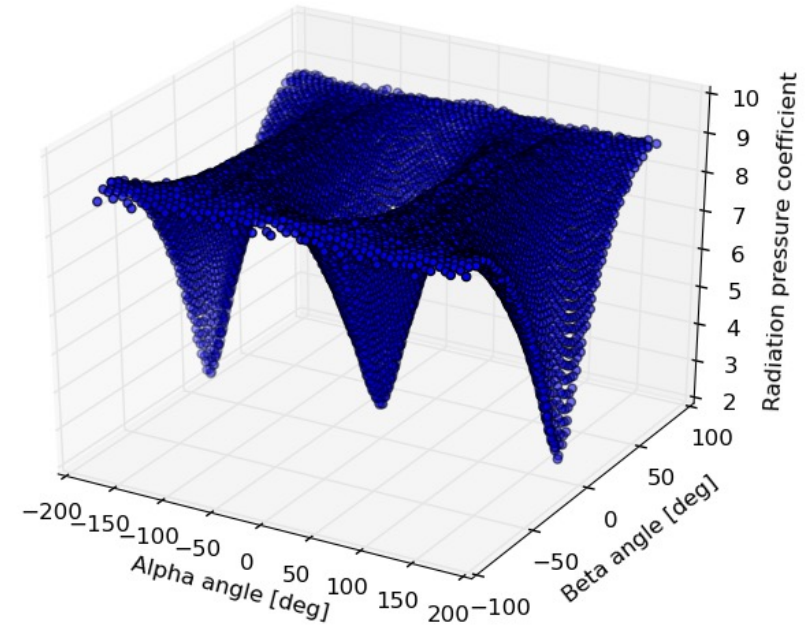




IOTA Aerodynamic coefficient database

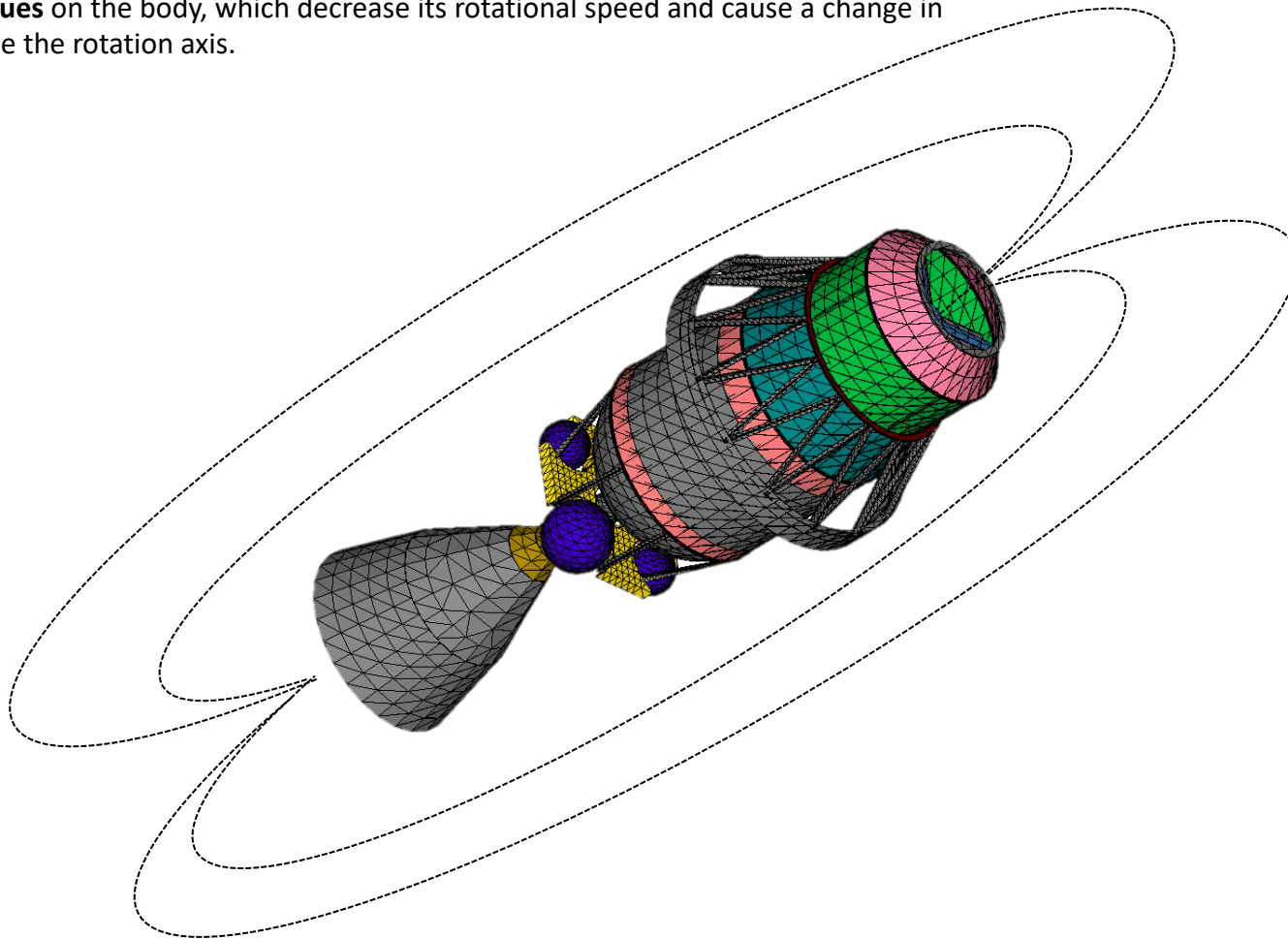


IOTA Radiation Pressure database



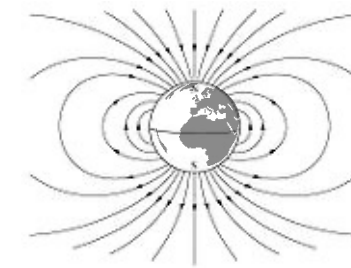
Magnetic tensor model

The S/C is seen as a rotating conductive body in the presence of magnetic field. This induce **eddy current torques** on the body, which decrease its rotational speed and cause a change in the direction of the the rotation axis.

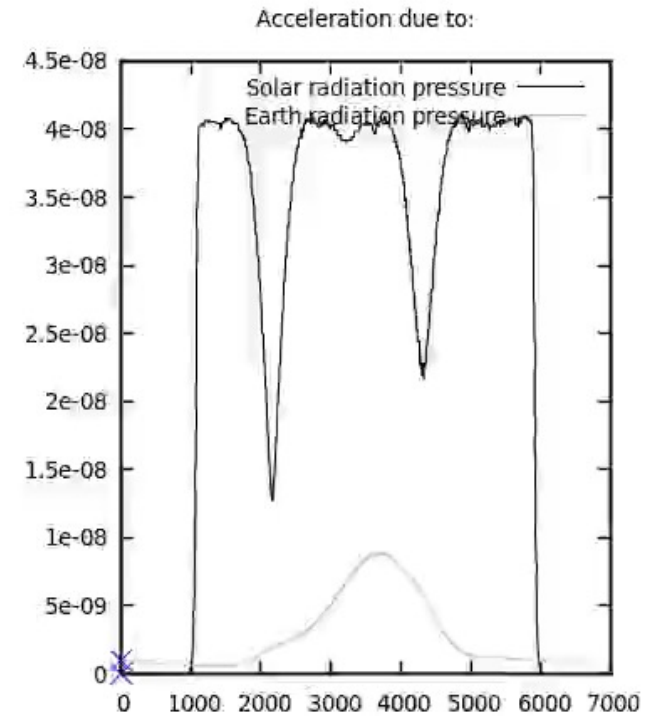
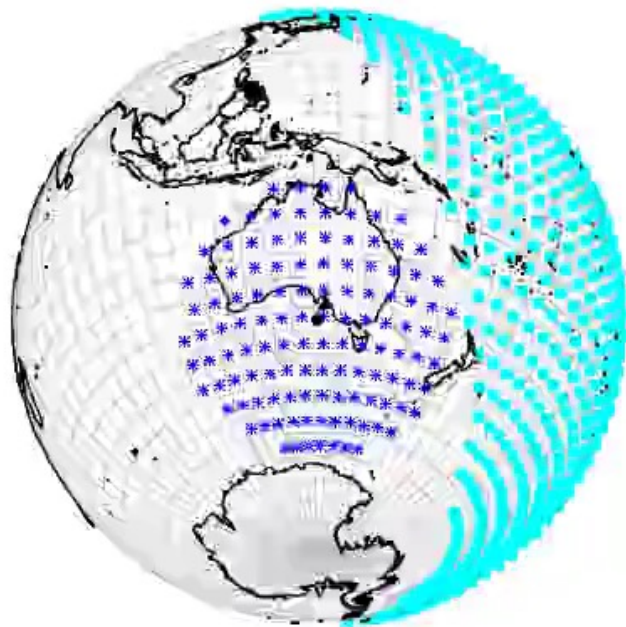


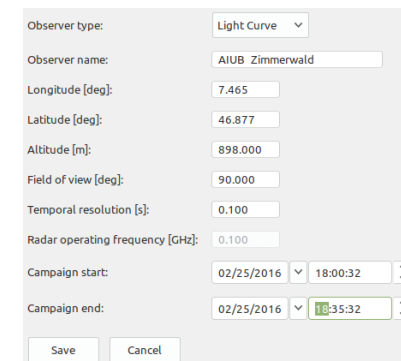
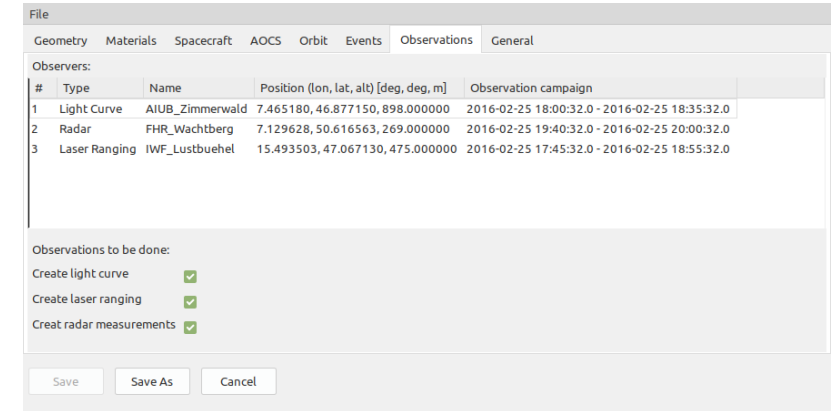
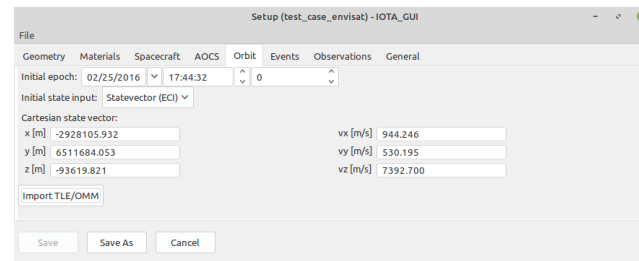
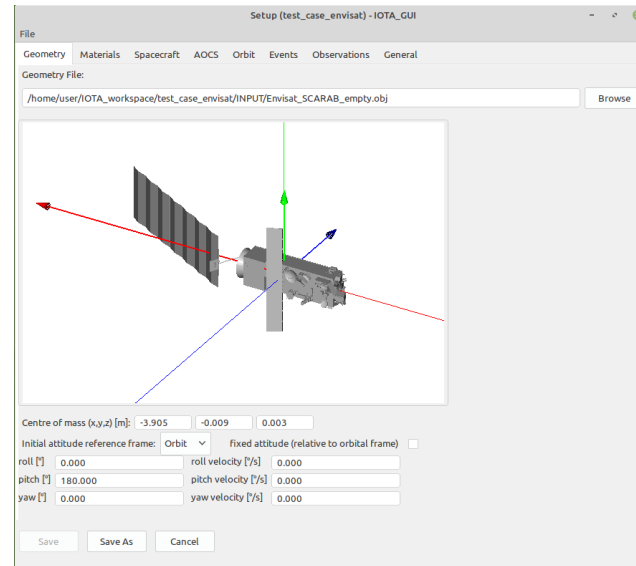
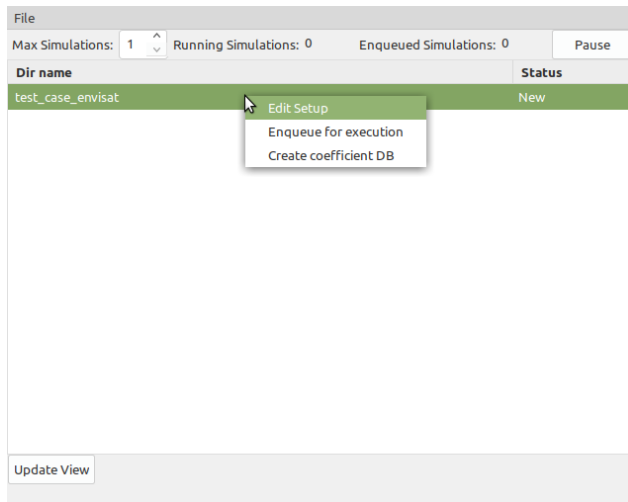
$$T_{eddy} = M(\bar{\omega} \times \bar{B}) \times \bar{B}$$

M ~ Magnetic tensor
 \bar{B} ~ Magnetic field
 $\bar{\omega}$ ~ Angular velocity

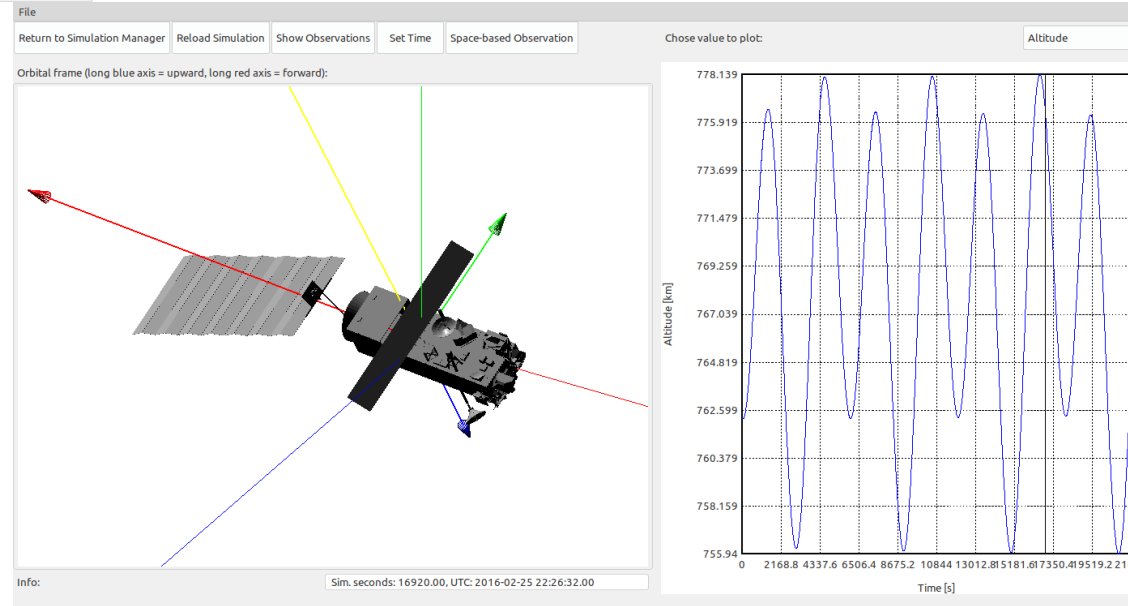
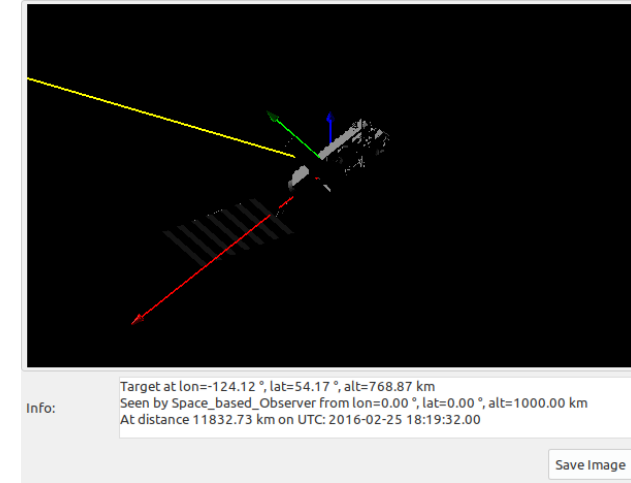
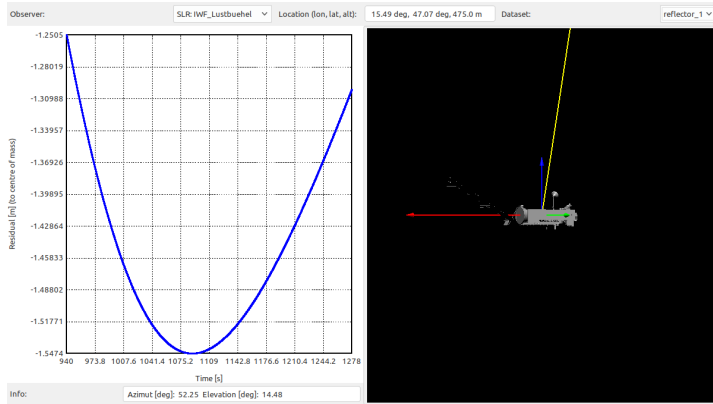


Illumination of Earth surface with acceleration due to ERP and SRP





IOTA GUI result view



- General

- Uncertainty Quantification via MC for simulation input (similar to SCARAB4)
- pyIOTA – Python wrapper (similar to pyDRAMA/pyMASTER)
- Write full precision snapshots to allow continuation of a simulation
 - Recover from a crash or continue with modified settings

- Models and methods

- Extend eddy current damping for slow tumbling objects (DD-0002 Eq. 6)
- Parallelisation of the coefficient DB calculation
- Rotating S/C parts, e.g. solar arrays (similar to DMF-03)
- Use pre-computed coefficients for light curve generation (similar to DMF-05)
- Improve RCS model (similar to DMF-05, model TBD)
- Implement MIRAD micro-particle impact model
- Consider YORP effect

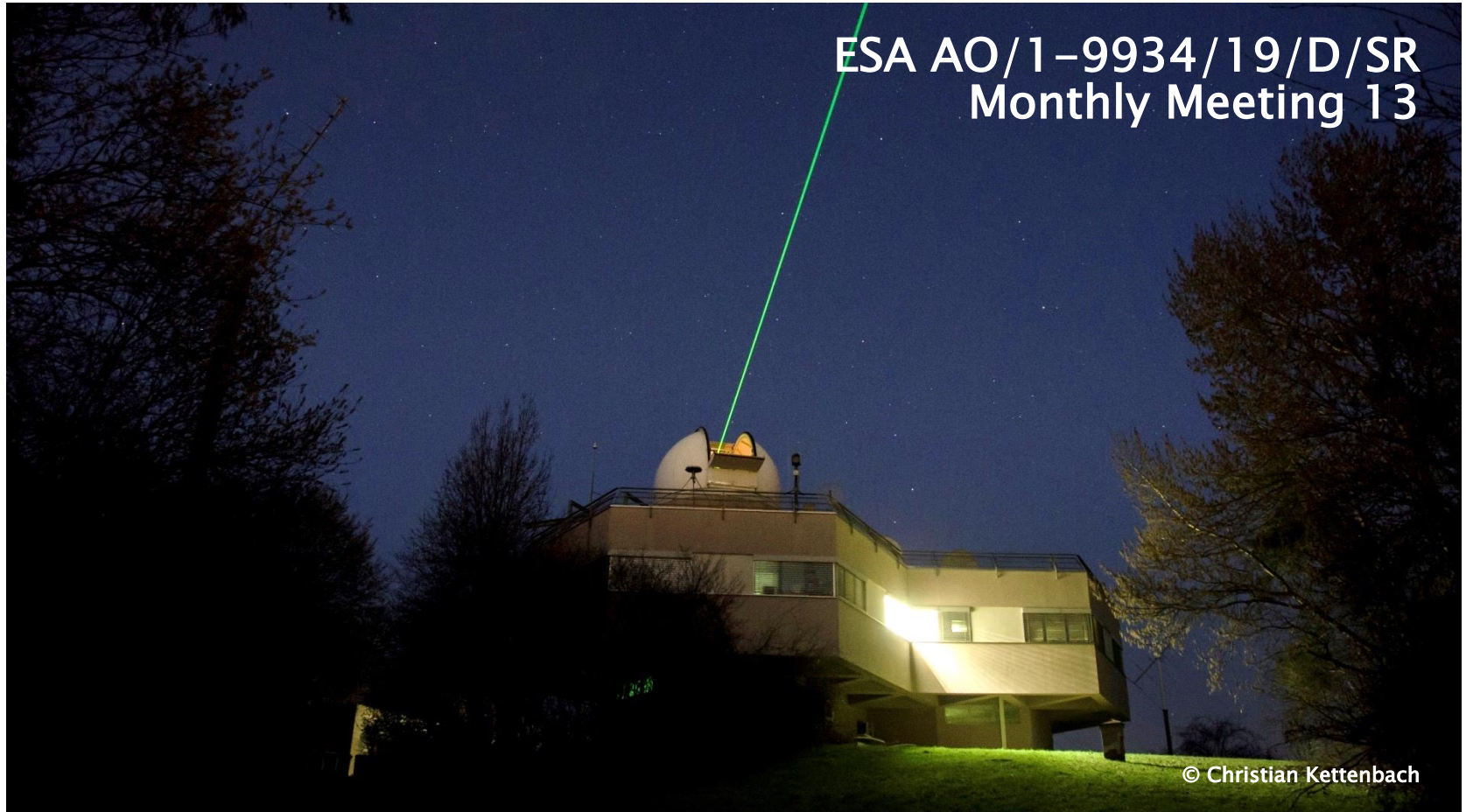
```
1 import pyIOTA
2 import os
3
4 # creates a configuration
5 setup = pyIOTA.get_basic_job_setup()
6
7 # change path to geometry in basic setup
8 setup["geometry"] = os.path.abspath( "/home/user/path/to/geom.obj" )
9
10 # run job with setup
11 job = pyIOTA.run(setup)
12
13 # extract results from job
14 time = pyIOTA.data_processing.get_simulation_time(job)
15 angular_momentum = pyIOTA.data_processing.get_angular_momentum(job)
16
17 # plot
18 pyIOTA.plot.ground_track( job )
```

Thank you for listening

Any questions?

TUMBLING MOTION ASSESSMENT FOR SPACE DEBRIS OBJECTS

ESA AO/1-9934/19/D/SR
Monthly Meeting 13



Michael Steindorfer, Daniel Kucharski, Franz Koidl, Georg Kirchner, Peiyuan Wang
Institut für Weltraumforschung, Österreichische Akademie der Wissenschaften

OBSERVATION CAMPAIGN OVERVIEW

- 1) Historic data: 2018-2020, especially from former ILRS targets
- 2) Pre selection campaign 2020: analysing current rotation behavior and „quality“ of light curves
- 3) Based on 2): reduction of target catalog to „object of interest“ catalog: 26 targets
- 4) Graz extensive observation campaign of 26 targets mainly performed in 2021
- 5) Few targets selected for joint tracking campaign with different sensors

Target	NORAD ID	Graz ID
Jason 2	33105	JA2
Envisat	27386	ENV
SL-16 rocket body	25407	335

OBJECTS OF INTEREST LARGE LIST

Orbit type	Number
<1500 km LEO objects	6
1500 – 3000 km LEO	2
MEO	5
MEO elliptical	1
GTO	3
GEO	3
Special Interest / LEO	3

Eccentricity	Number
< 0.02	18
> 0.5	5

Inclination	Number
80-100 °	6
60-80°	5
40-60°	6
20-40°	0
0-20°	6

Category	NORAD	CospaID	Name	Photometric pattern periodicity [s]	Observation techniques			Attitude determination methods			Launch date	Apogee Perigee			Orbital		
					Optical imaging	Hypertemporal photon counting	Laser ranging	Amplitude methods	Epoch methods	Laser range analysis		RCS [m2]	altitude [km]	altitude [km]	Eccentricity	Inclination [deg]	period [h]
1 LEO < 1500 km, circular, DEB	38345	2012-025F	h-2a_deb	40.0	Y	Y	Y	N	Y	Y	17-May-2012	2.7	628	621	0.0005	98.2	1.6
2 LEO < 1500 km, circular, DEB	38346	2012-025G	h-2a_deb	110.0	Y	Y	Y	N	Y	Y	17-May-2012	2.2	616	609	0.0005	98.2	1.6
3 LEO < 1500 km, circular, R/B	25407	1998-045B	sl-16_r_b	30.0	Y	Y	Y	Y	Y	Y	28-Jul-1998	11.3	836	823	0.0008	71.0	1.7
4 LEO < 1500 km, circular, P/L	18187	1987-060A	cosmos_1867	90.0	Y	Y	Y	N	Y	Y	10-Jul-1987	3.3	803	778	0.0017	65.0	1.7
5 LEO < 1500 km, circular, P/L	33105	2008-032A	jason_2	19.8	Y	Y	Y	N	Y	Y	20-Jun-2008	3.2	1310	1298	0.0008	66.0	1.9
6 LEO < 1500 km, circular, P/L	18123	1987-053A	dmsp_5d-2_f8_usa_26	9.2	Y	Y	Y	N	Y	Y	9-Jun-1905	3.0	827	809	0.0013	98.7	1.7
7 LEO 1500-3000 km, P/L	25770	1999-031A	globalstar_m025	2.5	Y	Y	N	N	Y	N	10-Jun-1999	2.3	2045	2043	0.0002	52.0	2.1
8 LEO 1500-3000 km, P/L	26083	2000-008C	globalstar_m060	28.4	Y	Y	N	N	Y	N	8-Feb-2000	2.0	1612	1604	0.0005	52.0	2.0
9 MEO, R/B	39167	2013-023B	atlas_5_centaur_r_b	6.7	Y	Y	N	Y	Y	N	15-May-2013	0.5	21421	20554	0.0158	56.2	12.5
10 MEO, R/B	40535	2015-013B	delta_4_r_b	209.0	Y	Y	N	Y	Y	N	7-Jul-1905	26.2	20960	20785	0.0032	52.2	12.4
11 MEO, R/B, highly elliptical	44867	2019-090D	cz-3b_r_b	27.5	Y	Y	N	Y	Y	N	16-Dec-2019	?	17767	230	0.5703	55.2	5.3
12 MEO, P/L	23205	1994-050C	cosmos_2288_glonass	9.2	Y	Y	Y	N	Y	Y	11-Aug-1994	3.9	19133	19124	0.0002	65.0	11.3
13 MEO, P/L	23833	1996-019A	navstar_37_usa_117	8.9	Y	Y	N	N	Y	N	28-Mar-1996	4.0	20747	20697	0.0009	53.5	12.3
14 MEO, P/L	28112	2003-056A	cosmos_2404_glonass	125.0	Y	Y	Y	N	Y	Y	10-Dec-2003	3.4	19235	19027	0.0041	63.7	11.3
15 GTO, R/B	20947	1990-100C	ariane_42p_r_b	11.9	Y	Y	N	Y	Y	N	20-Nov-1990	28.4	21739	278	0.6172	5.1	6.3
16 GTO, R/B	25405	1998-044B	cz-3b_r_b	26.6	Y	Y	N	Y	Y	N	18-Jul-1998	21.9	35729	607	0.7154	17.5	10.6
17 GTO, R/B	38094	2012-009B	atlas_5_centaur_r_b	20.8	Y	Y	N	Y	Y	N	24-Feb-2012	17.7	35203	3265	0.6235	19.1	11.3
18 GEO, P/L	15484	1985-007A	gorizont_11	24.3	Y	Y	N	N	Y	N	18-Jan-1985	5.0	35796	35772	0.0003	9.8	23.9
19 GEO, P/L	22653	1993-031A	astra_1c	190.0	Y	Y	N	N	Y	N	15-Jun-1905	14.1	36230	36156	0.0009	12.5	24.3
20 GEO, P/L, highly elliptical	23230	1994-056A	ets_6	11.1	Y	Y	N	N	Y	N	28-Aug-1994	38.6	38794	8447	0.5058	16.7	14.4
21 Special Interest Object	23560	1995-021A	ers_2	?	Y	Y	Y	N	Y	Y	21-Apr-1995	9.5	487	480	0.0005	98.6	1.6
22 Special Interest Object	24277	1996-046A	adeos	?	Y	Y	Y	N	Y	Y	17-Aug-1996	22.2	799	796	0.0002	98.9	1.7
23 Special Interest Object	27386	2002-009A	envisat	?	Y	Y	Y	N	Y	Y	1-Mar-2002	18.6	767	766	0.0001	98.1	1.7

GRAZ LARGE CAMPAIGN STATISTICS

Graz campaign data summary including historic data

- 2018 - 2022
- Data uploaded on servers
- Light curves from all of the large catalog targets
- Best tracking coverage: Jason 2 -> ~ 200 LC + 600 SLR passes

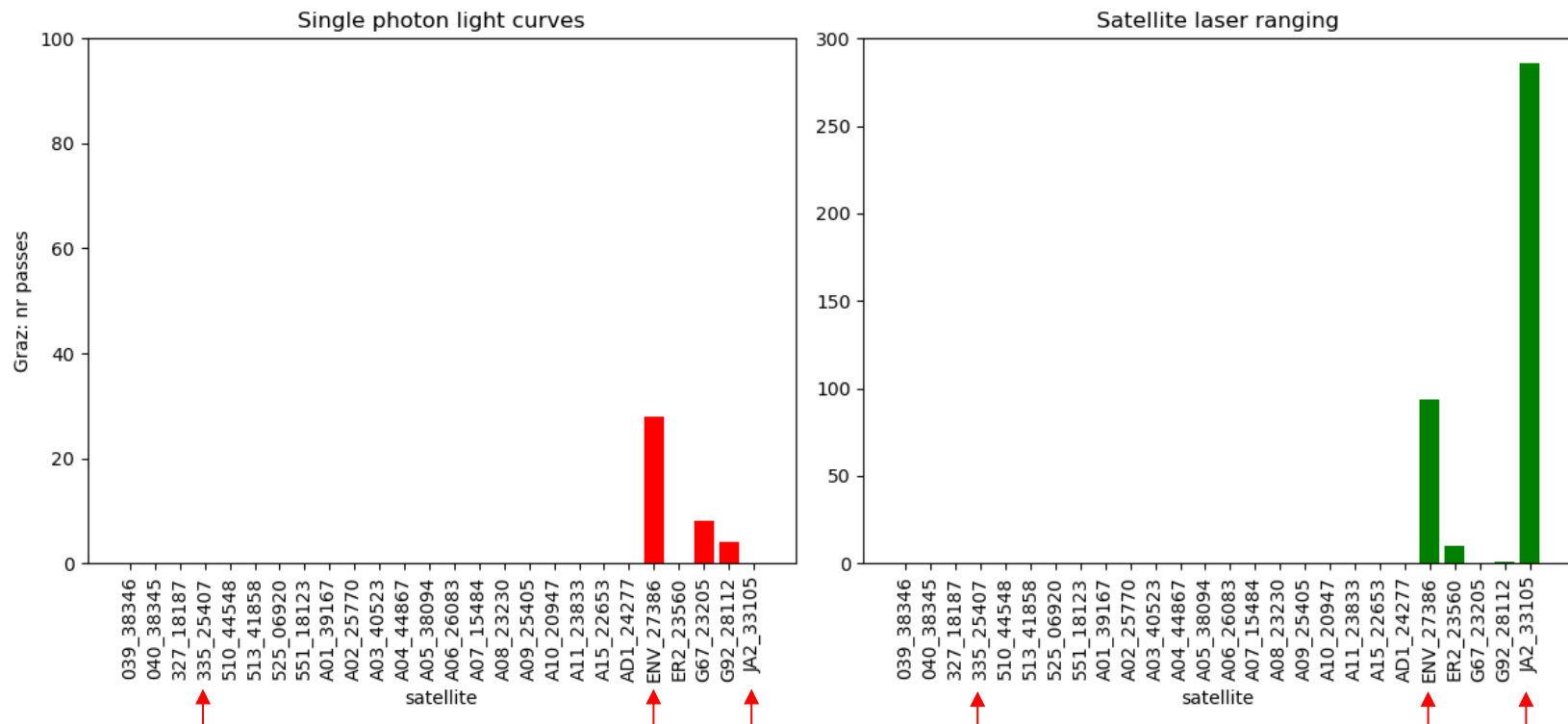
Year	LC	SLR	SDLR	Comment
2018	40	391	11	Historic data, mainly JA2, ENV, ER2, G67, G92
2019	47	278	14	Historic data, mainly JA2, ENV, ER2, G67, G92
2020	75	41	22	Mainly historic data
2021	421	102	24	Campaign data
2022	82	79	4	Some targets of interest continued, JA2, ENV

STATISTICS 2018

Historic data: mainly

- Envisat, Jason-2, ERS-2, 2 Glonass satellites

2018: Graz priority target: Tracking statistics
LC: 40, SLR: 391, SDLR: 11



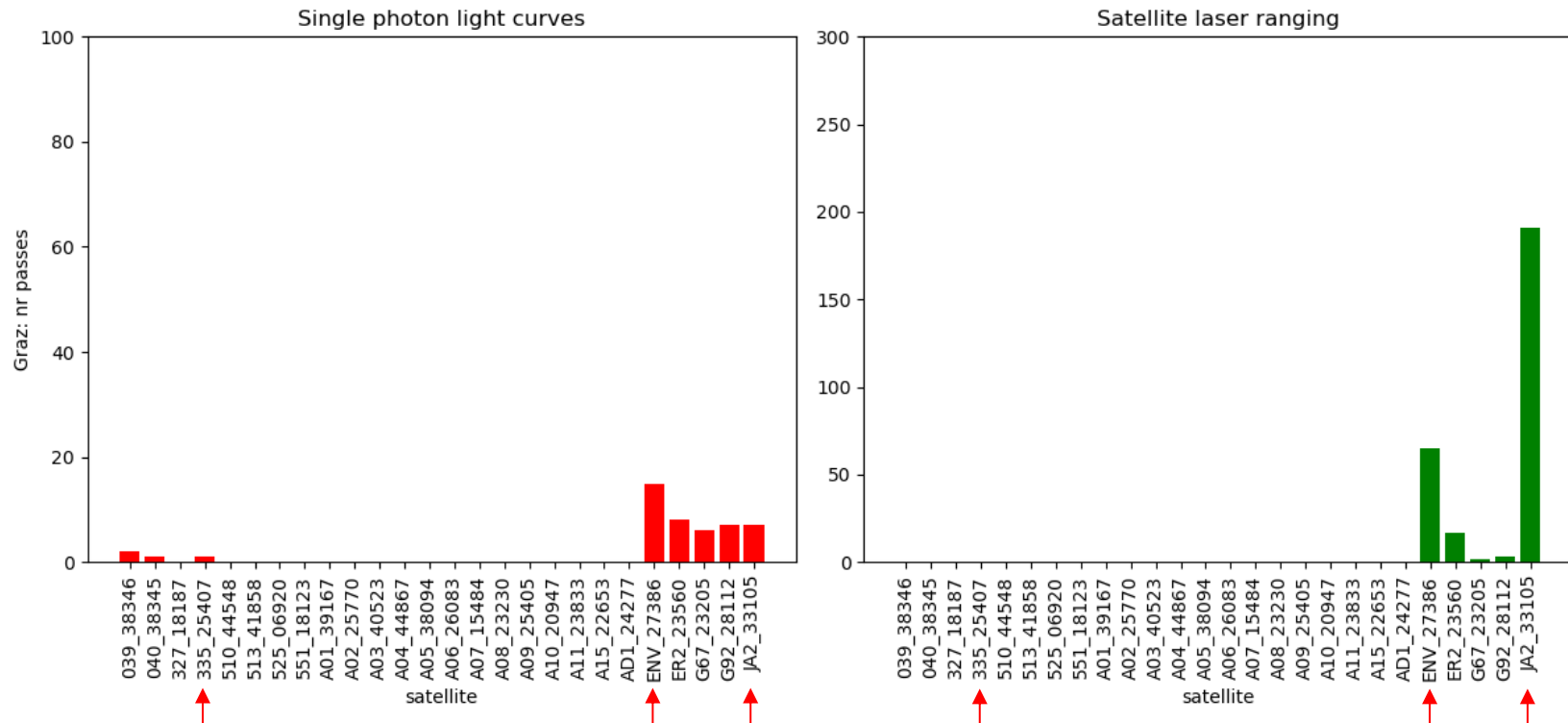


STATISTICS 2019

Historic data: mainly

- Envisat, Jason-2, ERS-2, 2 Glonass satellites

2019: Graz priority target: Tracking statistics
LC: 47, SLR: 278, SDLR: 14

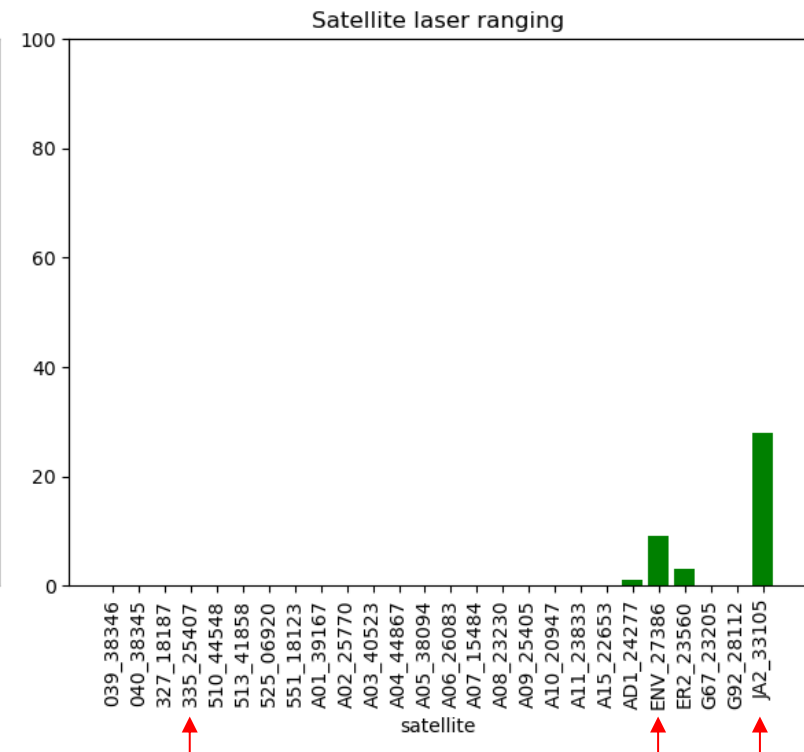
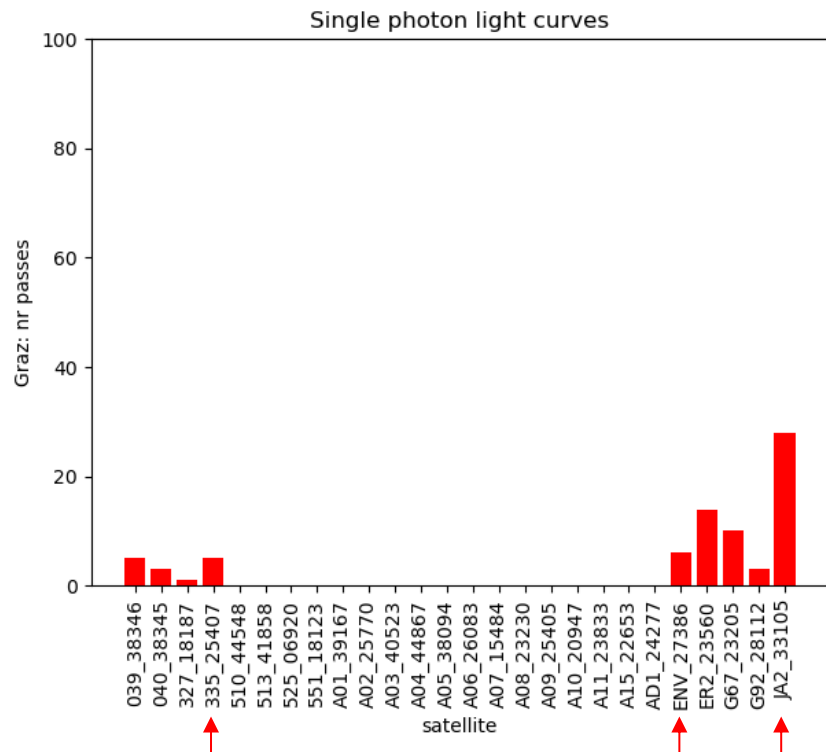


STATISTICS 2020

Historic data: mainly

- Envisat, Jason-2, ERS-2, 2 Glonass satellites

2020: Graz priority target: Tracking statistics
LC: 75, SLR: 41, SDLR: 22

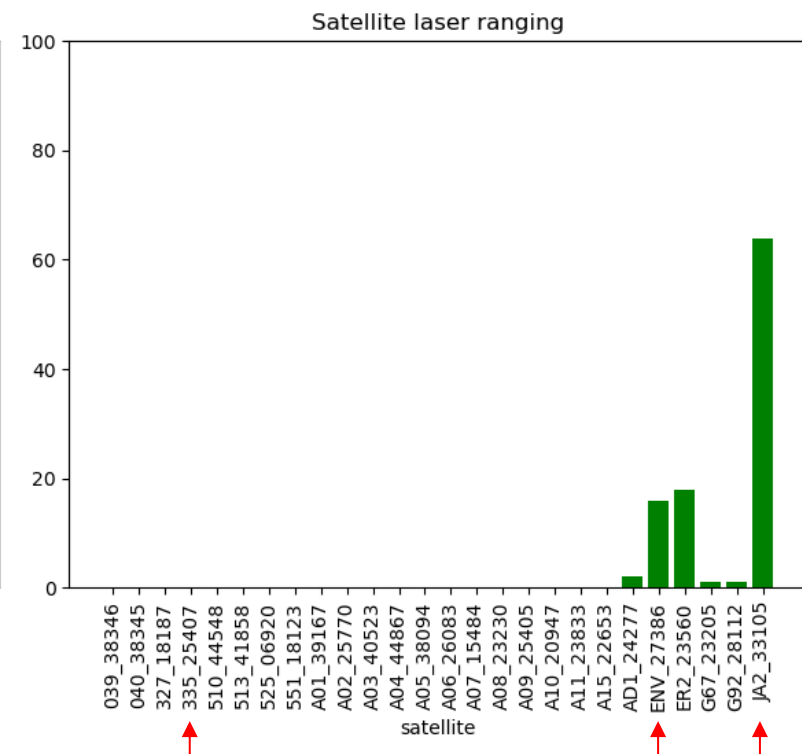
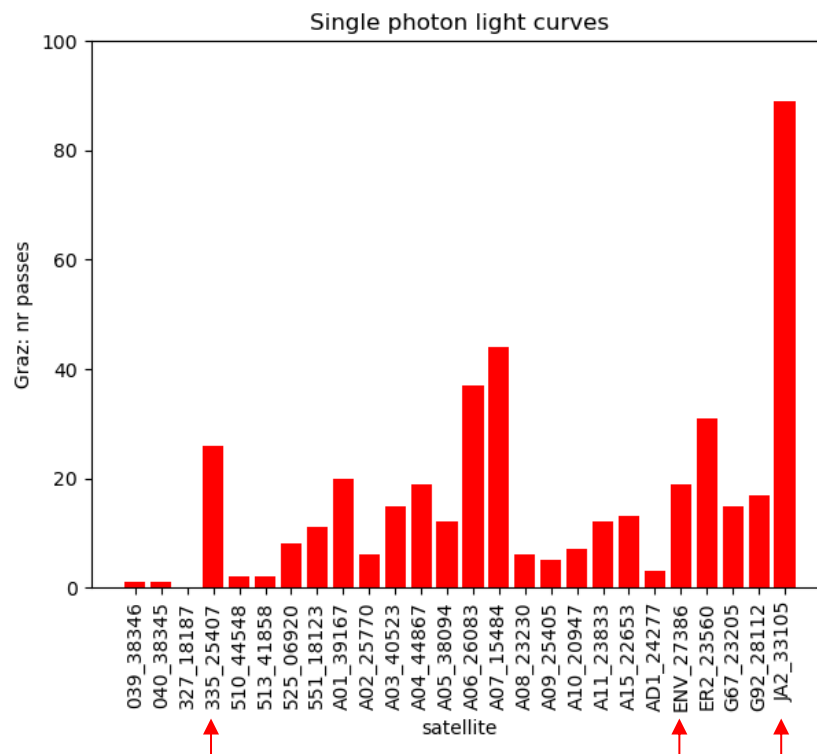


STATISTICS 2021

Observation campaign

- Mainly SLR + LC
- 421 light curves, 102 SLR, 24 SDLR

2021: Graz priority target: Tracking statistics
LC: 421, SLR: 102, SDLR: 24

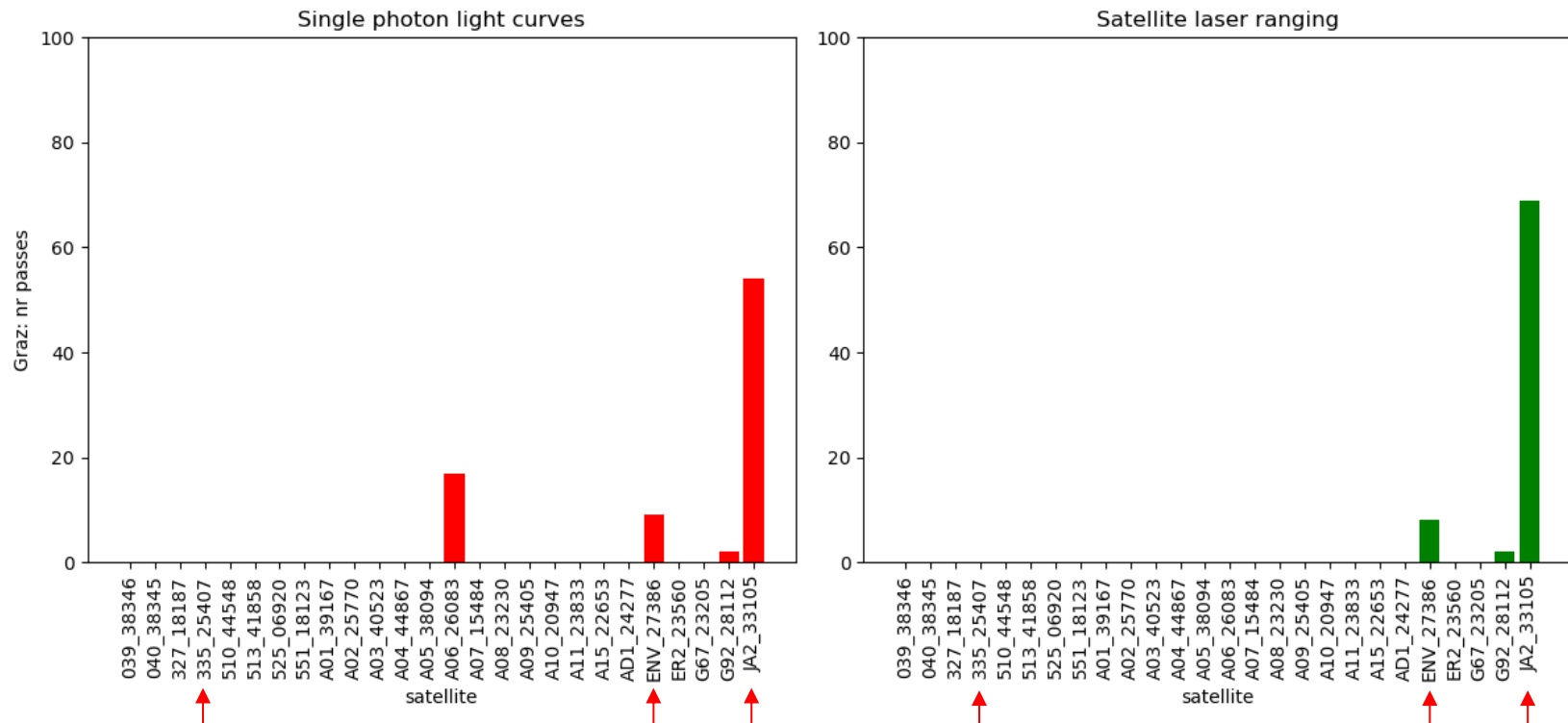


STATISTICS 2022

Some targets still monitored by IWF

- Jason-2, Envisat

2022: Graz priority target: Tracking statistics
LC: 82, SLR: 79, SDLR: 4



JOINT TRACKING CAMPAIGN

Selected targets:

- Jason-2
- Envisat
- SL-16 rocket body

Summary:

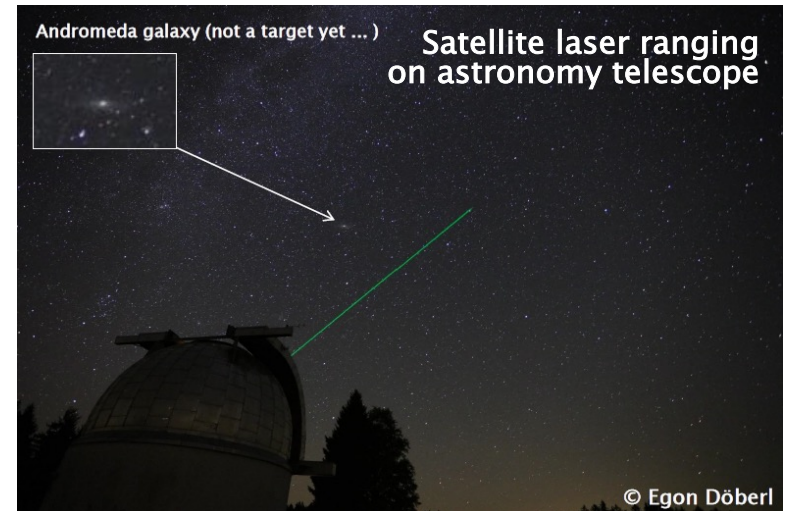
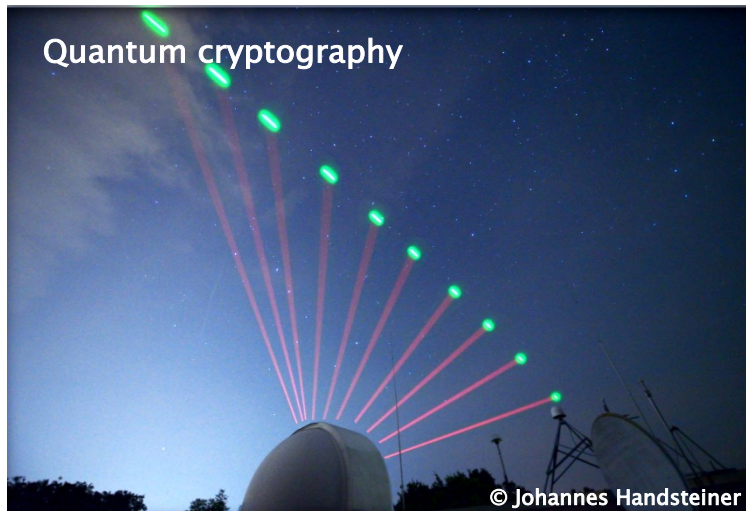
- Observation sessions including one TIRA pass, maximum 24 hours / session
- Each session: Graz, Zimmerwald collects data of all possible passes, TIRA collects one radar pass
- Simultaneous tracking not necessary but welcome
- Observation schedules were distributed to partners

JOINT TRACKING CAMPAIGN

Preferred observation characteristics: sensor

	TIRA	Graz	Zimmerwald
Available or most efficient operation time (in terms of SNR)	Daytime	After dusk	After dusk
Clear sky needed	No	Yes	Yes
Satellite visibility	Not required	Sunlit	Sunlit
Tracking topocentric elevation range (avoid tracking through keyhole)	5° - 85°	5° - 85°	5° - 85°
Closest approach topocentric elevation angle	≥ 50°	≥ 30°	≥ 30°
Minimum Moon distance (topocentric angle) to avoid optical noise	0°	5° at night	5° at night
Sat. closest approach position fully sunlit? (to assure for longer visibility of the target)	No	Preferred	Preferred
Data generated	Imaging radar RCS	Satellite laser ranging Space debris laser ranging Single photon light curve	Light curve Satellite laser ranging

! DANKE FÜR IHRE AUFMERKSAMKEIT !





 **Fraunhofer**
FHR

Fraunhofer Institute for
High Frequency Physics and
Radar Techniques FHR

D. Cerutti-Maori, C. Carloni, J. Rosebrock, I. Maouloud

Radars observations with the TIRA system



01

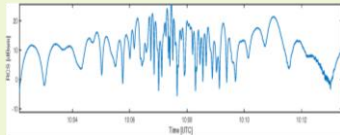
Tracking and Imaging Radar TIRA

Tracking and Imaging Radar (TIRA)

Different kinds of measurements for attitude motion estimation

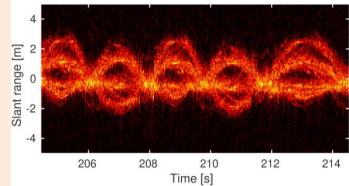
L-band tracking radar

RCS curve

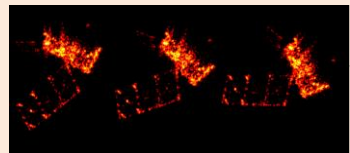


Ku-band imaging radar

RTI plot



Series of ISAR images

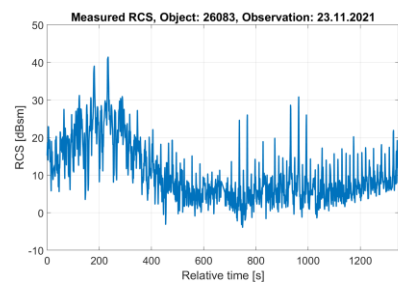
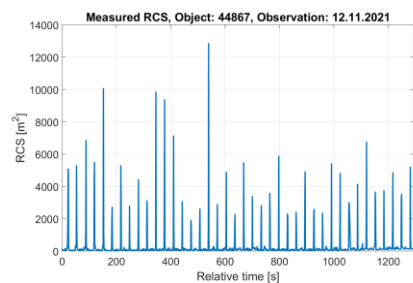


Tracking and Imaging Radar (TIRA)

Different kinds of measurements for attitude motion estimation

RCS curve (L-band tracking radar)

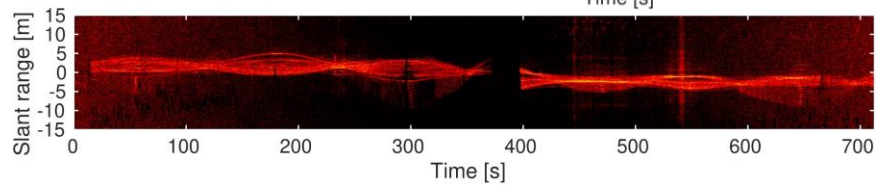
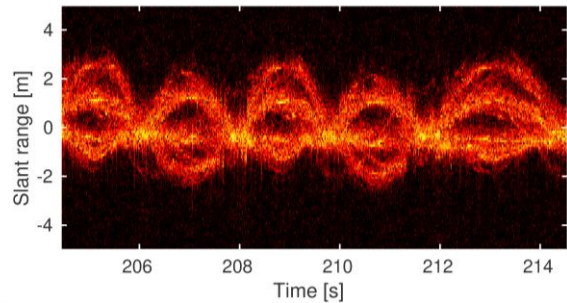
- Estimation of the apparent angular velocity through spectral analysis
- **Applicability**
 - At least 2 periods have to be observed
 - Works well for (fast) tumbling objects with single rotation axis



Tracking and Imaging Radar (TIRA)

Different kinds of measurements for attitude motion estimation

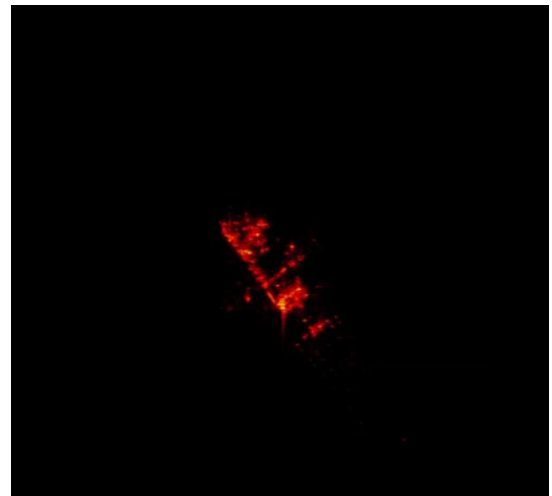
- RCS curve (L-band tracking radar)
- **RTI plot (Ku-band imaging radar)**
 - Estimation of the apparent angular velocity through spectral analysis
- **Applicability**
 - At least 1 period has to be observed
 - Works well for (fast) tumbling objects



Tracking and Imaging Radar (TIRA)

Different kinds of measurements for attitude motion estimation

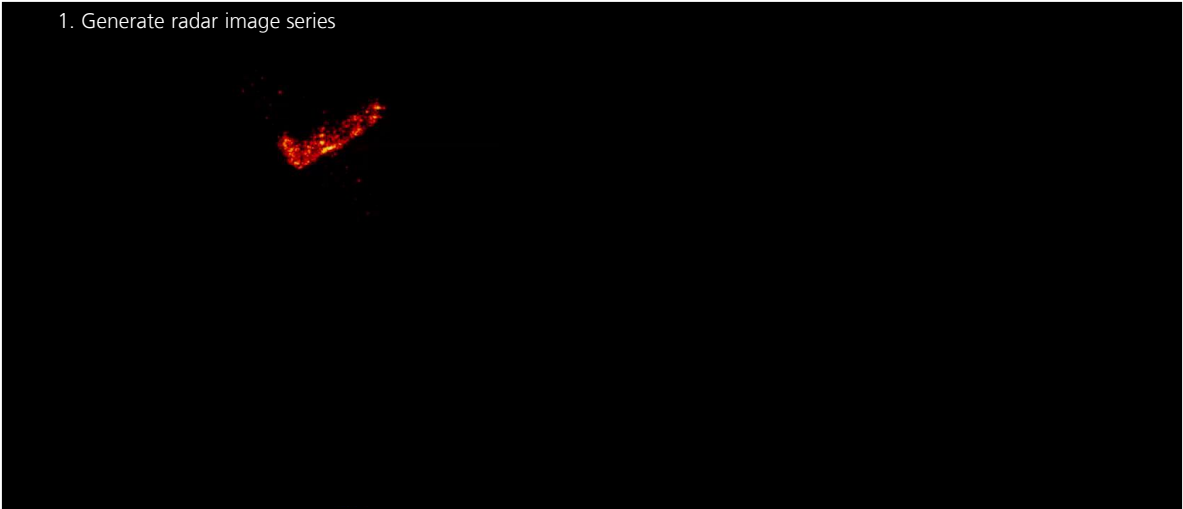
- RCS curve (L-band tracking radar)
- RTI plot (Ku-band imaging radar)
- **Series of ISAR images (Ku-band imaging radar)**
 - Estimation of the velocity vector (angular velocity and spin axis) and of the initial state through a ML approach
- **Applicability**
 - Slow tumbling objects (to avoid under-sampling issues in the ISAR images)
 - Assumes a constant rotational velocity vector during the observation time (or at least a part of it)



Estimation of the attitude motion of space objects

Ku-band imaging radar

1. Generate radar image series



Conducted measurements

L-band tracking radar and Ku-band imaging radar

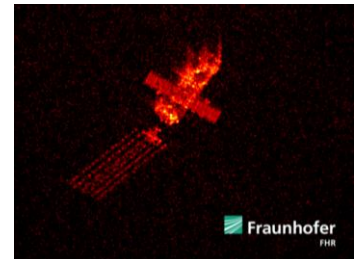
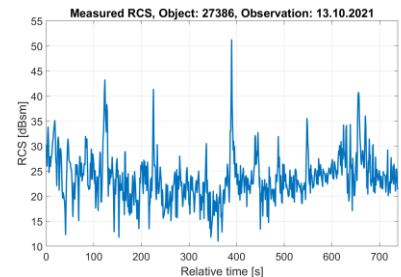
- TRKI (13.10): 02009a, 27386, Envisat
- TRKI (12.11): 02009a, 27386, Envisat
- TRKI (13.10): 08032a, 33105, Jason-2
- TRKI (29.10): 19063b, 44548, R/B CZ-2D
- TRK (12.11): 19090d, 44867, R/B CZ-3B (no imaging data)
- TRKI (23.11): 00008c, 26083, Globalstar M060

Conducted measurements

TRKI (13.10 / 12.11): Envisat



- L-band signature
 - Complex tumbling object
 - No clear periodicity could be observed
 - The attitude motion of Envisat changed and does not follow the observed decreasing rotational velocity with stable spin axis observed in the past years (2013-2017)
- ISAR image
 - No external damage could be observed
 - The change in the attitude motion is not caused by a major collision

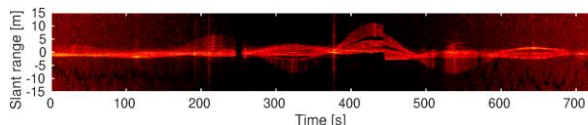


Conducted measurements

TRKI (13.10 / 12.11): Envisat

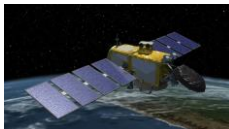


- RTI plot
 - Time between two nodes is approximately 110 s
 - Object rotates around 180 deg during this time
 - Apparent angular velocity is around 1.6 deg/s
- ISAR image series and WGM
 - The best match is achieved with the rotational velocity vector [0.5; 1.07; 1.2] deg/s (ECI J2000); estimated angular velocity: 1.68 deg/s
 - Decreasing match with the projected WGM towards the end of the passage
 - Complex tumbling motion, assumption of constant rotational velocity vector is not fulfilled

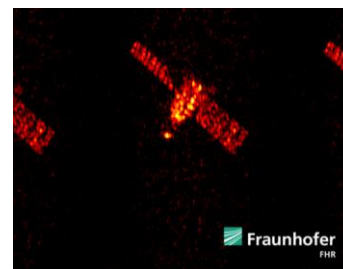
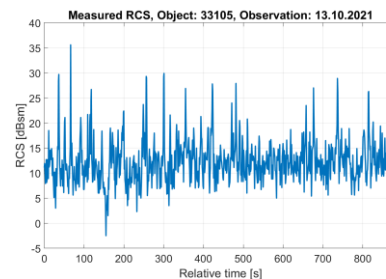


Conducted measurements

TRKI (13.10): Jason-2



- L-band signature
 - Complex tumbling object, no clear periodicity could be observed
- ISAR image
 - No external damage could be observed
 - Solar panel planes appear to be orthogonal to each other
 - The image was produced assuming a rotational velocity of 9 deg/s about an axis orthogonal to the orbit plane. In the inertial system the assumed rotational velocity vector is [-5.5; 6.1; -3.7] deg/s (ECI J2000)
 - The cross-range scaling of the image confirms this assumption
- RTI plot
 - Apparent angular velocity is around 9 deg/s

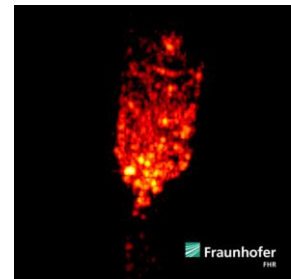
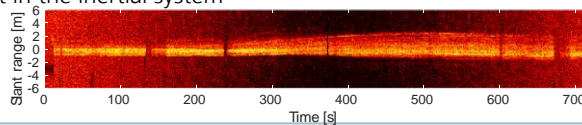
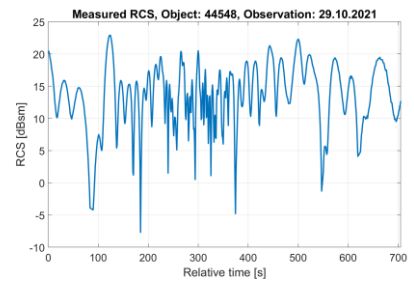


Conducted measurements

TRKI (29.10): R/B CZ-2D



- L-band signature
 - Slow rotating object
 - No periodicity could be observed
- ISAR image
 - No external damage could be observed
 - The image was produced assuming the object to rest in the inertial system
- RTI plot
 - Apparently no rotation
 - Object assumed to rest in the inertial system

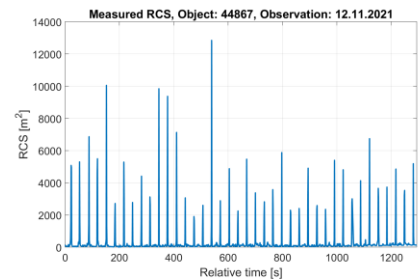


Conducted measurements

TRK (12.11): R/B CZ-3B

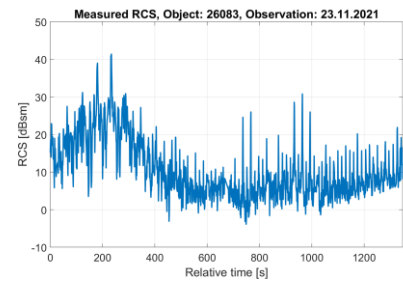


- L-band signature
 - Fast tumbling object
 - Probably single axis motion
 - Apparent spin frequency: $0.031 \text{ Hz} / 2$ due to the object symmetry
 - Apparent rotation period: $32.6 \text{ s} * 2$ due to the object symmetry

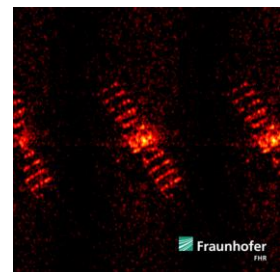


Conducted measurements

TRKI (23.11): Globalstar M060



- L-band signature
 - Fast tumbling object with complex motion
 - Region 1, $t < 600$ s, apparent rotational velocity ~ 6.8 deg/s
 - Region 2, $t > 600$ s, apparent rotational velocity ~ 13 deg/s
- ISAR image
 - Undersampling in the cross-range direction causes aliasing
 - The image was produced at relative time 480 s assuming a rotational velocity of 6 deg/s with rotation axis orthogonal to the orbit plane ($[-4.7; -0.8; -3.7]$ deg/s, ECI J2000)
- RTI plot
 - Very faint maximal range spans at approx. 448 s and 508 s lead to an apparent rotational velocity of 6 deg/s confirming the order of magnitude of the L-band result



Thank you for your attention! Questions?



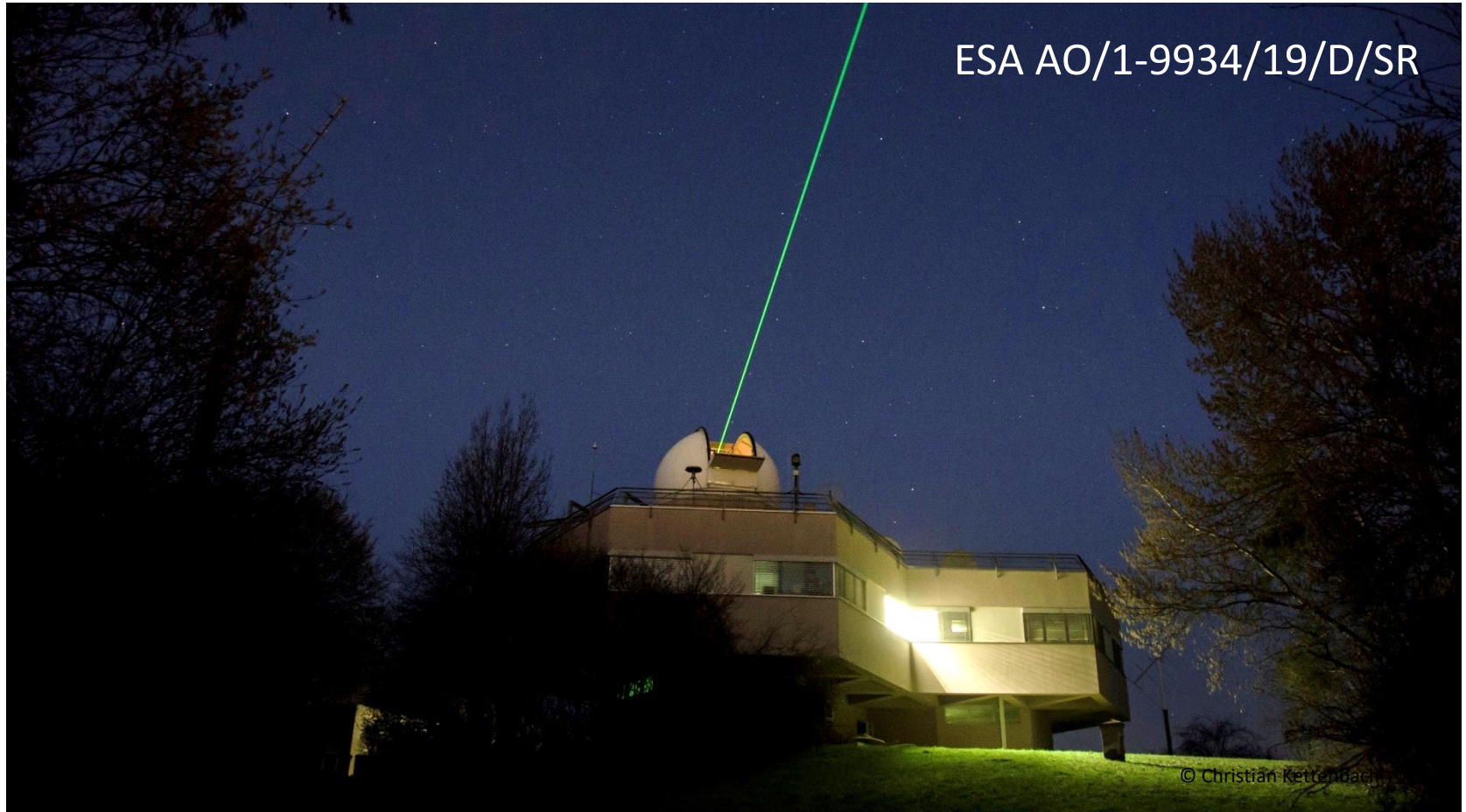


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Tumbling Motion Assessment for Space Debris Objects



Daniel Kucharski, Michael Steindorfer, Franz Koidl, Georg Kirchner, Peiyuan Wang
 Institut für Weltraumforschung, Österreichische Akademie der Wissenschaften

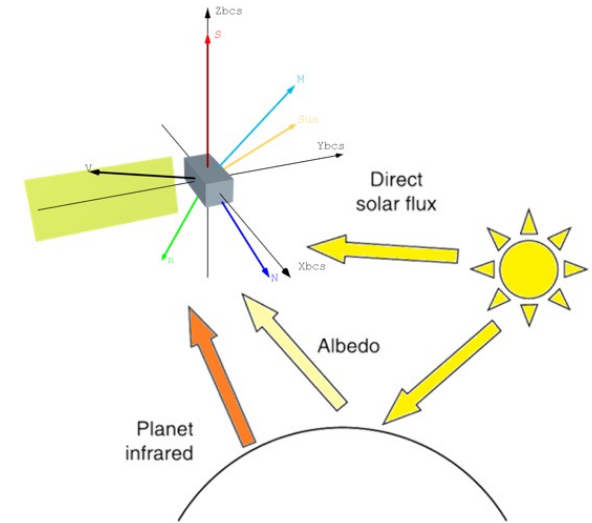
Environmental characteristics

The selection process of objects-of-interest was based on environmental characteristics analysis.

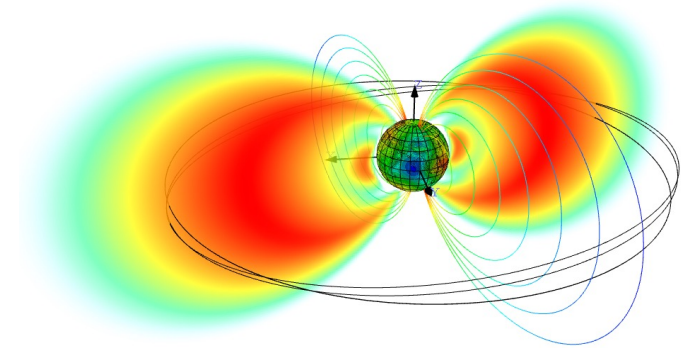
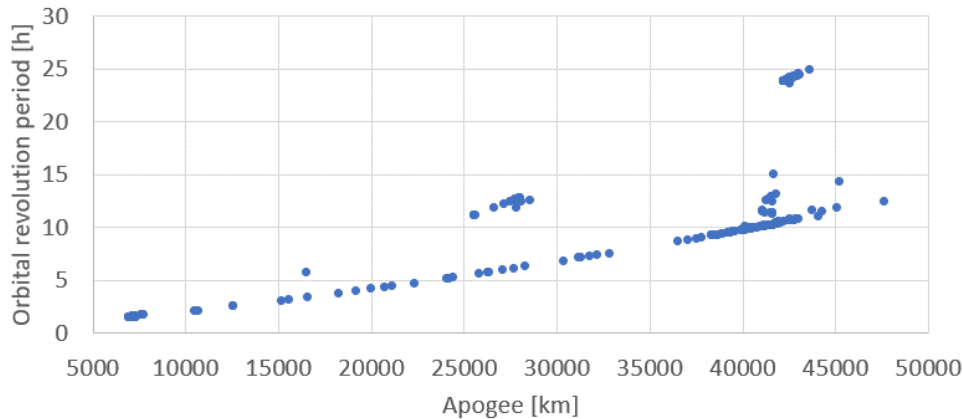
Estimation of the environmental forces and torques is not exactly possible without knowing how an artificial object interacts with the external factors, but we can characterize environmental phenomena that lead to and affect the tumbling motion.

The environmental forces and torques can be divided into separate categories:

1. Monopolar gravitational field, the force vector changes once per orbital revolution.
2. Bipolar magnetic field, force vector changes twice per revolution.
3. Directional solar irradiation flux, charged particle flux, residual atmospheric particles. The flux direction can change periodically.



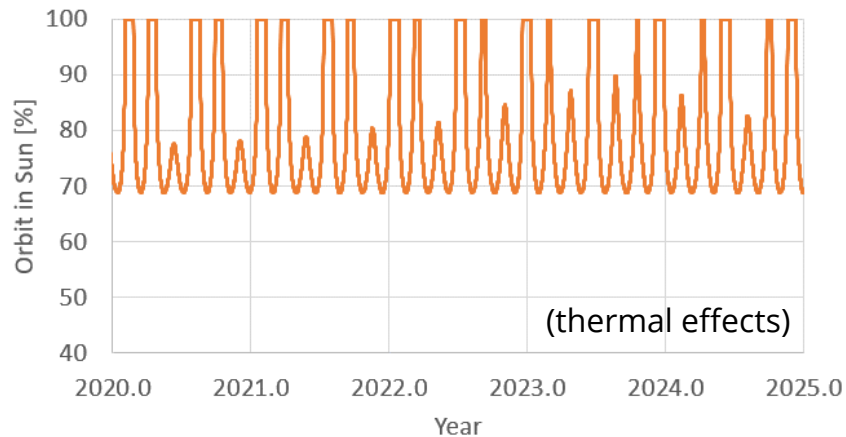
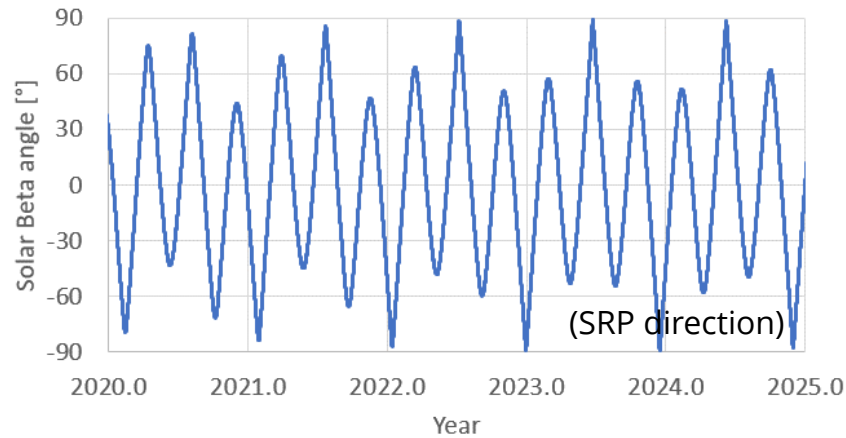
Objects-of-Interest



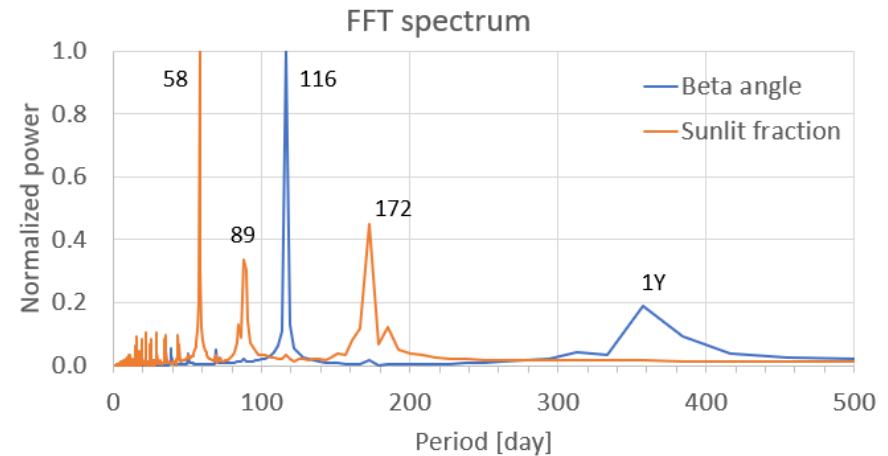
Environmental characteristics

Periodical effects on spinning Jason-2

- Use TLE to simulate orbital configuration at constant time step (0.1 day)
- Apply FFT for identification of the periodical signals in the simulated time series
- The change of the orbital configuration wrt the Sun stimulates tumbling motion



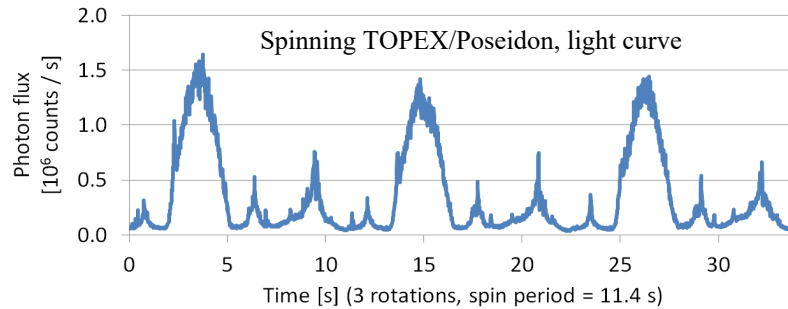
All ~300 objects-of-interest is processed and characterized in terms of expected periodicity in the tumbling motion due to the changes in the orbital configuration.



Satellite attitude observation

High rate laser and optical tracking allows for accurate attitude measurement of the active and passive satellites.

Graz SLR station simultaneously performs laser ranging and light curve measurements (since 2015).

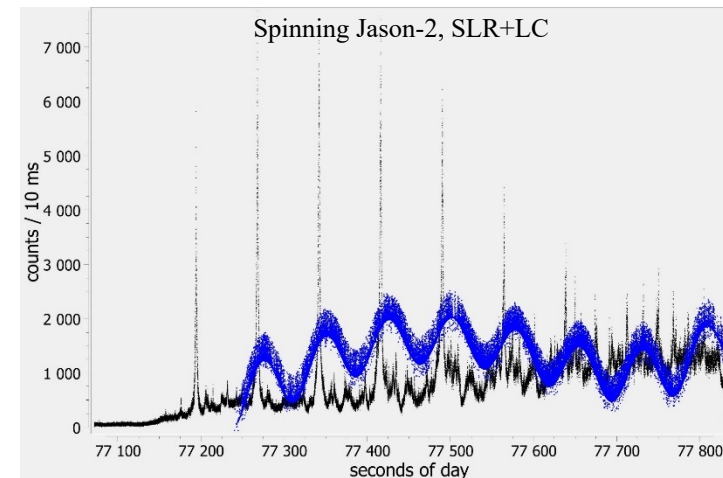
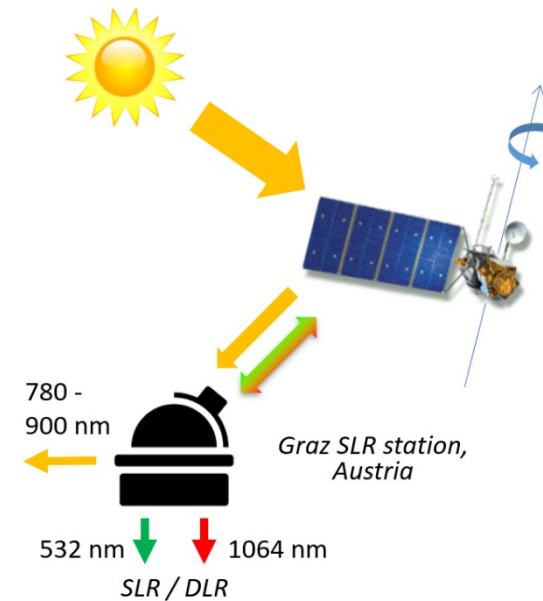


The hypertemporal light curves measured at the single photon resolution present a high level of details due to the fact that the photon counters do not integrate the incoming optical signal (as opposed to the CCD technology).

The high-rate SLR allows for the mm-accuracy absolute range measurements that reveal motion of the individual corner cube retroreflectors wrt the satellite CoM.

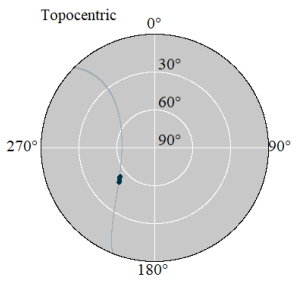
The pattern of the data distribution depends on:

- LC: the angular configuration Sun-Satellite-Observer
- SLR: the range between the Satellite and Observer optical reflection and reference points

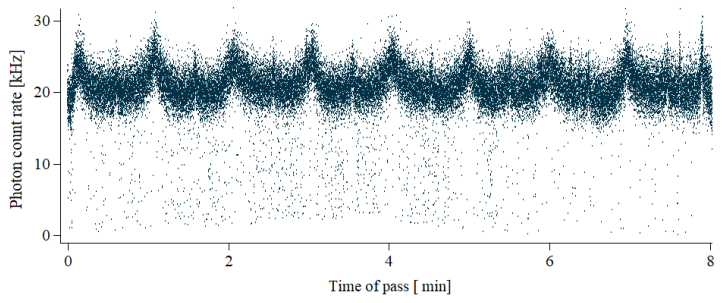


M. Steindorfer

Object: COSMOS 2276 [GLONASS] Result summary
 NORAD: 23043
 Cospar: 1994-021a
 Launch: 11 April, 1994
 Apogee alt.: 19180 km
 Perigee alt.: 19081 km
 Inclination: 64.8°
 Orb. period: 11.3 h
 RCS: 4.1 m2
 Object age: 21.2 y
 Exp. spin status: -
 Mag: -

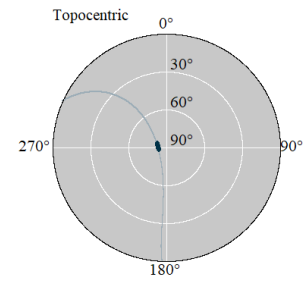


Single-photon light curve (1994-021a), Graz 28 June 2015, 22:31:26

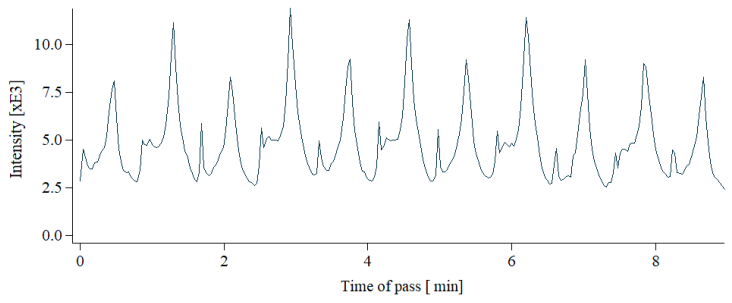


Pass information
 Duration: 8m:02s
 Data points: 47227
 Data rate: 100.0 Hz
 Closest approach
 Topo el: 54.4°
 Slant range: 20025 km
 Phase angle: 55.5°
 Sim. app. mag.: 11.8
 Solar Beta angle: -3.5°

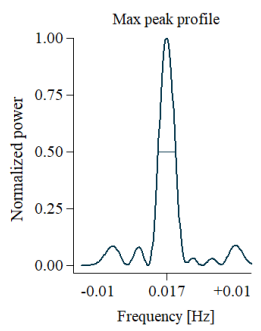
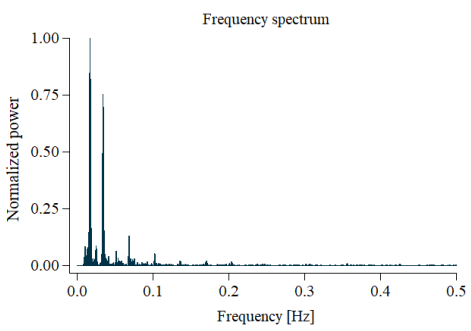
Object: COSMOS 2276 [GLONASS] Result summary
 NORAD: 23043
 Cospar: 1994-021a
 Launch: 11 April, 1994
 Apogee alt.: 19180 km
 Perigee alt.: 19081 km
 Inclination: 64.8°
 Orb. period: 11.3 h
 RCS: 4.1 m2
 Object age: 24.9 y
 Exp. spin status: -
 Mag: -



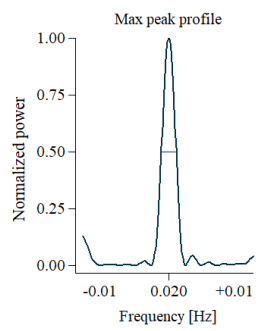
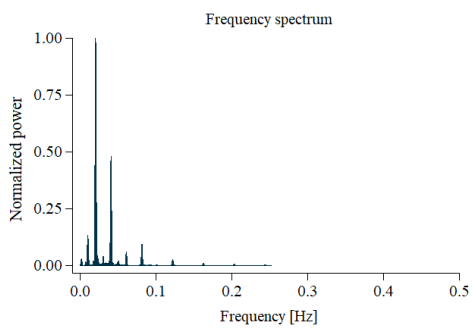
Light curve (1994-021a), Zimmerwald 17 March 2019, 0:29:02



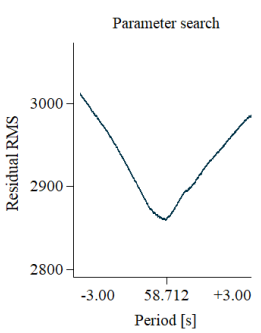
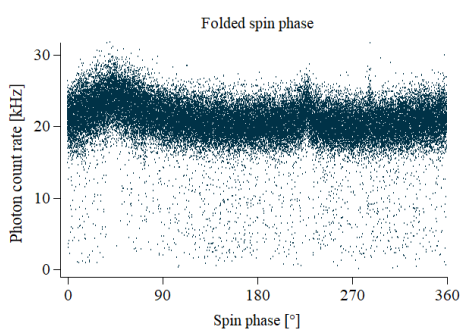
Pass information
 Duration: 8m:58s
 Data points: 269
 Data rate: 0.5 Hz
 Closest approach
 Topo el: 83.8°
 Slant range: 19144 km
 Phase angle: 43.8°
 Sim. app. mag.: 11.5
 Solar Beta angle: 32.2°



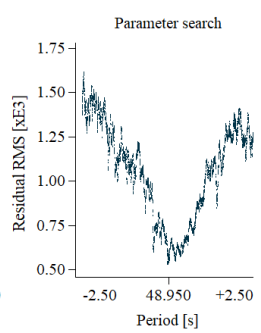
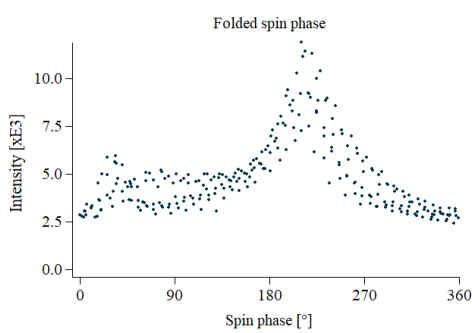
Frequency analysis
 Frequency: 0.017 Hz
 Period: 59.172 s
 FWHM: 1.891 mHz
 SNR: 21.0
 CoVar: 4.751 %



Frequency analysis
 Frequency: 0.020 Hz
 Period: 49.261 s
 FWHM: 1.696 mHz
 SNR: 28.2
 CoVar: 3.548 %



PDM
 Period: 58.712 s

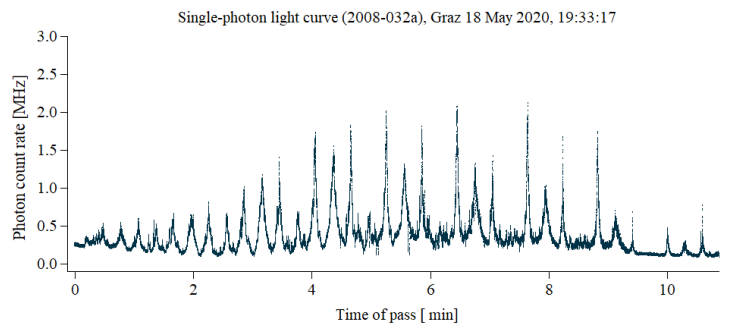
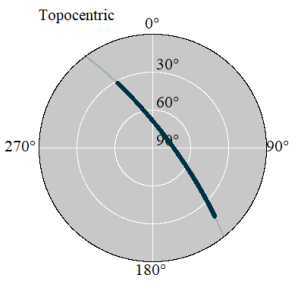


PDM
 Period: 48.950 s

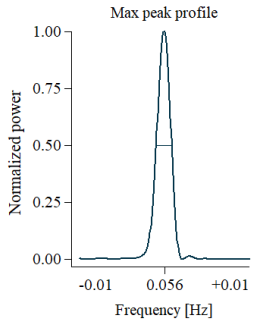
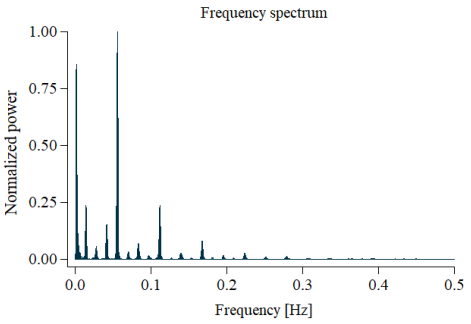
Object: **JASON-2 (OSTM)**
 NORAD: 33105
 Cospar: 2008-032a
 Launch: 20 June, 2008
 Apogee alt.: 1317 km
 Perigee alt.: 1305 km
 Inclination: 66.0°
 Orb. period: 1.9 h
 RCS: 3.2 m2
 Object age: 11.9 y
 Exp. spin status: aperiodic
 Mag: 0.34 .. 8.35, 5.70, se 0.77

Result summary
 Frequency analysis
 Period: 17.921 s
 Spin phase folding
 Period: 17.917 s

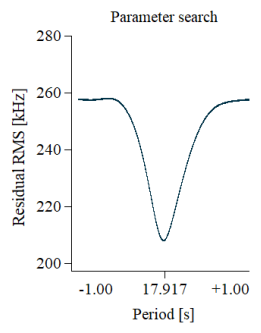
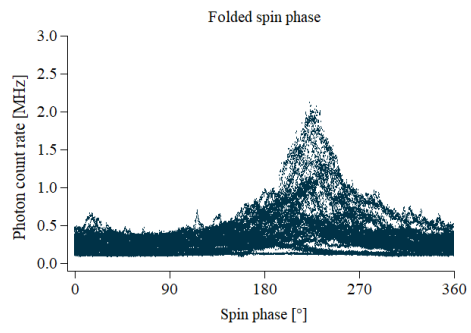
x4=71.6 s



Pass information
 Duration: 10m:53s
 Data points: 65306
 Data rate: 100.0 Hz
 Closest approach
 Topo el: 76.6°
 Slant range: 1352 km
 Phase angle: 78.5°
 Sim. app. mag.: 6.4
 Solar Beta angle: -6.2°



Frequency analysis
 Frequency: 0.056 Hz
 Period: 17.921 s
 FWHM: 1.657 mHz
 SNR: 79.3
 CoVar: 1.261 %

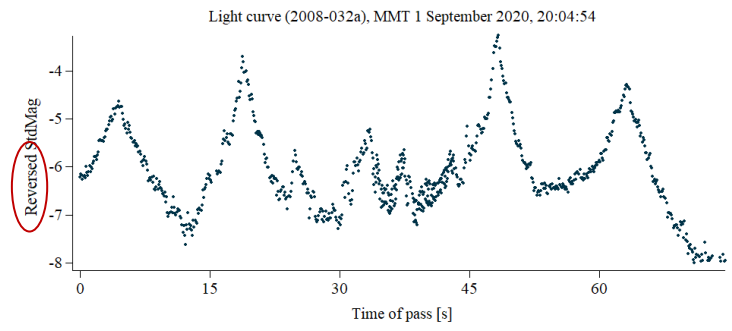
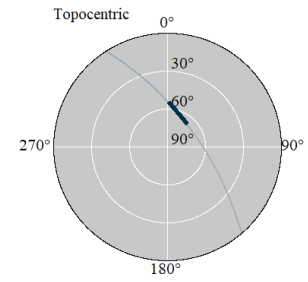


PDM
 Period: 17.917 s

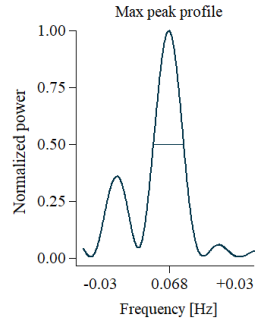
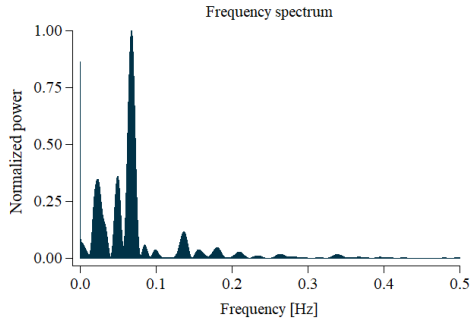
Object: JASON-2 (OSTM)
 NORAD: 33105
 Cospar: 2008-032a
 Launch: 20 June, 2008
 Apogee alt.: 1317 km
 Perigee alt.: 1305 km
 Inclination: 66.0°
 Orb. period: 1.9 h
 RCS: 3.2 m2
 Object age: 12.2 y
 Exp. spin status: aperiodic
 Mag: 0.34 .. 8.35, 5.70, se 0.77

Result summary
 Frequency analysis
 Period: 14.815 s
 Spin phase folding
 Period: 14.764 s

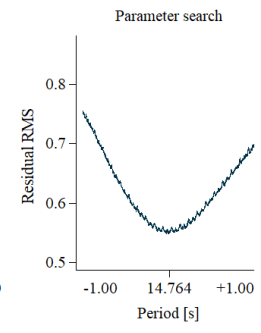
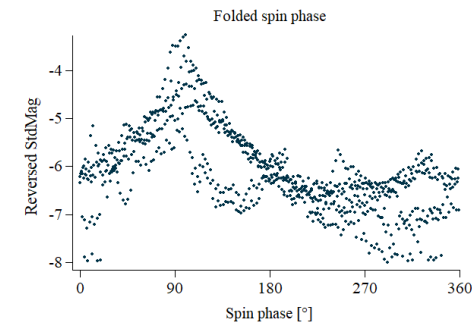
x4=59.2 s



Pass information
 Duration: 1m:15s
 Data points: 830
 Sampling rate: 10.0 Hz
 Closest approach
 Topo el: 66.0°
 Slant range: 1421 km
 Phase angle: 68.0°
 Sim. app. mag.: 999.0
 Solar Beta angle: 17.1°



Frequency analysis
 Frequency: 0.068 Hz
 Period: 14.815 s
 FWHM: 10.521 mHz
 SNR: 15.1
 CoVar: 6.619 %



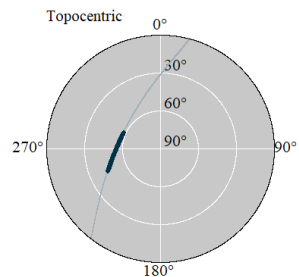
PDM
 Period: 14.764 s

Object: **NOAA 3**
 NORAD: 6920
 Cospar: 1973-086a
 Launch: 6 November, 1973
 Apogee alt.: 1510 km
 Perigee alt.: 1500 km
 Inclination: 102.1°
 Orb. period: 1.9 h
 RCS: 2.5 m2
 Object age: 46.0 y
 Exp. spin status: periodic, 54.4 s
 Mag: 4.79 .. 10.30, 7.97, se 0.56

Result summary

Frequency analysis
 Period: 25.840 s
 Spin phase folding
 Period: 25.339 s

x2=51.6 s

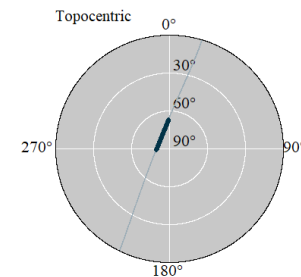


Object: NOAA 3
 NORAD: 6920
 Cospar: 1973-086a
 Launch: 6 November, 1973
 Apogee alt.: 1510 km
 Perigee alt.: 1500 km
 Inclination: 102.1°
 Orb. period: 1.9 h
 RCS: 2.5 m2
 Object age: 47.0 y
 Exp. spin status: periodic, 54.4 s
 Mag: 4.79 .. 10.30, 7.97, se 0.56

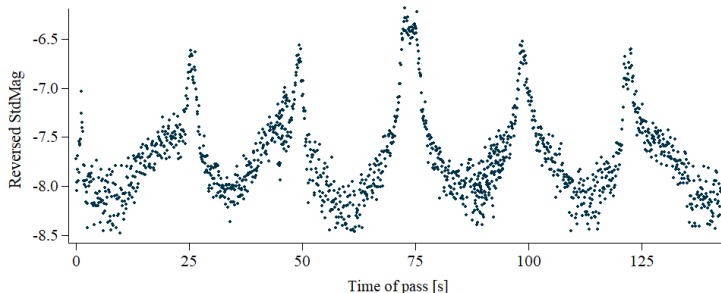
Result summary

Frequency analysis
 Period: 26.385 s
 Spin phase folding
 Period: 25.959 s

x2=52.8 s

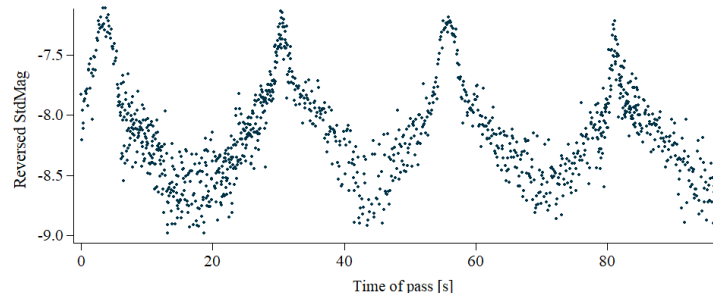


Light curve (1973-086a), MMT 24 October 2019, 2:10:54

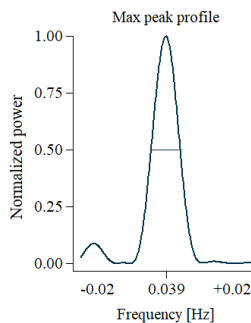
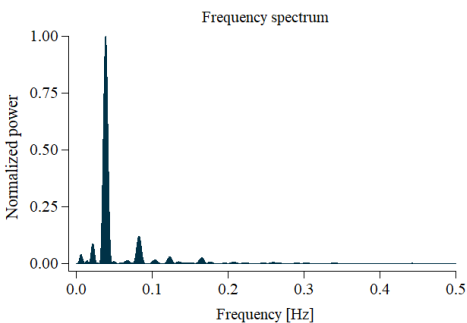


Pass information
 Duration: 2m:23s
 Data points: 1497
 Sampling rate: 10.0 Hz
 Closest approach
 Topo el: 58.5°
 Slant range: 1716 km
 Phase angle: 45.3°
 Sim. app. mag.: 6.3
 Solar Beta angle: 57.5°

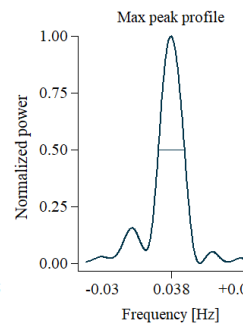
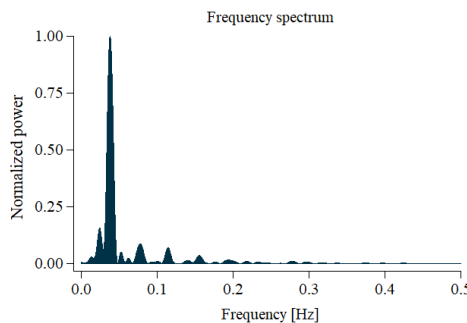
Light curve (1973-086a), MMT 9 November 2020, 1:54:17



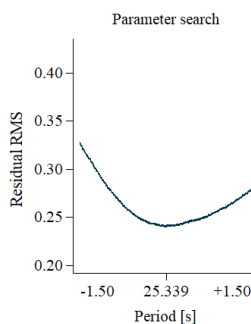
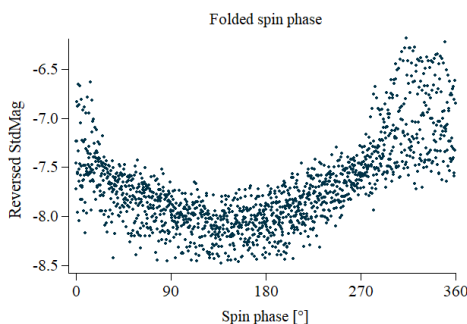
Pass information
 Duration: 1m:38s
 Data points: 1243
 Sampling rate: 10.0 Hz
 Closest approach
 Topo el: 81.0°
 Slant range: 1529 km
 Phase angle: 58.7°
 Sim. app. mag.: 6.3
 Solar Beta angle: 60.7°



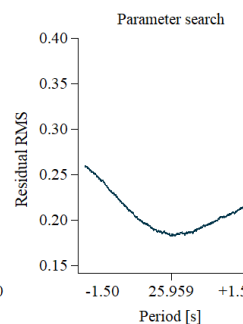
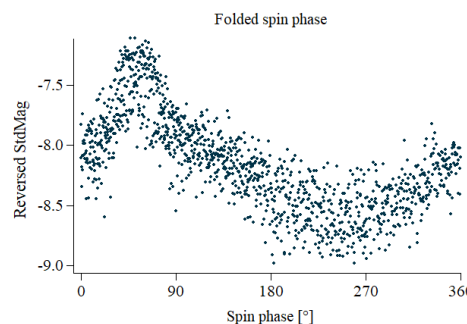
Frequency analysis
 Frequency: 0.039 Hz
 Period: 25.840 s
 FWHM: 6.809 mHz
 SNR: 13.4
 CoVar: 7.472 %



Frequency analysis
 Frequency: 0.038 Hz
 Period: 26.385 s
 FWHM: 9.150 mHz
 SNR: 9.8
 CoVar: 10.253 %



PDM
 Period: 25.339 s



PDM
 Period: 25.959 s

Description of rotation

Spin vector definition

Rotational motion of a rigid object about its center of mass can be characterized by the spin angular momentum vector \mathbf{L} oriented in an external, right-handed Cartesian coordinate system \mathbb{R}^3 about which the body spins in a counter-clockwise direction at an angular rate equal to the magnitude of \mathbf{L} .

During an overhead pass of a satellite above a ground tracking system, it is possible to predict an inertial, satellite-centered unit direction vector $\hat{\mathbf{T}}$ toward the telescope. Transformation of $\hat{\mathbf{T}}$ from an external \mathbb{R}^3 ($\hat{\mathbf{T}}_{\mathbb{R}^3}$) to a spacecraft body-centered and -fixed coordinate system (BCS) can be realized by $\hat{\mathbf{T}}_{BCS} = \mathbf{A}\hat{\mathbf{T}}_{\mathbb{R}^3}$, where the attitude tensor \mathbf{A} between the external and the embedded reference frames is a rotation matrix computed by:

$$\mathbf{A} = \mathbf{R}_2(-x_P)\mathbf{R}_1(-y_P)\mathbf{R}_3(\gamma)\mathbf{R}_1\left(\frac{\pi}{2} - \delta\right)\mathbf{R}_3\left(\frac{\pi}{2} + \alpha\right)$$

where the \mathbf{R}_1 , \mathbf{R}_2 and \mathbf{R}_3 are the standard rotation matrices about the x, y and z-axis respectively.

Description of rotation

Spin vector definition

The orientation of \mathbf{L} in \mathbb{R}^3 ($\mathbf{L}_{\mathbb{R}^3}$) is defined by spherical angles α , δ corresponding to right ascension and declination in the case of Earth Centered Inertial reference frame (ECI, J2000) being an external system.

Rotation angle γ increases at the rate of $\omega = \|\mathbf{L}\|$, while the pole coordinates x_P and y_P describe the relative position of \mathbf{L} with respect to the satellite body axis $+Z_{BCS}$. In this work we assume that the body's pole axis coincides with the axis of rotation, thus x_P and y_P are 0.

For a given spin vector $\mathbf{L}_{\mathbb{R}^3}$ it is possible to find the relative orientation of $\hat{\mathbf{T}}_L$ (satellite-centered observer direction vector) by:

$$\hat{\mathbf{T}}_L = \mathbf{R}_1 \left(\frac{\pi}{2} - \delta \right) \mathbf{R}_3 \left(\frac{\pi}{2} + \alpha \right) \hat{\mathbf{T}}_{\mathbb{R}^3}$$

where the azimuthal motion of $\hat{\mathbf{T}}_L$ about \mathbf{L} during a satellite pass is known as an apparent rotation effect.

Timeseries analysis – epoch methods

Several methods can support high level of automation of the tumbling motion determination processes:

- Savitzky-Golay filters – for timeseries approximation, detrending with time derivatives.
- Spectral analysis (Lomb) – for apparent rate determination.
- Phase Dispersion Minimization – for apparent and inertial periodicity determination.
- Autocorrelation – for apparent periodicity.
- Inertial PDM mapping – for spin vector orientation.

Apparent PDM

Typical, apparent periodicity as seen by the observer.

The phase coordinate of a data point is computed as a fraction of a fixed (tested) period.

Inertial PDM

Based on the attitude tensor transformation.

The phase coordinate of a data point is derived through the inertial-to-body transformation of a satellite-centered phase vector. The body-fixed azimuth of the transformed phase vector is the phase coordinate of the observed data point.

This method requires modelling a time-dependent, satellite attitude tensor that takes the satellite spin vector \bar{L} as an input. Optimization of \bar{L} (orientation and magnitude) minimizes dispersion of the folded pattern (measured by the variance ratio Θ).

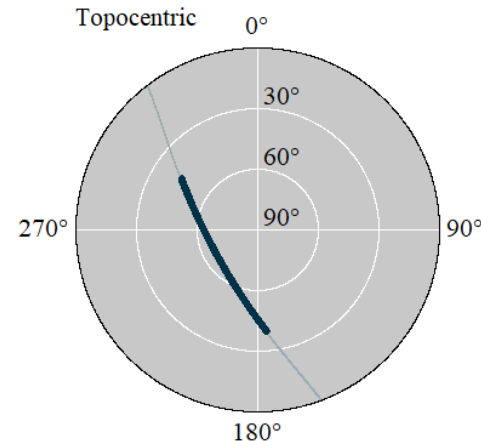
Apparent and Inertial Phase Folding – Lightcurve



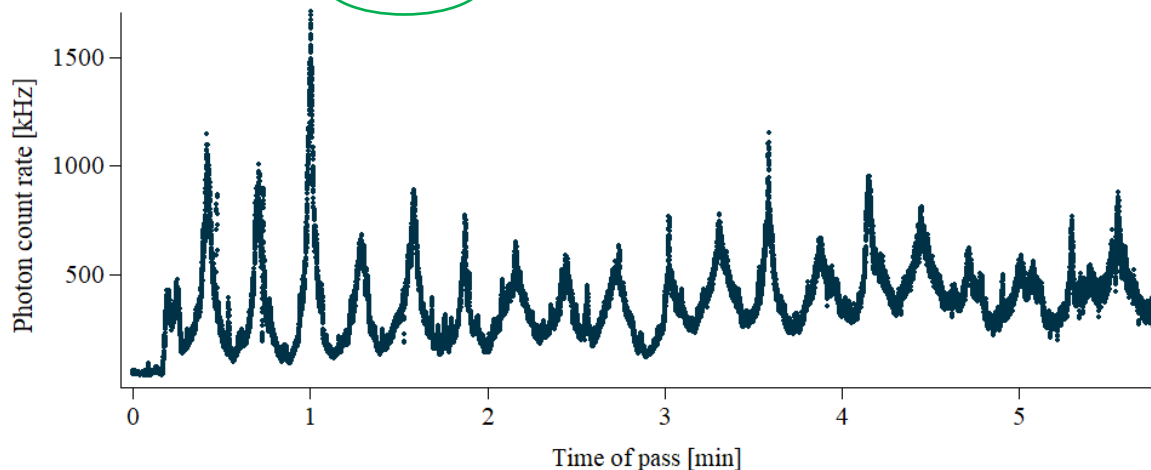
CNES

Object: JASON-2 (OSTM)
 NORAD: 33105
 Cospar: 2008-032A
 Launch: 20 June, 2008
 Apogee alt.: 1317 km
 Perigee alt.: 1305 km
 Inclination: 66.0°
 Orb. period: 1.9 h
 RCS: 3.2 m²
 Object age: 12.6 y
 Exp. spin status: aperiodic
 Mag: -1.17 .. 8.38, 5.70, se 0.81
 datafile: JA202017_CT

Result summary
 Frequency analysis
 Period: 17.194 s
 Spin phase folding
 Period: 0.000 s



Light-curve (2008-032A), Graz 20 January 2021, 17:30:16



Pass information
 Duration: 5m:49s
 Data points: 34864
 Data rate: 100.0 Hz

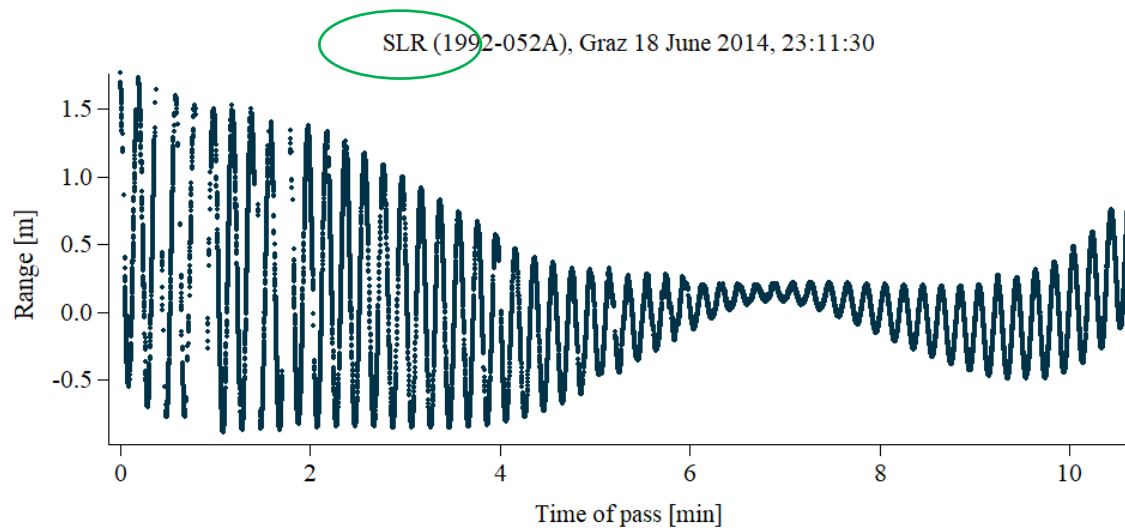
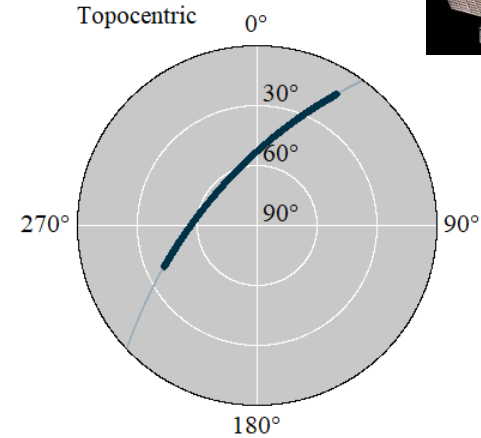
Closest approach
 Topo el: 66.4°
 Slant range: 1419 km
 Phase angle: 94.0°
 Sim. app. mag.: 7.0
 Solar Beta angle: -64.7°

Apparent and Inertial Phase Folding – SLR



CNES

Object: TOPEX/POSEIDON	Result summary
NORAD: 22076	
Cospar: 1992-052A	Frequency analysis
Launch: 10 August, 1992	Period: 11.810 s
Apogee alt.: 1344 km	
Perigee alt.: 1331 km	Spin phase folding
Inclination: 66.0°	Period: 0.000 s
Orb. period: 1.9 h	
RCS: 7.8 m ²	
Object age: 21.9 y	
Exp. spin status: periodic, 9.6 s	
Mag: -2.75 .. 9.31, 5.01, se 1.16	
datafile: top16923	



Pass information
 Duration: 10m:44s
 Data points: 476170
 Data rate: 1705.2 Hz

Closest approach
 Topo el: 64.3°
 Slant range: 1468 km
 Phase angle: 87.1°
 Sim. app. mag.: 6.8
 Solar Beta angle: 38.7°

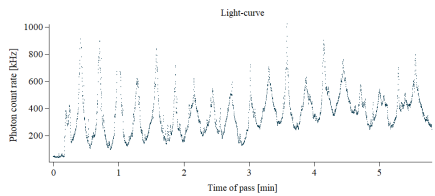
Timeseries analysis, Jason-2 light curve

A. Lomb

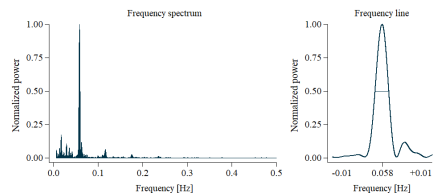
B. Apparent PDM

C. Inertial PDM

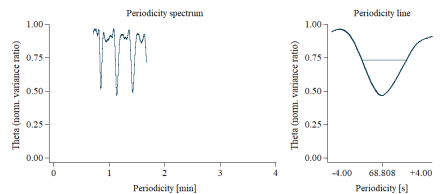
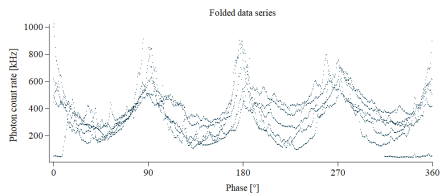
1. Averaged data



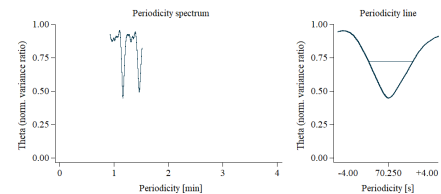
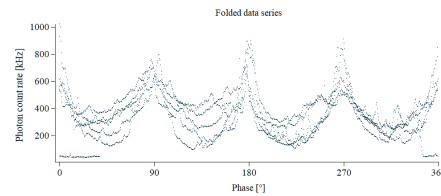
Data averaged
RMS: 155681 Hz



Frequency analysis Solution profile
Power: 697.8
Frequency: 0.058 Hz
Period: 17.194 s
FWHM: 2.750 mHz
StdErr: 1.168 mHz
SNR: 49.8
CoVar: 2.008 %

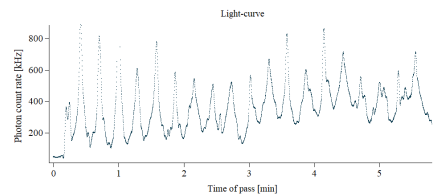


PDM Solution profile
Theta: 0.466
Period: 68.808 s
Frequency: 0.015 Hz
FWHM: 3.548 s
StdErr: 1.507 s
SNR: 45.7
CoVar: 2.190 %

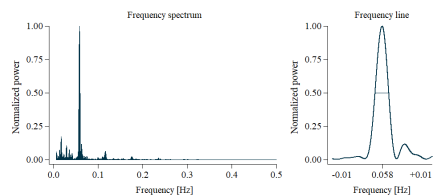


PDM Solution profile
Theta: 0.448
Period: 70.250 s
Frequency: 0.014 Hz
FWHM: 3.540 s
StdErr: 1.503 s
SNR: 46.7
CoVar: 2.140 %

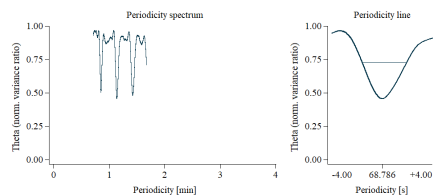
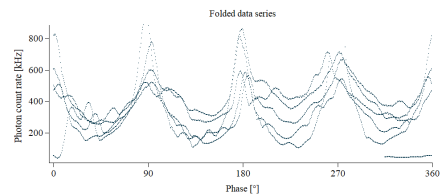
2. Approximated data



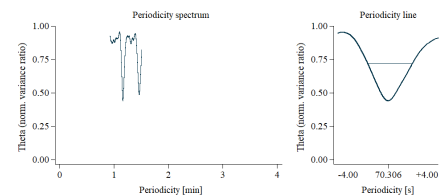
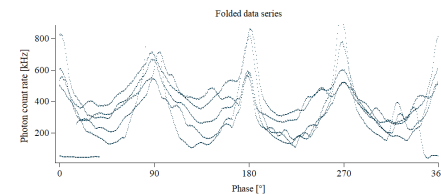
Data approximated
RMS: 153585 Hz



Frequency analysis Solution profile
Power: 717.3
Frequency: 0.058 Hz
Period: 17.194 s
FWHM: 2.748 mHz
StdErr: 1.167 mHz
SNR: 49.8
CoVar: 2.007 %

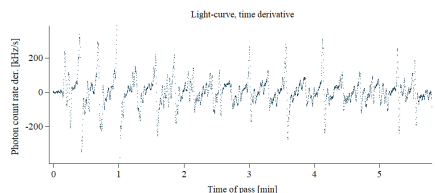


PDM Solution profile
Theta: 0.456
Period: 68.786 s
Frequency: 0.015 Hz
FWHM: 3.543 s
StdErr: 1.505 s
SNR: 45.7
CoVar: 2.187 %

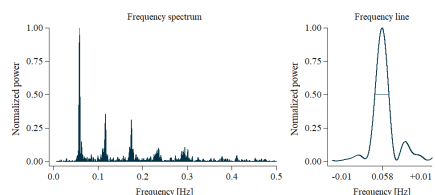


PDM Solution profile
Theta: 0.441
Period: 70.306 s
Frequency: 0.014 Hz
FWHM: 3.579 s
StdErr: 1.520 s
SNR: 46.3
CoVar: 2.162 %

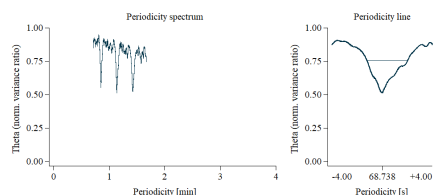
3. 1st derivative



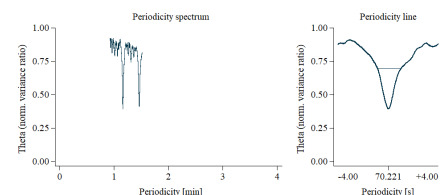
Time derivative, 1st degree
RMS: 80521 Hz/s



Frequency analysis Solution profile
Power: 372.9
Frequency: 0.058 Hz
Period: 17.213 s
FWHM: 2.817 mHz
StdErr: 1.196 mHz
SNR: 48.6
CoVar: 2.059 %



PDM Solution profile
Theta: 0.513
Period: 68.738 s
Frequency: 0.015 Hz
FWHM: 3.245 s
StdErr: 1.378 s
SNR: 49.9
CoVar: 2.005 %



PDM Solution profile
Theta: 0.392
Period: 70.221 s
Frequency: 0.014 Hz
FWHM: 1.859 s
StdErr: 0.790 s
SNR: 88.9
CoVar: 1.124 %

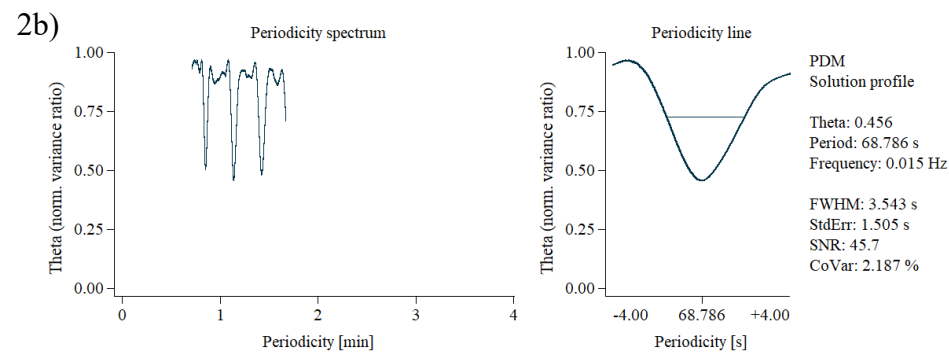
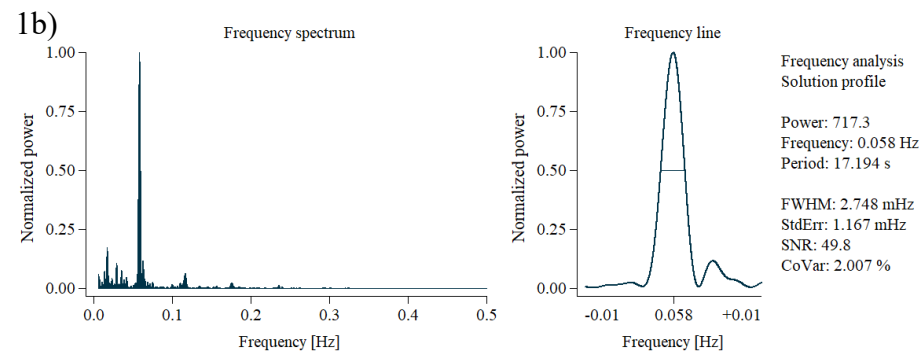
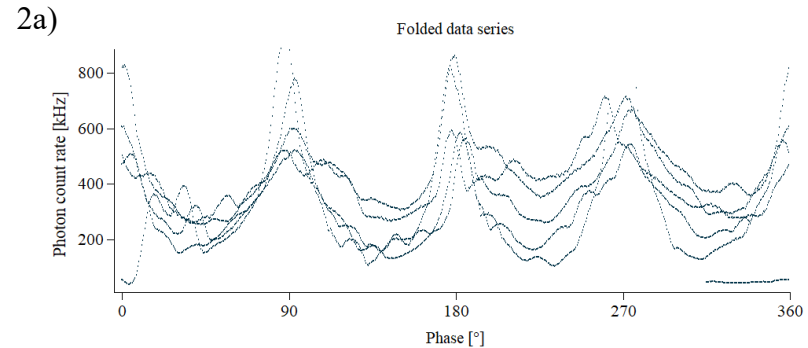
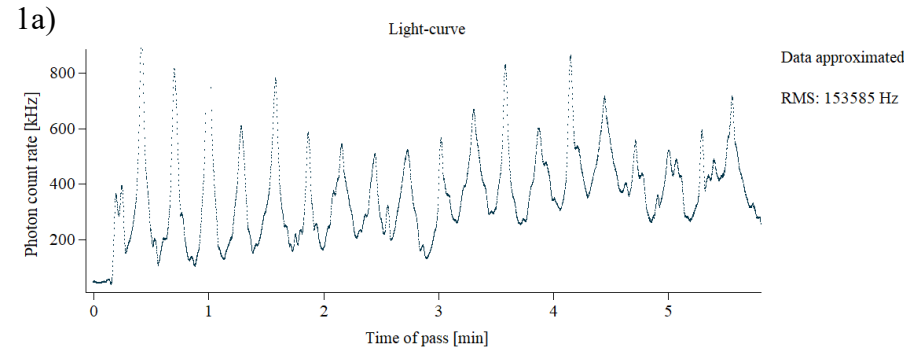
Timeseries analysis, Jason-2 light curve



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Approximated data, with 3° moving polynomial fit (SGF).

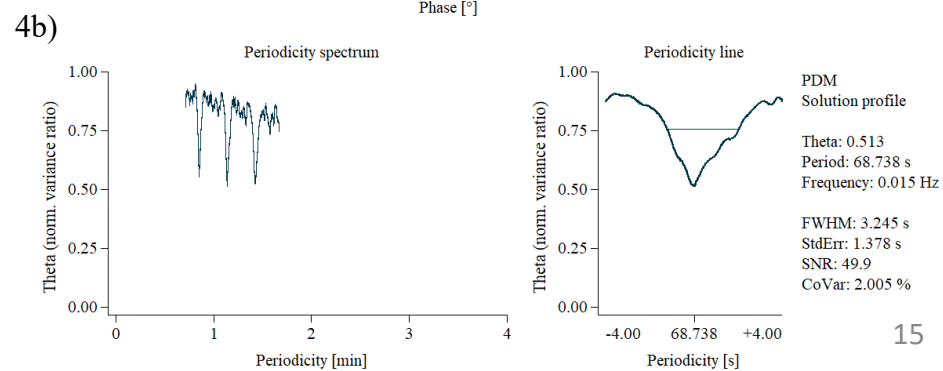
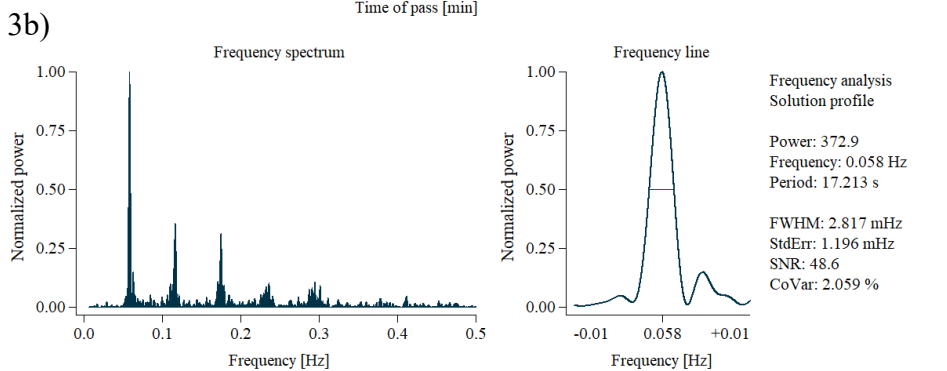
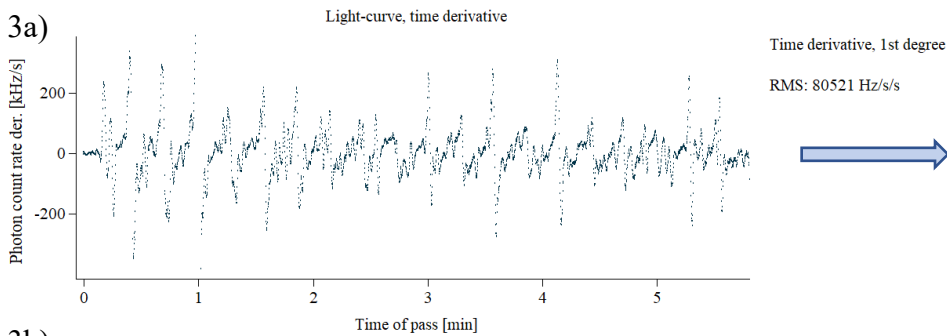
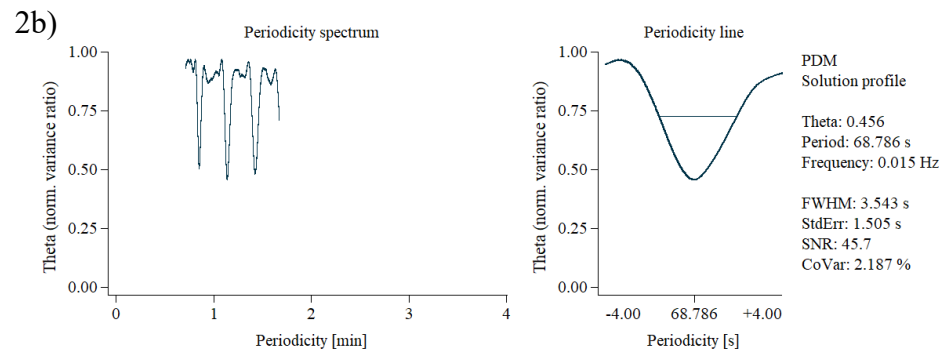
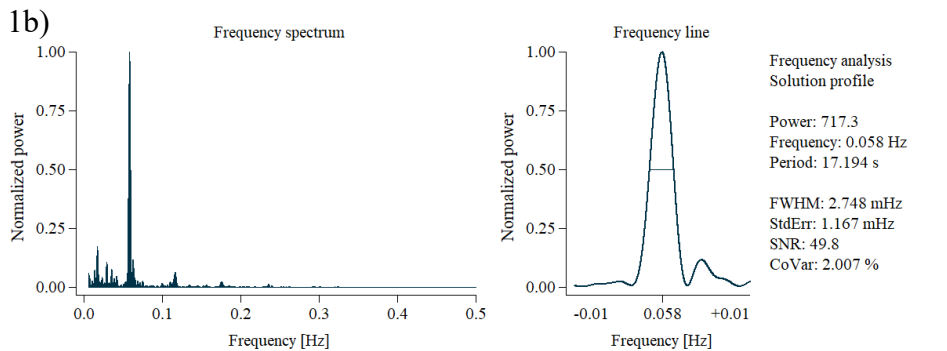
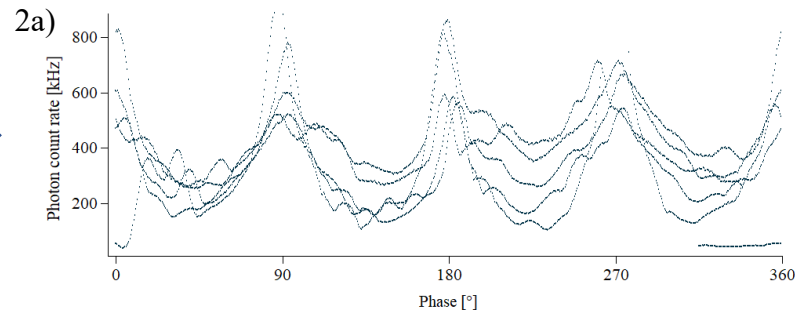
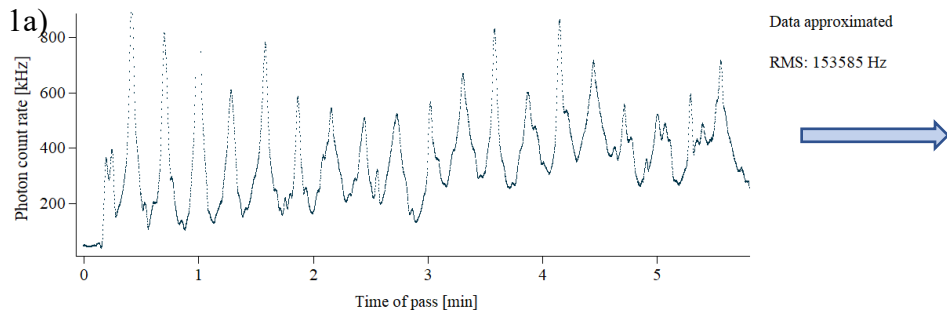
Polynomial window 3 s, step 0.1s; Orthogonal polynomial fit (Gram method).



The profile of the solution line allows finding the statistical parameters of the dominant signal:

- Full Width at Half-Maximum (FWHM) of the solution line represents the errors and inaccuracies of the analysis related to the sampling noise or apparent rotation and frequency shift effects.
- Signal-to-Noise Ratio (SNR). Assuming Gaussian distribution of the solution (f_{max}) error it is possible to calculate its standard deviation as $\sigma = \frac{FWHM}{2\sqrt{\ln(4)}}$ and $SNR = \frac{f_{max}}{\sigma}$. Here, the standard deviation represents the noise and other negative interference to the measured signal.
- Coefficient of Variation (CoVar) measures relative variability and is defined as the standard deviation divided by the mean, multiplied by 100 %: $CoVar = \frac{\sigma}{f_{max}} \cdot 100\%$.

Comparison of Lomb and PDM on approximated and derivative timeseries (Jason-2 lightcurve)

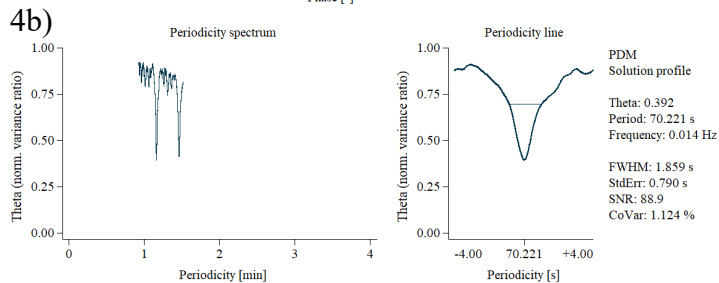
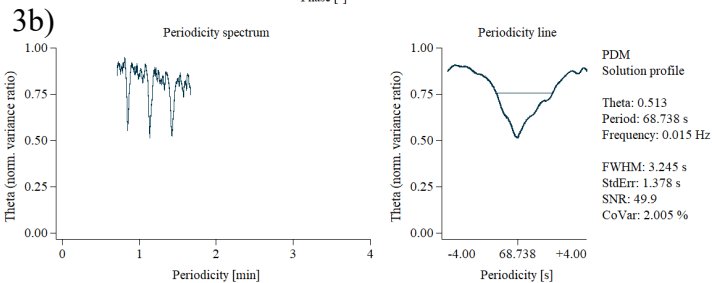
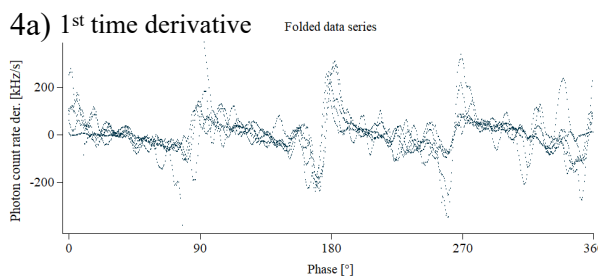
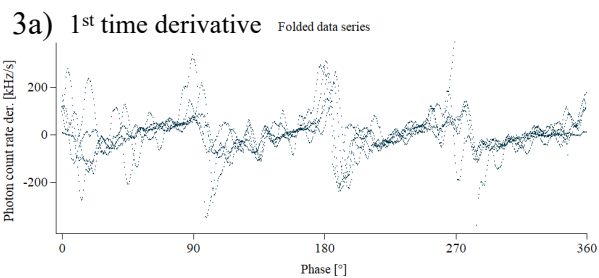
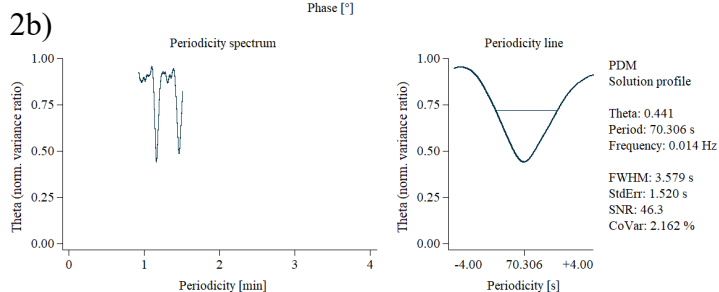
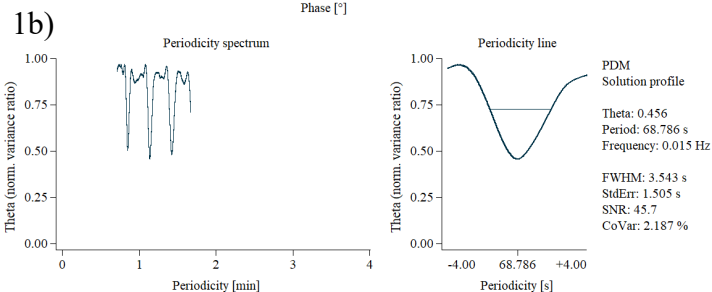
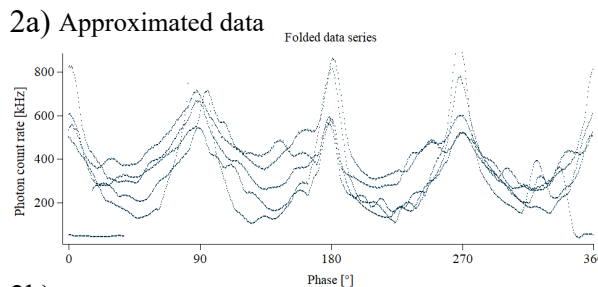
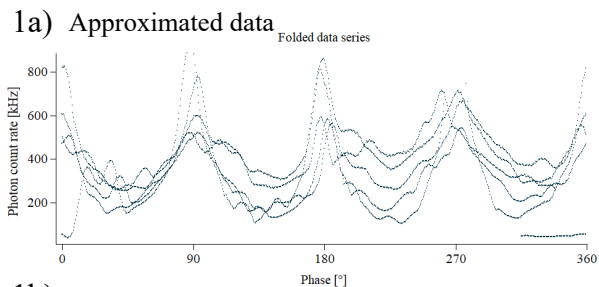


Timeseries analysis, Jason-2 light curve

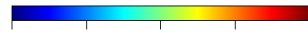
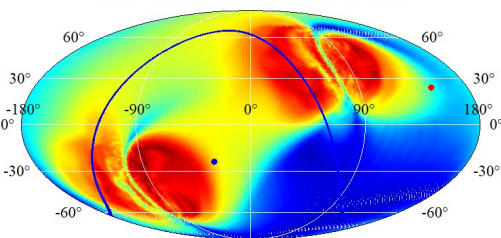


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Apparent vs. Inertial PDM



PDM inertial, Theta (Mean \pm 2 RMS), ECI J2000



Theta: mean \pm 2 RMS
(RMS of all Theta points on the map)

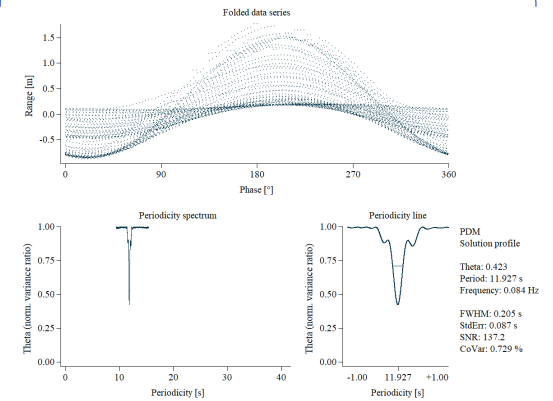
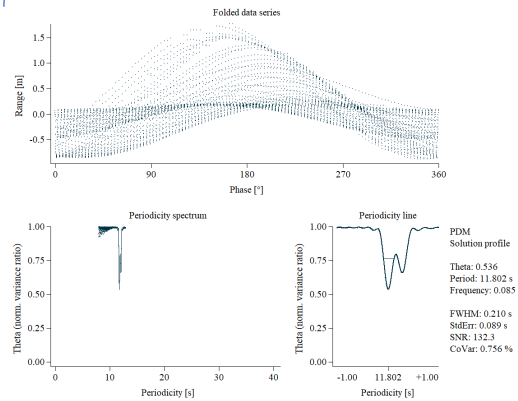
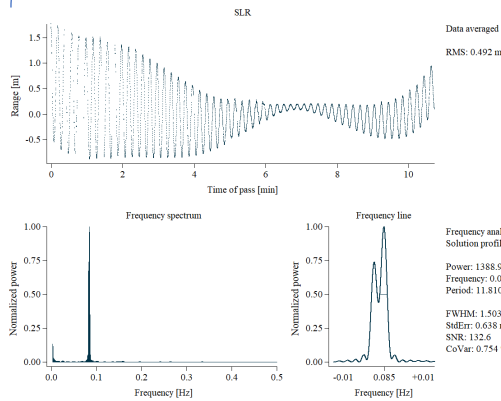
Timeseries analysis, TOPEX SLR

A. Lomb

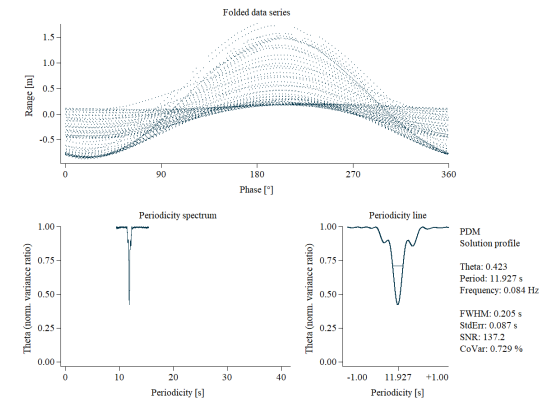
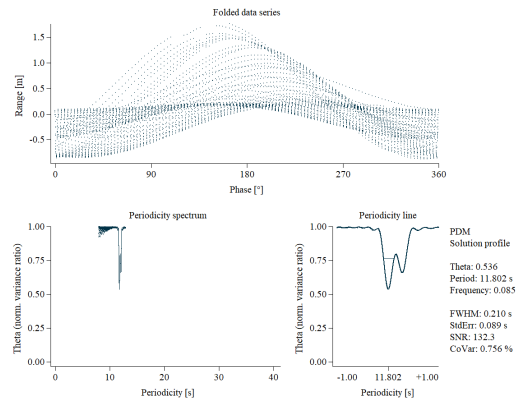
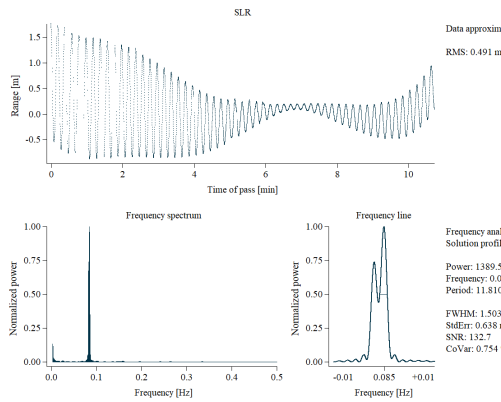
B. Apparent PDM

C. Inertial PDM

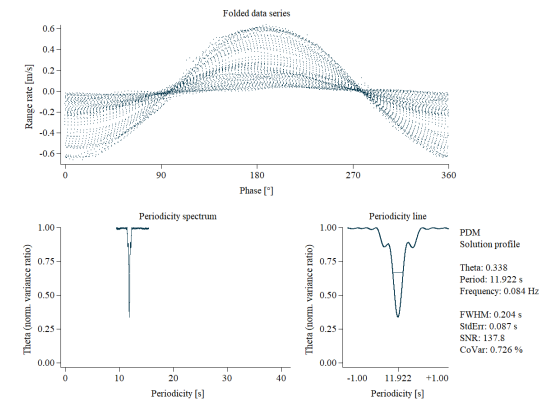
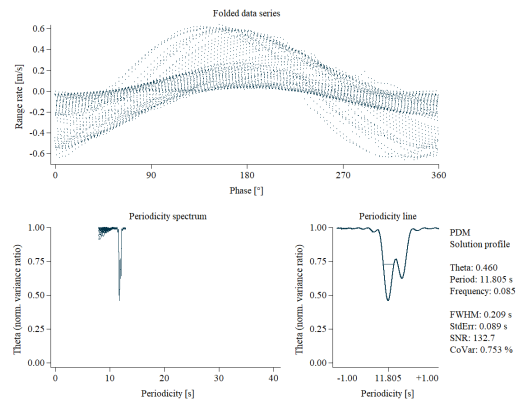
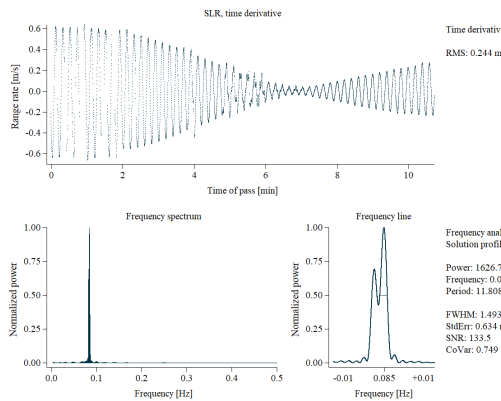
1. Averaged data



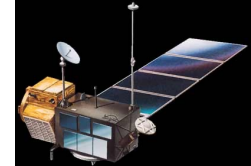
2. Approximated data



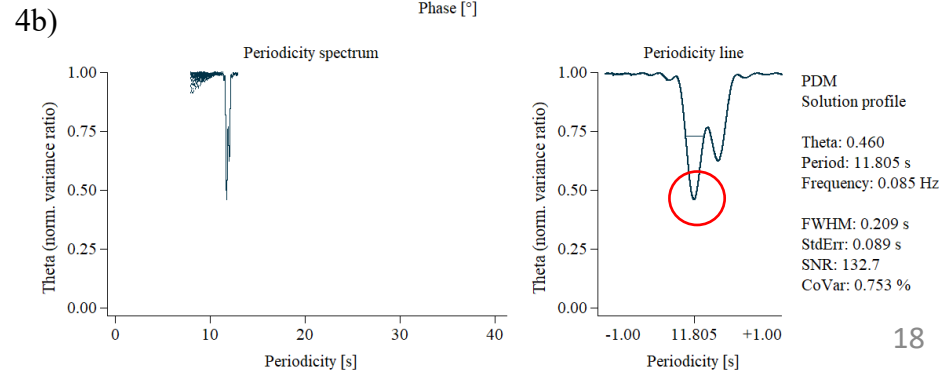
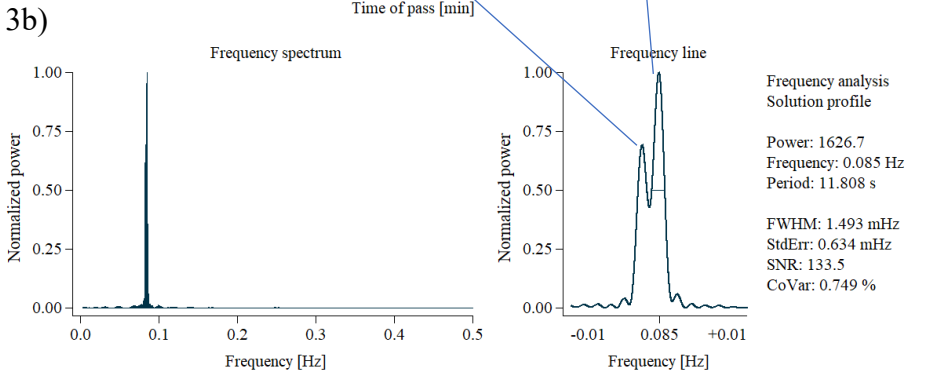
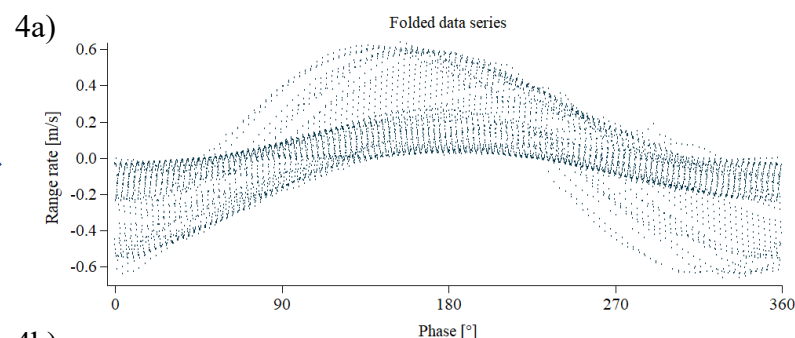
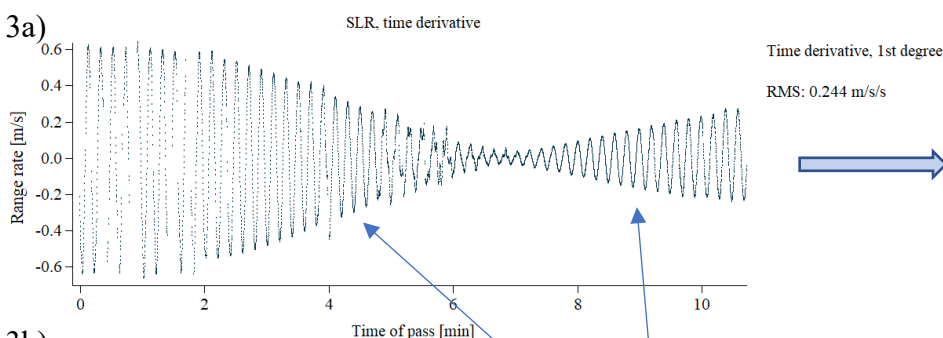
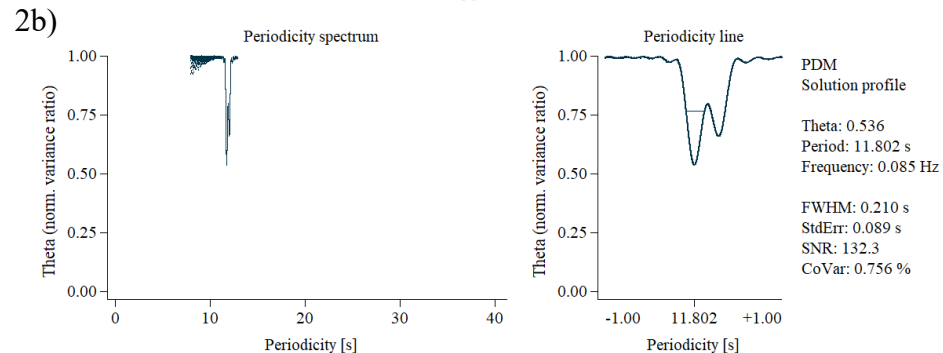
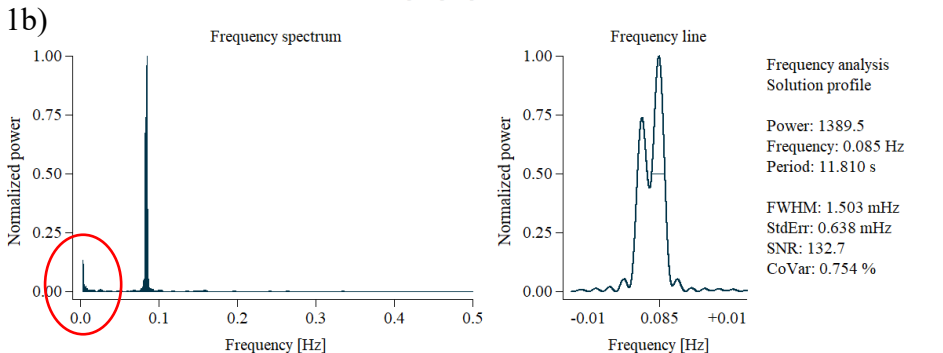
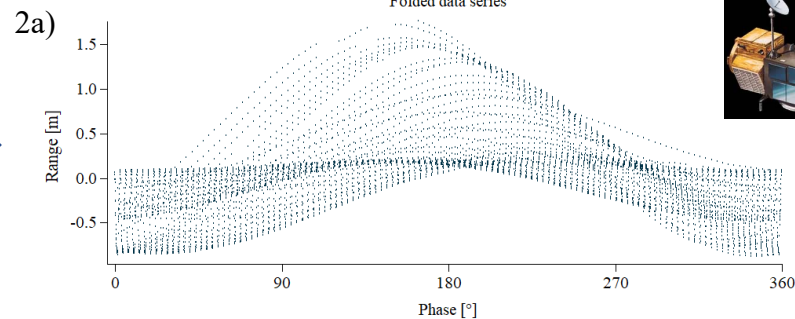
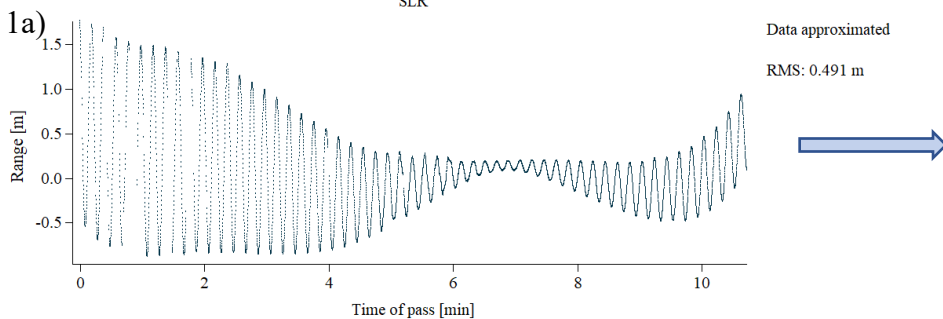
3. 1st derivative



Comparison of Lomb and PDM on approximated and derivative timeseries (Topex SLR)



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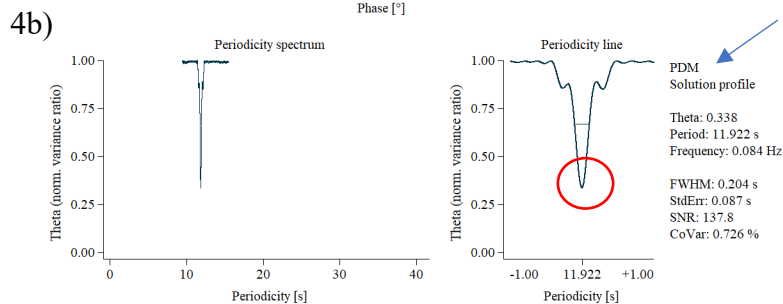
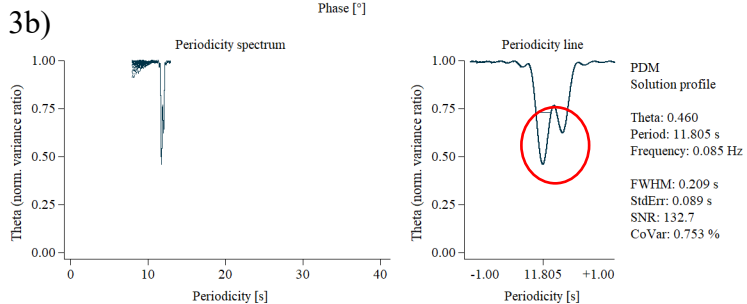
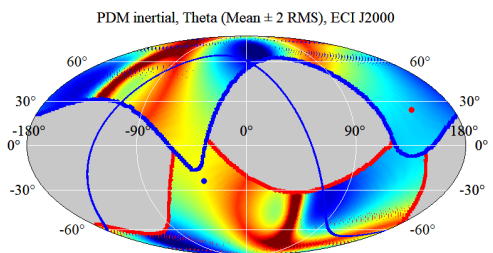
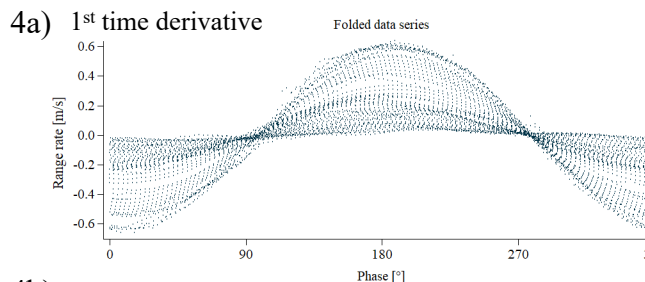
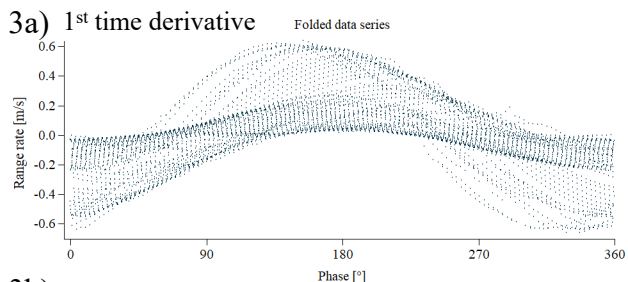
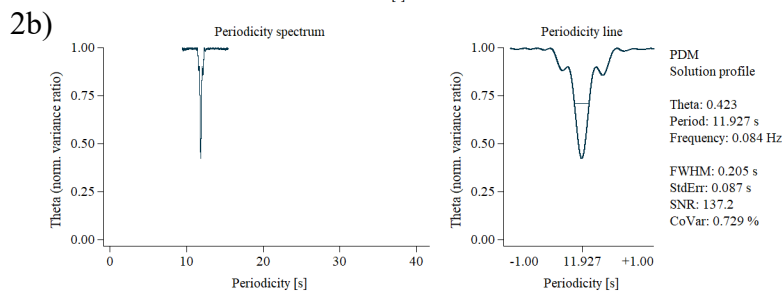
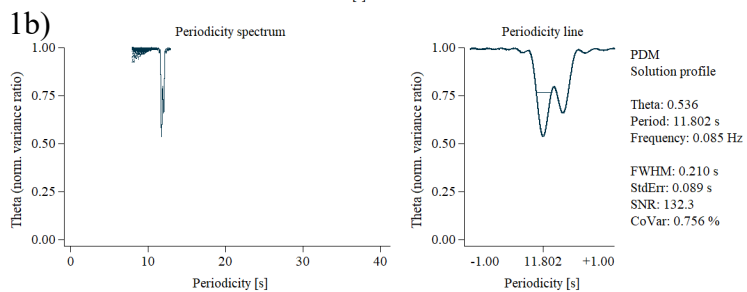
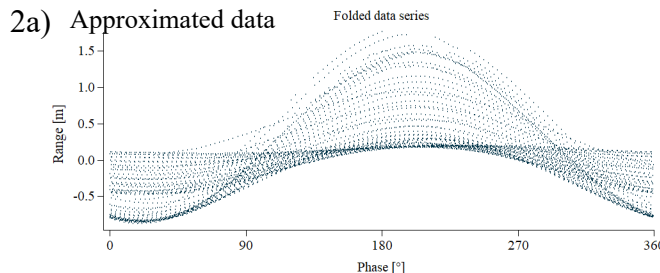
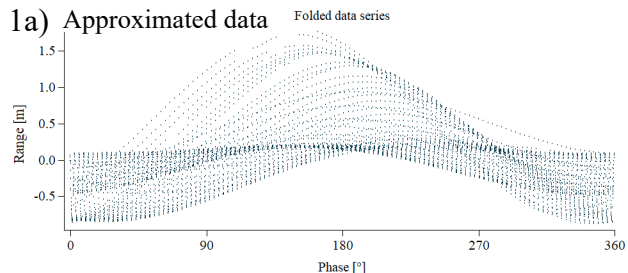


Timeseries analysis, TOPEX SLR



CNES

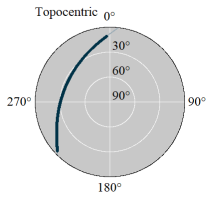
Apparent vs. Inertial PDM



Theta: mean ± 2 RMS
(RMS of all Theta points on the map)

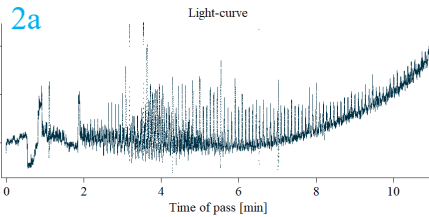
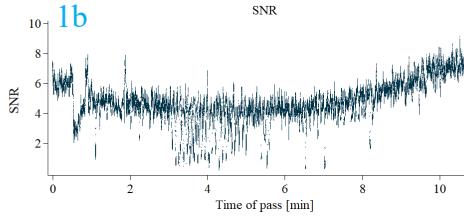
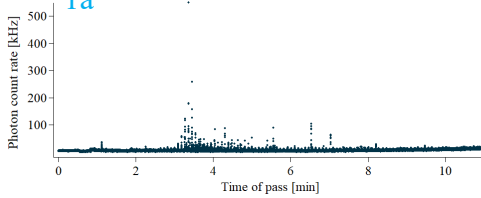
Results of the epoch methods

Object: DMSP 5D-2 F8 (USA 26)
 NORAD: 18123
 Cospar: 1987-053A
 Launch: 20 June, 1987
 Apogee alt.: 839 km
 Perigee alt.: 820 km
 Inclination: 98.7°
 Orb. period: 1.7 h
 RCS: 3.0 m²
 Object age: 34.3 y
 Exp. spin status: periodic, 10.9 s
 Mag: -1.28 .. 9.92, 5.46, se 0.89
 datafile: 55128218_ct

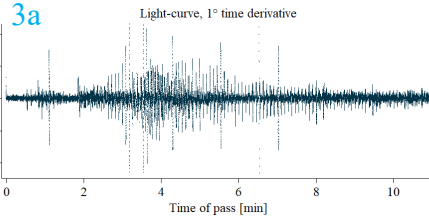
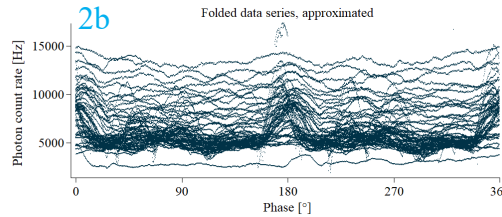


1a Light-curve (1987-053A), Graz 9 October 2021, 18:25:08

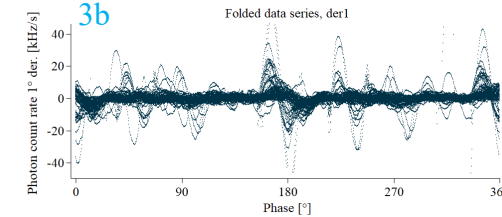
Pass information
 Duration: 10m:57s
 Data points: 65567
 Data rate: 100.0 Hz
 Closest approach
 Topo el: 34.2°
 Slant range: 1338 km
 Phase angle: 122.5°
 Sm. app. mag.: 8.0
 Solar Beta angle: -74.3°



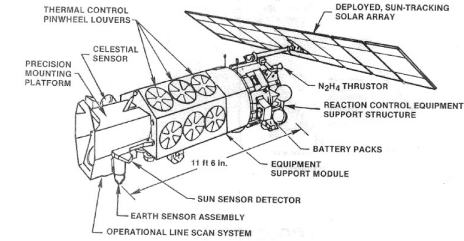
Data approximated
 RMS: 2401 Hz



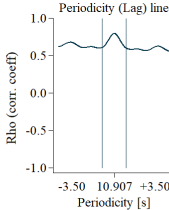
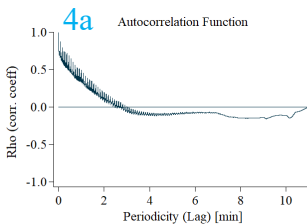
Time derivative, 1°
 RMS: 4706 Hz/s



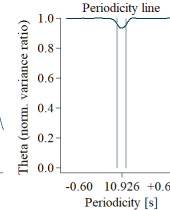
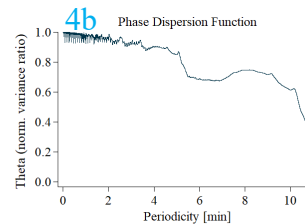
DMSP 5 D-2 SATELLITE



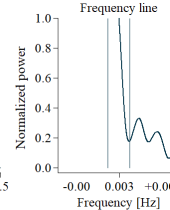
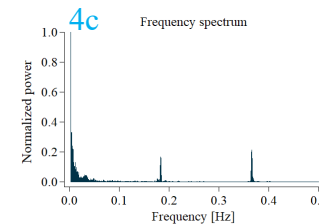
Approximated data: 2-a,b, 4-a,b,c
 Time derivative: 3-a,b, 5-a,b,c



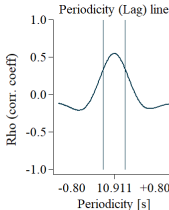
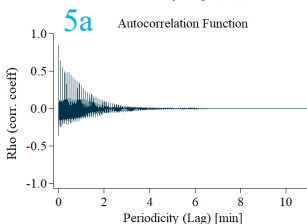
ACF Solution profile
 Rho: 0.792
 Period: 10.907 s
 Frequency: 0.092 Hz
 FWHM: 1.736 s
 StdDev: 0.737 s
 SNR: 14.8
 CoVar: 6.759 %



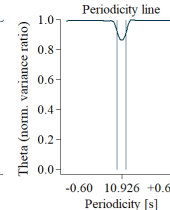
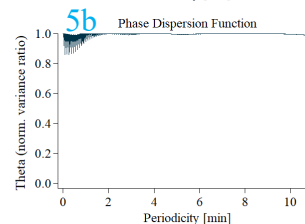
PDM Solution profile
 Theta: 0.932
 Period: 10.926 s
 Frequency: 0.092 Hz
 FWHM: 0.115 s
 StdDev: 0.049 s
 SNR: 223.9
 CoVar: 0.447 %



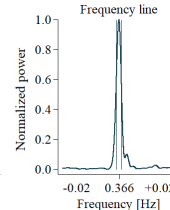
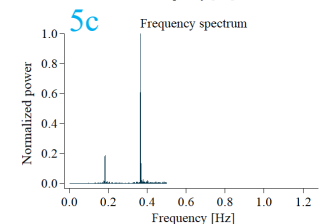
Frequency analysis Solution profile
 Power: 5567.6
 Frequency: 0.093 Hz
 Period: 10.903 s
 FWHM: 1.826 mHz
 StdDev: 0.776 mHz
 SNR: 3.9
 CoVar: 25.476 %



ACF Solution profile
 Rho: 0.555
 Period: 10.911 s
 Frequency: 0.092 Hz
 FWHM: 0.364 s
 StdDev: 0.155 s
 SNR: 70.6
 CoVar: 1.416 %

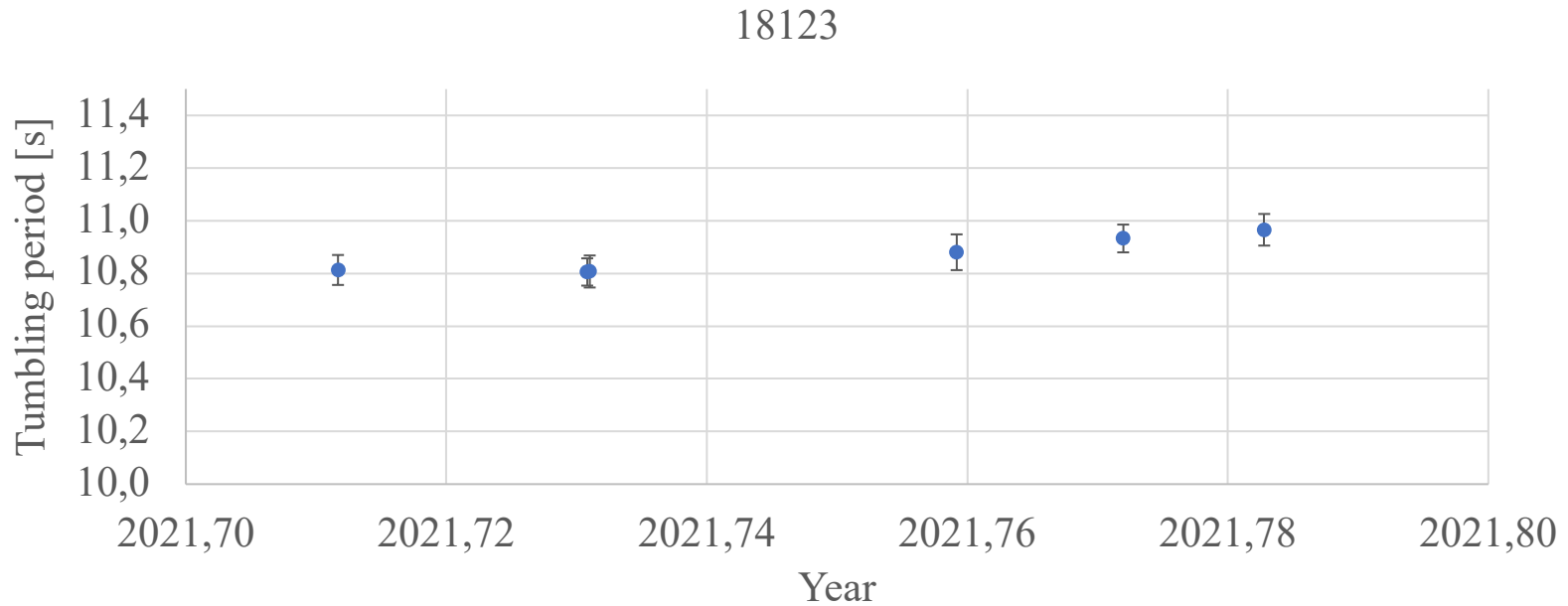


PDM Solution profile
 Theta: 0.861
 Period: 10.926 s
 Frequency: 0.092 Hz
 FWHM: 0.110 s
 StdDev: 0.047 s
 SNR: 234.4
 CoVar: 0.427 %



Frequency analysis Solution profile
 Power: 1551.3
 Frequency: 0.366 Hz
 Period: 2.733 s
 FWHM: 2.070 mHz
 StdDev: 0.879 mHz
 SNR: 416.2
 CoVar: 0.240 %

Results of the epoch methods



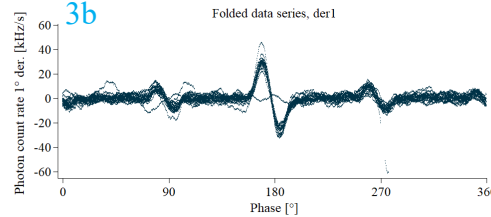
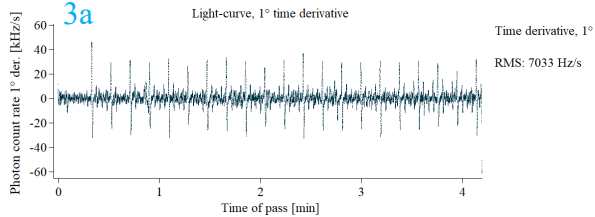
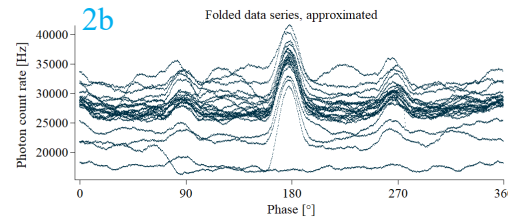
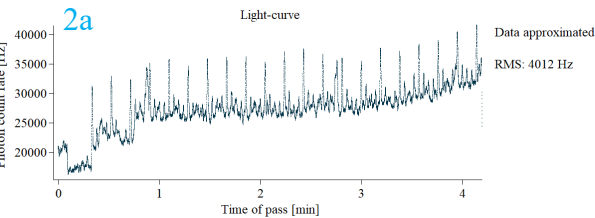
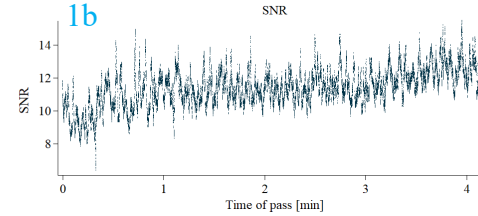
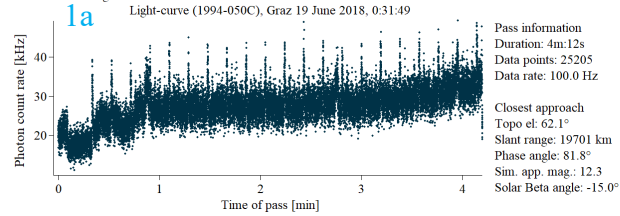
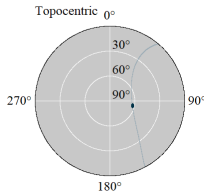
SatCat	
Name	DMSP Block 5D-2 F8
COSPAR	1987-053A
NORAD	18123
Apogee_altitude_[km]	839
Perigee_altitude_[km]	820
Orbital_period_[h]	1.692
Inclination_[deg]	98.7
RCS_[m2]	2.96
Launch_date	6/20/1987
Age_[year]	34.7
Source_or_ownership	us

DISCOS	
Class	Payload
Shape	Cyl + 1 Pan
CS_min_[m2]	1.767
CS_max_[m2]	10.5
CS_ave_[m2]	9.012
Mass_[kg]	815.7
Span_[m]	6.9
Height_[m]	6.9
Diameter_[m]	1.5
Depth_[m]	0
Width_[m]	0

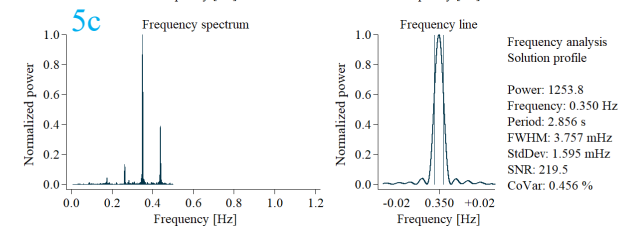
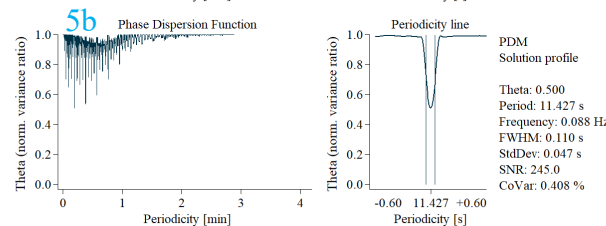
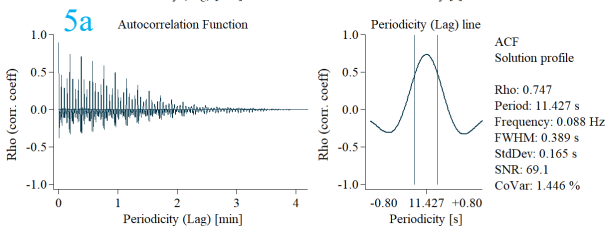
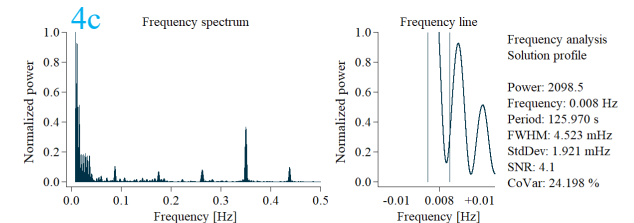
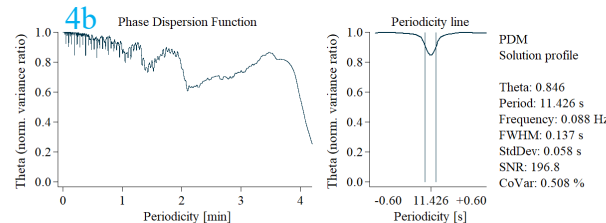
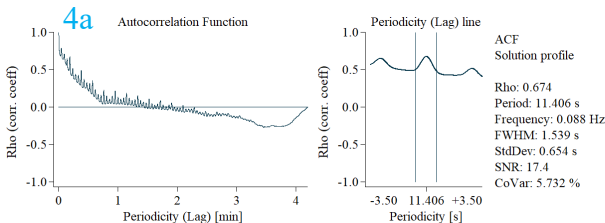
MMT	
Variability	periodic
Period_[s]	10.9
StdMag_Clear	5.5

Results of the epoch methods

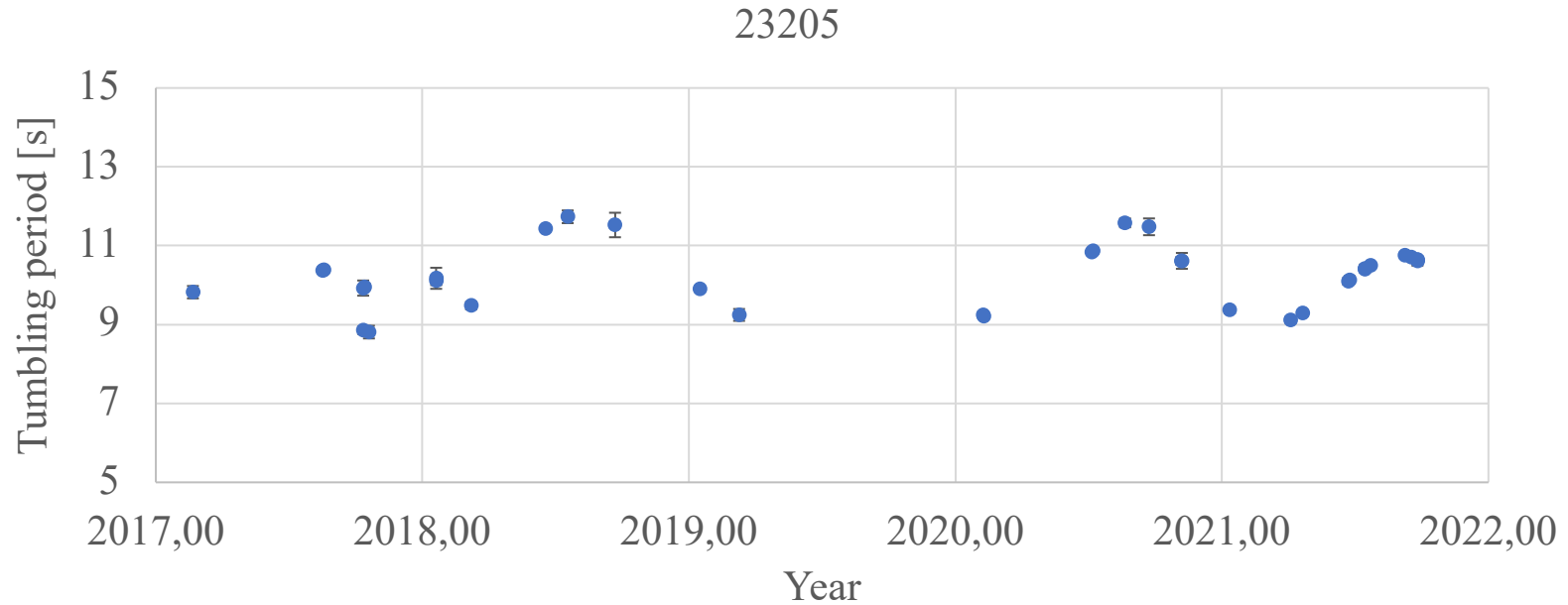
Object: COSMOS 2288 [GLONASS]
 NORAD: 23205
 Cospar: 1994-050C
 Launch: 11 August, 1994
 Apogee alt.: 19134 km
 Perigee alt.: 19125 km
 Inclination: 65.6°
 Orb. period: 11.3 h
 RCS: 3.9 m2
 Object age: 23.9 y
 Exp. spin status: -
 Mag: -
 datafile: g6716923



Approximated data: 2-a,b, 4-a,b,c
 Time derivative: 3-a,b, 5-a,b,c



Results of the epoch methods



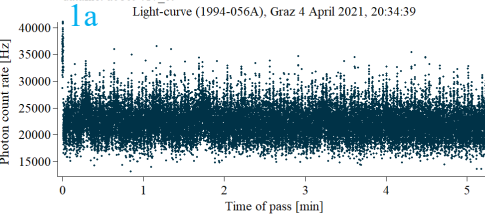
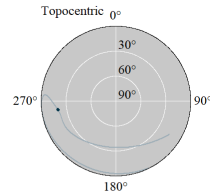
SatCat	
Name	Cosmos-2289
COSPAR	1994-050C
NORAD	23205
Apogee_altitude_[km]	19135
Perigee_altitude_[km]	19125
Orbital_period_[h]	11.262
Inclination_[deg]	65.6
RCS_[m2]	3.948
Launch_date	8/11/1994
Age_[year]	27.5
Source_or_ownership	cis

DISCOS	
Class	Payload
Shape	Cyl + 2 Pan
CS_min_[m2]	4.052
CS_max_[m2]	24.972
CS_ave_[m2]	14.748
Mass_[kg]	1288.4
Span_[m]	7.8
Height_[m]	4
Diameter_[m]	1.5
Depth_[m]	0
Width_[m]	0

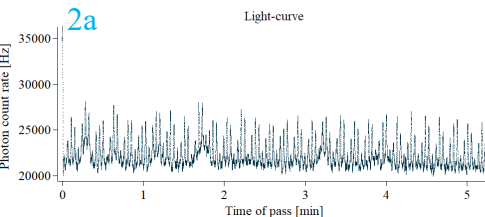
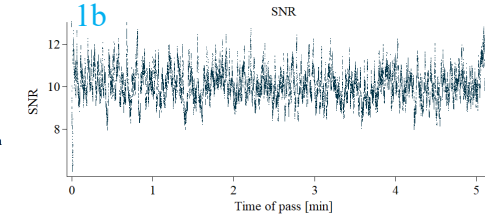
MMT	
Variability	N/A
Period_[s]	-1
StdMag_Clear	999

Results of the epoch methods

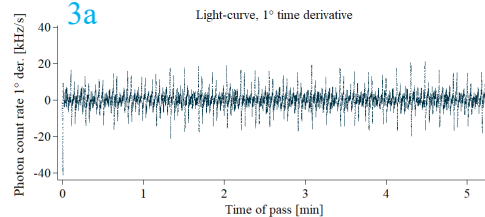
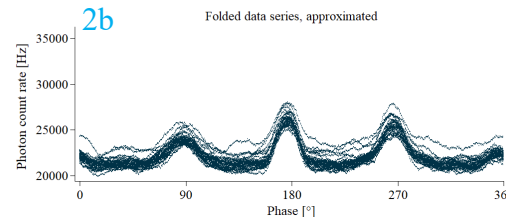
Object: KIKU-6 (ETS-VI)
 NORAD: 23230
 Cospar: 1994-056A
 Launch: 28 August, 1994
 Apogee alt.: 38772 km
 Perigee alt.: 8468 km
 Inclination: 15.3°
 Orb. period: 14.4 h
 RCS: 38.6 m2
 Object age: 26.6 y
 Exp. spin status: periodic, 5.3 s
 Mag: 0.61 .. 6.02, 4.47, se 0.63
 datafile: a0809417_ct



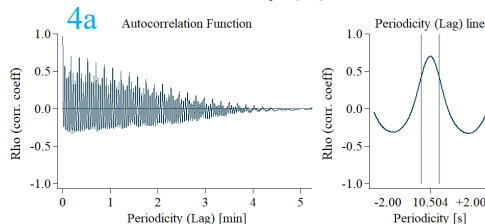
Pass information
 Duration: 5m:15s
 Data points: 31464
 Data rate: 100.0 Hz
 Closest approach
 Topo el: 19.9°
 Slant range: 41711 km
 Phase angle: 107.2°
 Sim. app. mag.: 14.8
 Solar Beta angle: 8.7°



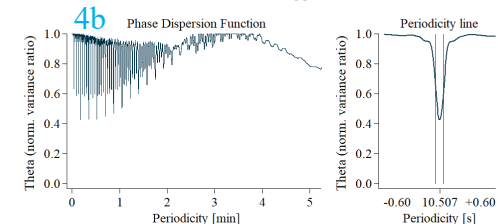
Data approximated
 RMS: 1511 Hz



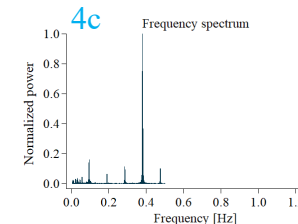
Time derivative, 1°
 RMS: 4892 Hz/s



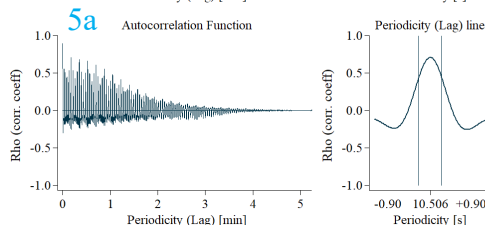
ACF Solution profile
 Rho: 0.703
 Period: 10.504 s
 Frequency: 0.095 Hz
 FWHM: 0.750 s
 StdDev: 0.319 s
 SNR: 33.0
 CoVar: 3.033 %



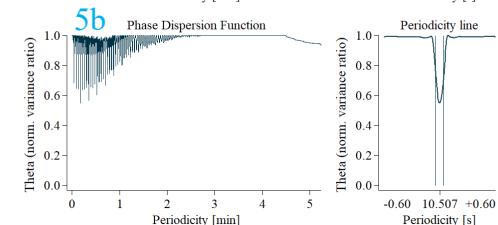
PDM Solution profile
 Theta: 0.423
 Period: 10.507 s
 Frequency: 0.095 Hz
 FWHM: 0.106 s
 StdDev: 0.045 s
 SNR: 233.8
 CoVar: 0.428 %



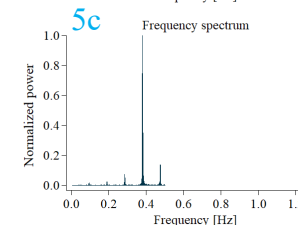
Frequency analysis Solution profile
 Power: 6371.3
 Frequency: 0.381 Hz
 Period: 2.627 s
 FWHM: 2.896 mHz
 StdDev: 1.230 mHz
 SNR: 309.5
 CoVar: 0.323 %



ACF Solution profile
 Rho: 0.716
 Period: 10.506 s
 Frequency: 0.095 Hz
 FWHM: 0.433 s
 StdDev: 0.184 s
 SNR: 57.2
 CoVar: 1.750 %



PDM Solution profile
 Theta: 0.540
 Period: 10.507 s
 Frequency: 0.095 Hz
 FWHM: 0.094 s
 StdDev: 0.040 s
 SNR: 262.1
 CoVar: 0.381 %



Frequency analysis Solution profile
 Power: 3548.1
 Frequency: 0.381 Hz
 Period: 2.626 s
 FWHM: 2.870 mHz
 StdDev: 1.219 mHz
 SNR: 312.5
 CoVar: 0.320 %

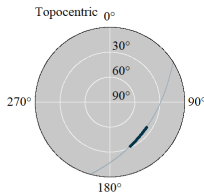


NASDA (JAXA)

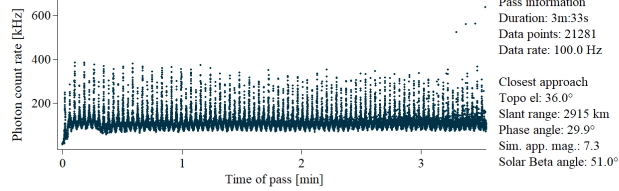
Approximated data: 2-a,b, 4-a,b,c
 Time derivative: 3-a,b, 5-a,b,c

Results of the epoch methods

Object: GLOBALSTAR M025
 NORAD: 25770
 Cospar: 1999-031A
 Launch: 10 June, 1999
 Apogee alt.: 2049 km
 Perigee alt.: 2046 km
 Inclination: 52.0°
 Orb. period: 2.1 h
 RCS: 2.3 m²
 Object age: 22.1 y
 Exp. spin status: periodic, 4.6 s
 Mag: -3.58 .. 9.43, 6.18, se 1.00
 datafile: a0219320_ct

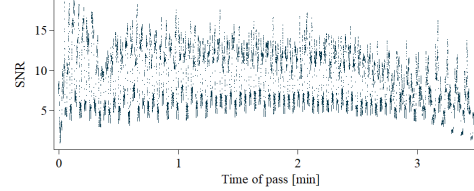


1a Light-curve (1999-031A), Graz 12 July 2021, 21:00:38

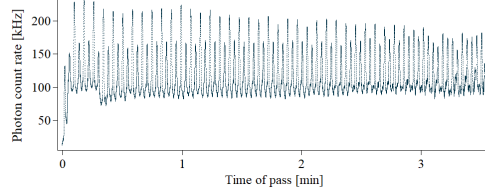


Pass information
 Duration: 3m:33s
 Data points: 21281
 Data rate: 100.0 Hz
 Closest approach
 Topo el: 36.0°
 Slant range: 2915 km
 Phase angle: 29.9°
 Sim. app. mag.: 7.3
 Solar Beta angle: 51.0°

1b SNR

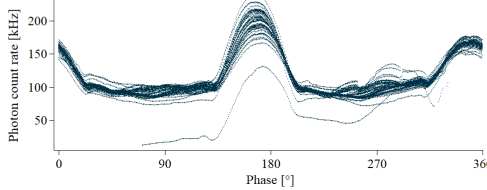


2a Light-curve

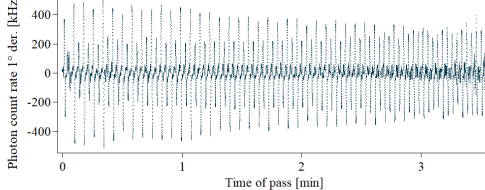


Data approximated
 RMS: 34533 Hz

2b Folded data series, approximated

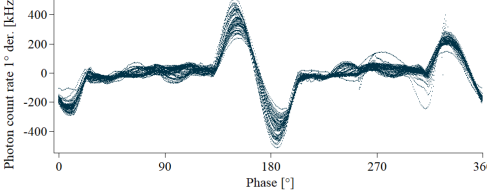


3a Light-curve, 1° time derivative



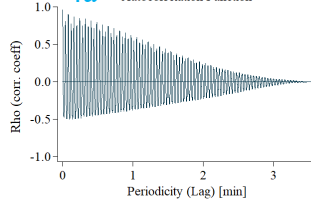
Time derivative, 1°
 RMS: 142004 Hz/s

3b Folded data series, der1

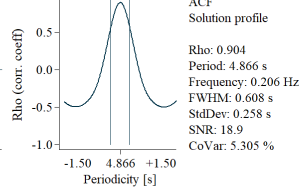


Approximated data: 2-a,b, 4-a,b,c
 Time derivative: 3-a,b, 5-a,b,c

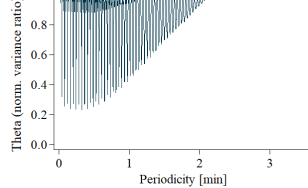
4a Autocorrelation Function



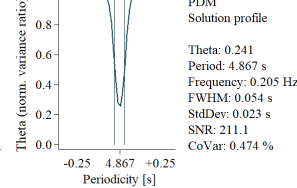
4b Phase Dispersion Function



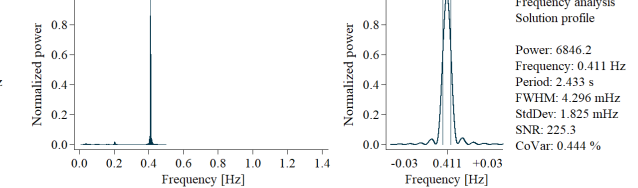
4c Frequency spectrum



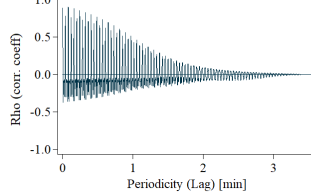
4d Periodicity line



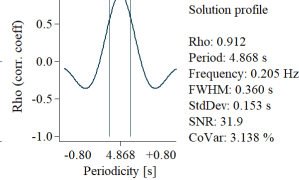
4e Frequency line



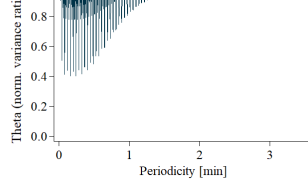
5a Autocorrelation Function



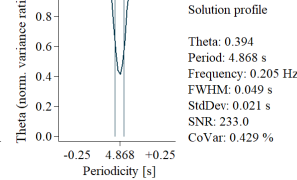
5b Phase Dispersion Function



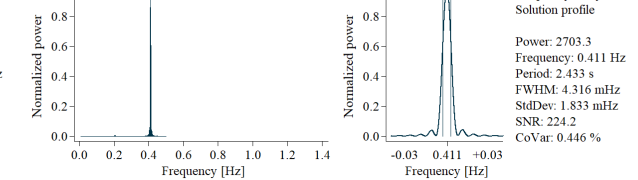
5c Frequency spectrum



5d Periodicity line

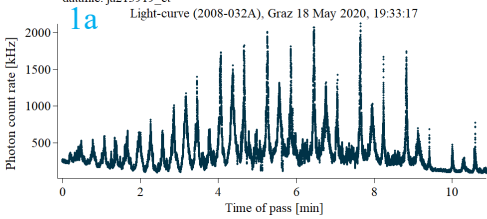
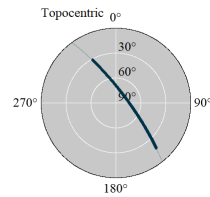


5e Frequency line



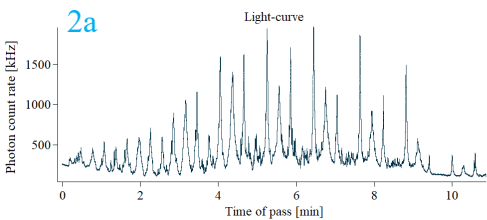
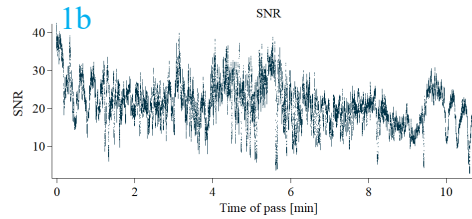
Results of the epoch methods

Object: JASON-2 (OSTM)
 NORAD: 33105
 Cospar: 2008-032A
 Launch: 20 June, 2008
 Apogee alt.: 1317 km
 Perigee alt.: 1305 km
 Inclination: 66.0°
 Orb. period: 1.9 h
 RCS: 3.2 m2
 Object age: 11.9 y
 Exp. spin status: aperiodic
 Mag: -1.17 .. 8.38, 5.70, se 0.81
 datafile: ja213919_ct

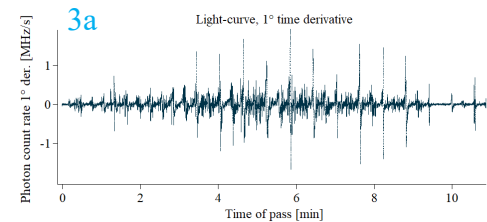
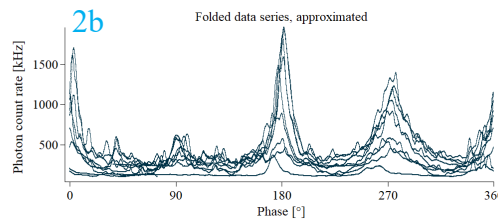


Pass information
 Duration: 10m:53s
 Data points: 65306
 Data rate: 100.0 Hz

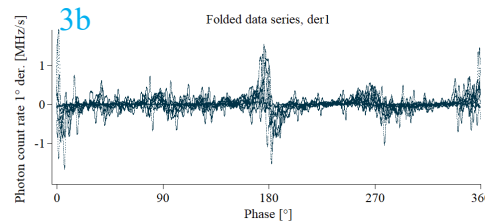
Closest approach
 Topo el: 77.0°
 Slant range: 1351 km
 Phase angle: 78.4°
 Sim. app. mag.: 6.4
 Solar Beta angle: -6.4°



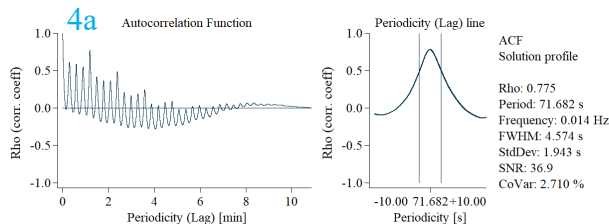
Data approximated
 RMS: 256361 Hz



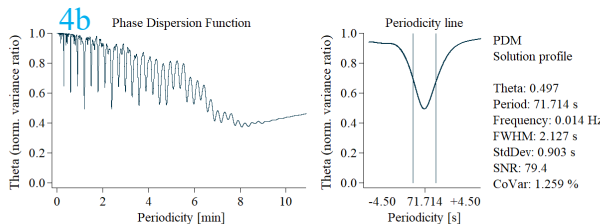
Time derivative, 1°
 RMS: 203327 Hz/s



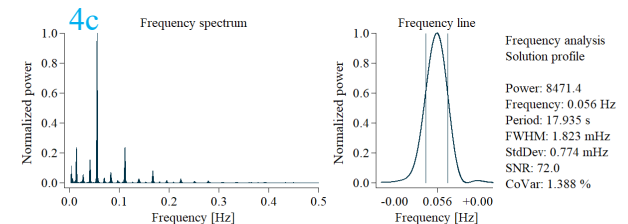
Approximated data: 2-a,b, 4-a,b,c
 Time derivative: 3-a,b, 5-a,b,c



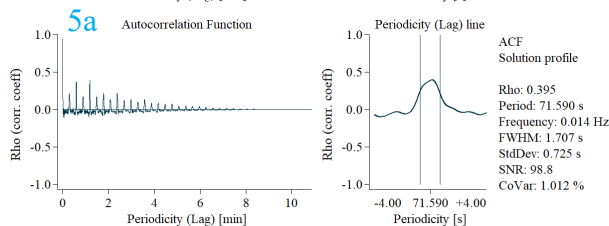
ACF Solution profile
 Rho: 0.775
 Period: 71.682 s
 Frequency: 0.014 Hz
 FWHM: 4.574 s
 StdDev: 1.943 s
 SNR: 36.9
 CoVar: 2.710 %



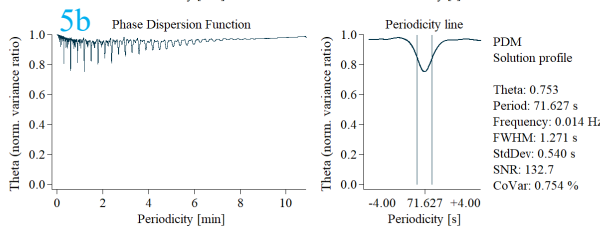
PDM Solution profile
 Theta: 0.497
 Period: 71.714 s
 Frequency: 0.014 Hz
 FWHM: 2.127 s
 StdDev: 0.903 s
 SNR: 79.4
 CoVar: 1.259 %



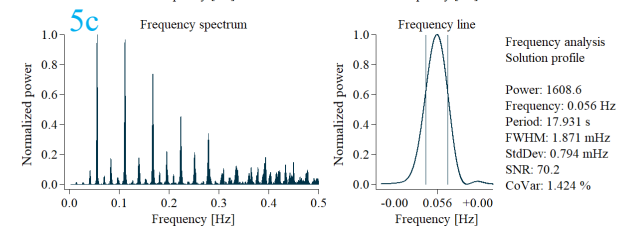
Frequency analysis Solution profile
 Power: 8471.4
 Frequency: 0.056 Hz
 Period: 17.935 s
 FWHM: 1.823 mHz
 StdDev: 0.774 mHz
 SNR: 72.0
 CoVar: 1.388 %



ACF Solution profile
 Rho: 0.395
 Period: 71.590 s
 Frequency: 0.014 Hz
 FWHM: 1.707 s
 StdDev: 0.725 s
 SNR: 98.8
 CoVar: 1.012 %

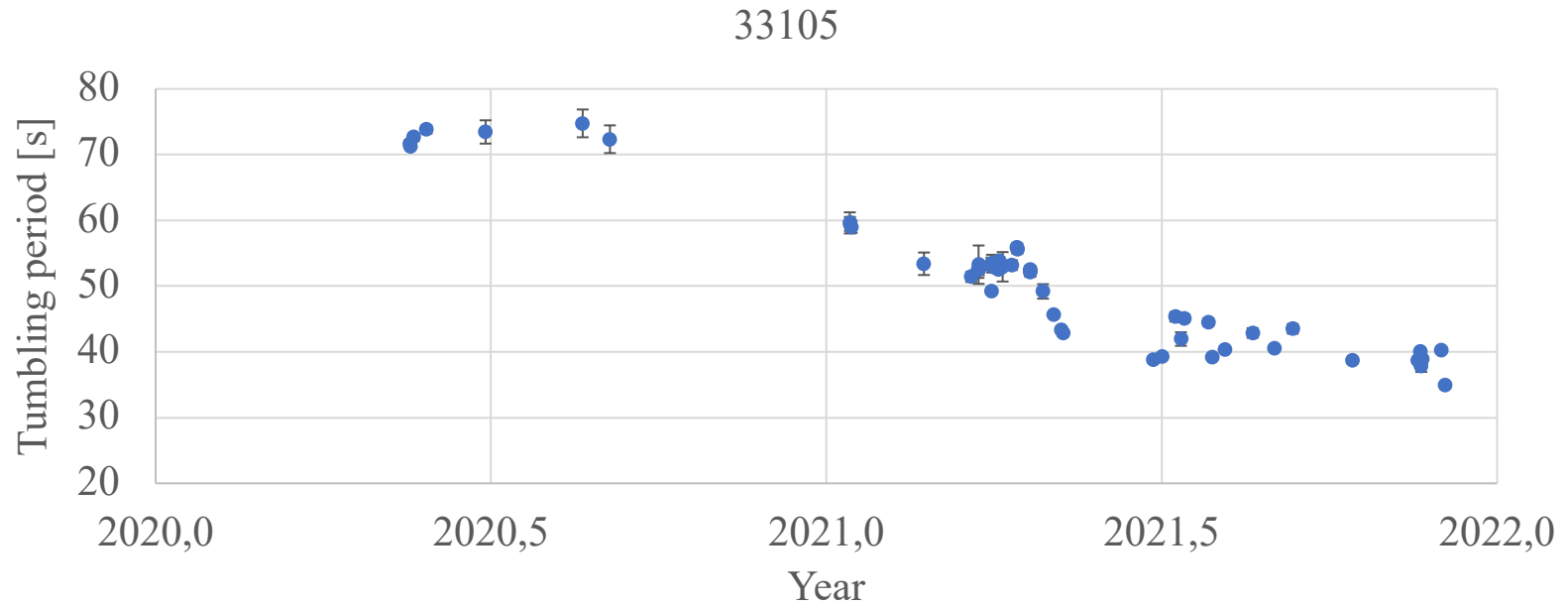


PDM Solution profile
 Theta: 0.753
 Period: 71.627 s
 Frequency: 0.014 Hz
 FWHM: 1.271 s
 StdDev: 0.540 s
 SNR: 132.7
 CoVar: 0.754 %



Frequency analysis Solution profile
 Power: 1608.6
 Frequency: 0.056 Hz
 Period: 17.931 s
 FWHM: 1.871 mHz
 StdDev: 0.794 mHz
 SNR: 70.2
 CoVar: 1.424 %

Results of the epoch methods



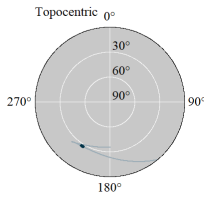
SatCat	
Name	Jason-2
COSPAR	2008-032A
NORAD	33105
Apogee_altitude_[km]	1317
Perigee_altitude_[km]	1305
Orbital_period_[h]	1.863
Inclination_[deg]	66
RCS_[m2]	3.162
Launch_date	6/20/2008
Age_[year]	13.7
Source_or_ownership	fr

DISCOS	
Class	Payload
Shape	Box + 2 Pan
CS_min_[m2]	3.61
CS_max_[m2]	22.433
CS_ave_[m2]	12.823
Mass_[kg]	506
Span_[m]	9.7
Height_[m]	3.7
Diameter_[m]	0
Depth_[m]	1.9
Width_[m]	0

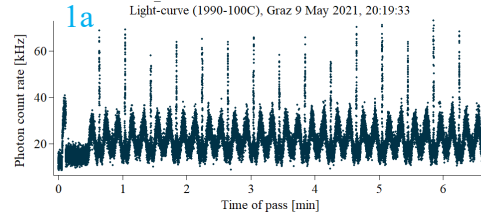
MMT	
Variability	aperiodic
Period_[s]	0
StdMag_Clear	5.7

Results of the epoch methods

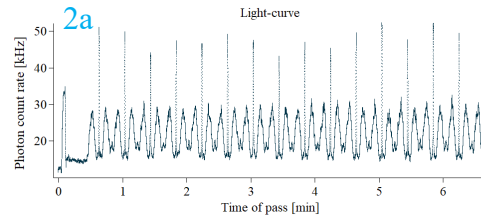
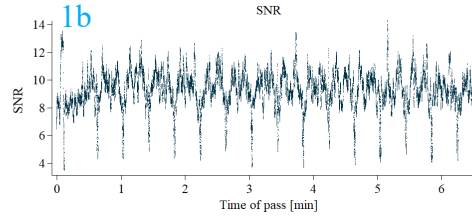
Object: ARIANE 42P R/B
 NORAD: 20947
 Cospar: 1990-100C
 Launch: 20 November, 1990
 Apogee alt.: 21424 km
 Perigee alt.: 215 km
 Inclination: 7.1°
 Orb. period: 6.2 h
 RCS: 28.4 m2
 Object age: 30.5 y
 Exp. spin status: periodic, 23.6 s
 Mag: 3.31 .. 6.34, 4.59, se 0.41
 datafile: a1012918_ct



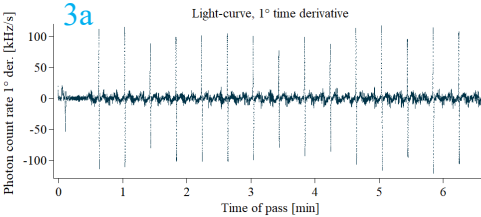
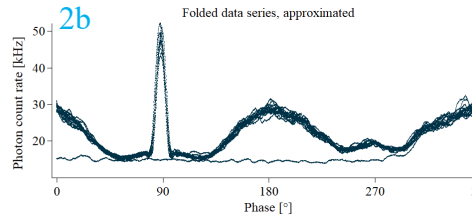
Approximated data: 2-a,b, 4-a,b,c
 Time derivative: 3-a,b, 5-a,b,c



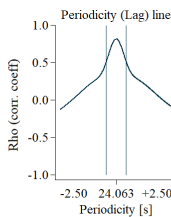
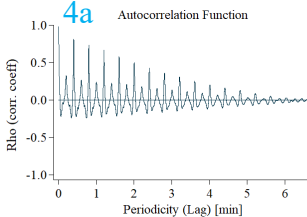
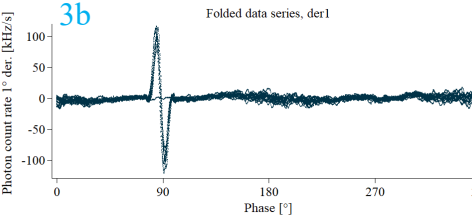
Pass information
 Duration: 6m:37s
 Data points: 39677
 Data rate: 100.0 Hz
 Closest approach
 Topo el: 27.4°
 Slant range: 15562 km
 Phase angle: 62.7°
 Sim. app. mag.: 11.4
 Solar Beta angle: 24.5°



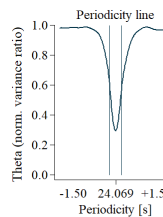
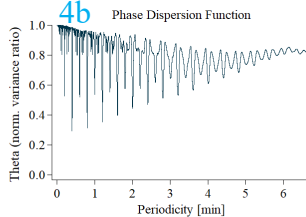
Data approximated
 RMS: 5739 Hz



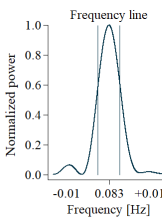
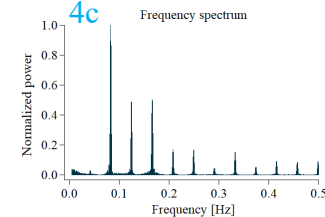
Time derivative, 1°
 RMS: 14970 Hz/s



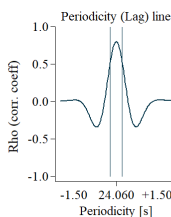
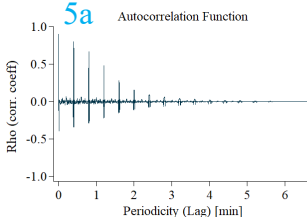
ACF Solution profile
 Rho: 0.814
 Period: 24.063 s
 Frequency: 0.042 Hz
 FWHM: 1.047 s
 StdDev: 0.445 s
 SNR: 54.1
 CoVar: 1.848 %



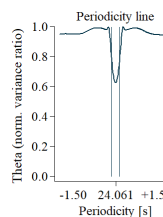
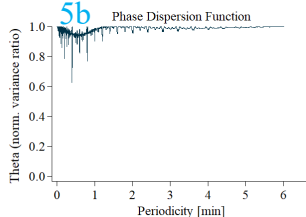
PDM Solution profile
 Theta: 0.293
 Period: 24.069 s
 Frequency: 0.042 Hz
 FWHM: 0.362 s
 StdDev: 0.154 s
 SNR: 156.7
 CoVar: 0.638 %



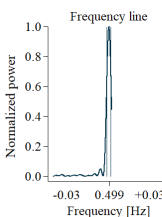
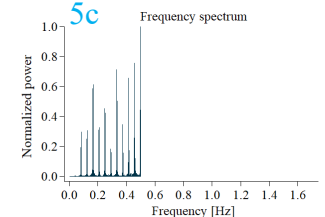
Frequency analysis Solution profile
 Power: 4775.6
 Frequency: 0.083 Hz
 Period: 12.053 s
 FWHM: 2.256 mHz
 StdDev: 0.958 mHz
 SNR: 86.6
 CoVar: 1.155 %



ACF Solution profile
 Rho: 0.813
 Period: 24.060 s
 Frequency: 0.042 Hz
 FWHM: 0.389 s
 StdDev: 0.165 s
 SNR: 145.6
 CoVar: 0.687 %



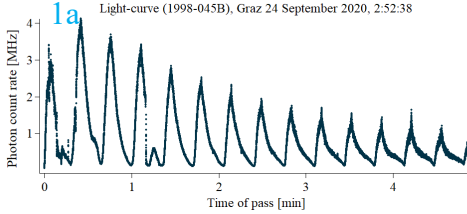
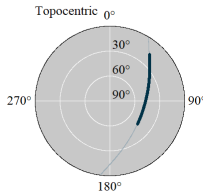
PDM Solution profile
 Theta: 0.620
 Period: 24.061 s
 Frequency: 0.042 Hz
 FWHM: 0.241 s
 StdDev: 0.102 s
 SNR: 235.2
 CoVar: 0.425 %



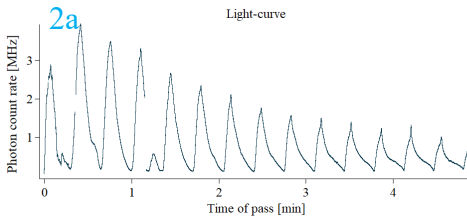
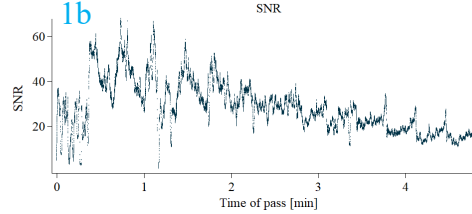
Frequency analysis Solution profile
 Power: 636.4
 Frequency: 0.499 Hz
 Period: 2.005 s
 FWHM: 2.311 mHz
 StdDev: 0.981 mHz
 SNR: 508.2
 CoVar: 0.197 %

Results of the epoch methods

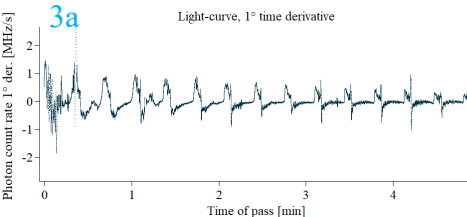
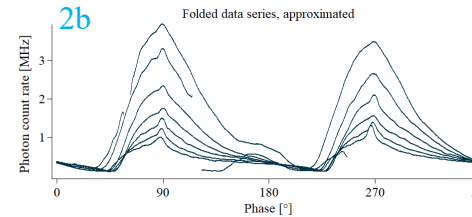
Object: SL-16 R/B
 NORAD: 25407
 Cospar: 1998-045B
 Launch: 28 July, 1998
 Apogee alt.: 844 km
 Perigee alt.: 835 km
 Inclination: 71.0°
 Orb. period: 1.7 h
 RCS: 11.3 m²
 Object age: 22.2 y
 Exp. spin status: -
 Mag: -
 datafile: 33526802_ct



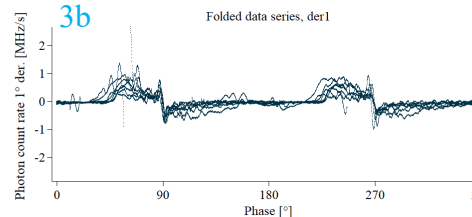
Pass information
 Duration: 4m:52s
 Data points: 29180
 Data rate: 100.0 Hz
 Closest approach
 Topo el: 48.4°
 Slant range: 1088 km
 Phase angle: 102.8°
 Sim. app. mag.: 6.7
 Solar Beta angle: -45.5°



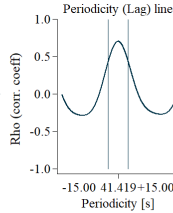
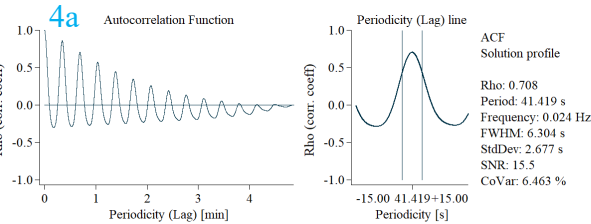
Data approximated
 RMS: 792149 Hz



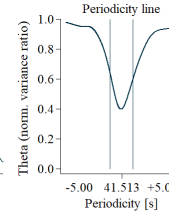
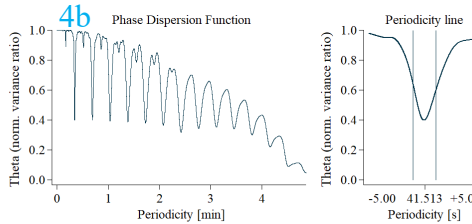
Time derivative, 1°
 RMS: 304280 Hz/s



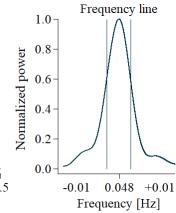
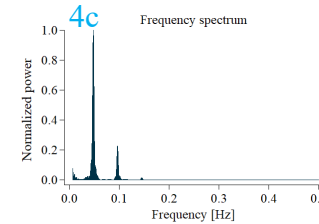
Approximated data: 2-a,b, 4-a,b,c
 Time derivative: 3-a,b, 5-a,b,c



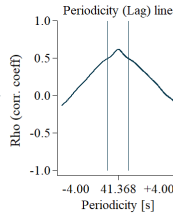
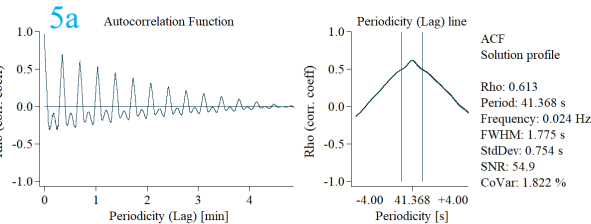
ACF Solution profile
 Rho: 0.708
 Period: 41.419 s
 Frequency: 0.024 Hz
 FWHM: 6.304 s
 StdDev: 2.677 s
 SNR: 15.5
 CoVar: 6.463 %



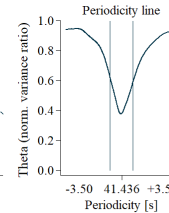
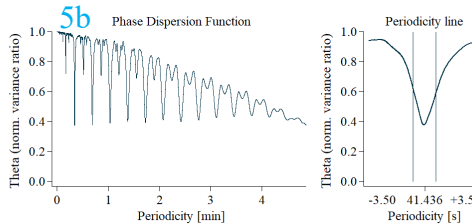
PDM Solution profile
 Theta: 0.401
 Period: 41.513 s
 Frequency: 0.024 Hz
 FWHM: 2.432 s
 StdDev: 1.033 s
 SNR: 40.2
 CoVar: 2.487 %



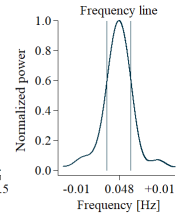
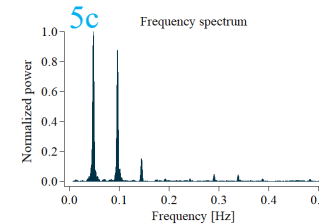
Frequency analysis Solution profile
 Power: 7037.0
 Frequency: 0.048 Hz
 Period: 20.784 s
 FWHM: 3.474 mHz
 StdDev: 1.475 mHz
 SNR: 32.6
 CoVar: 3.067 %



ACF Solution profile
 Rho: 0.613
 Period: 41.368 s
 Frequency: 0.024 Hz
 FWHM: 1.775 s
 StdDev: 0.754 s
 SNR: 54.9
 CoVar: 1.822 %

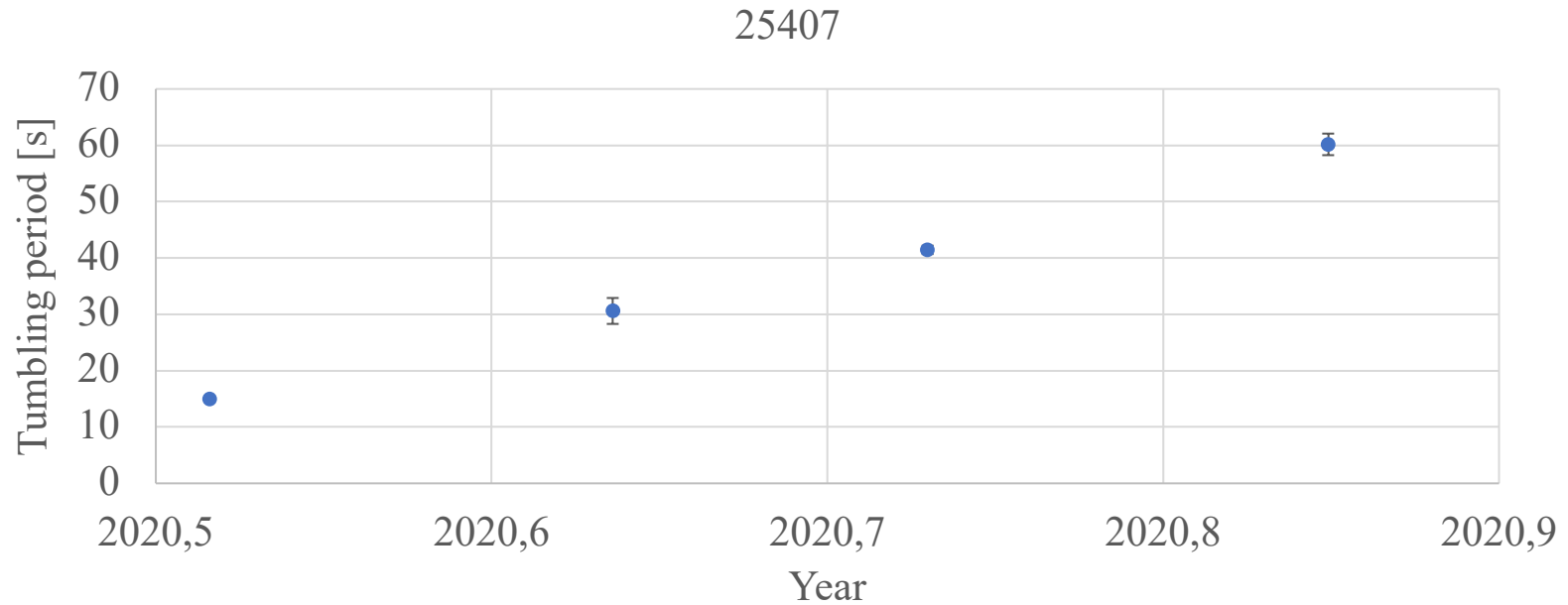


PDM Solution profile
 Theta: 0.390
 Period: 41.436 s
 Frequency: 0.024 Hz
 FWHM: 1.721 s
 StdDev: 0.731 s
 SNR: 56.7
 CoVar: 1.764 %



Frequency analysis Solution profile
 Power: 4072.6
 Frequency: 0.048 Hz
 Period: 20.767 s
 FWHM: 3.459 mHz
 StdDev: 1.469 mHz
 SNR: 32.8
 CoVar: 3.051 %

Results of the epoch methods



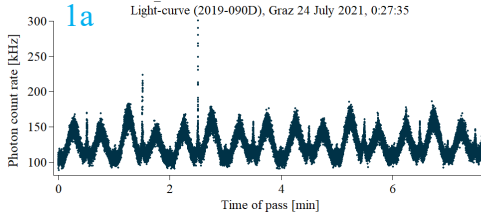
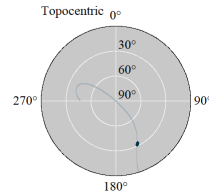
SatCat	
Name	Zenit-2 second stage
COSPAR	1998-045B
NORAD	25407
Apogee_altitude_[km]	844
Perigee_altitude_[km]	835
Orbital_period_[h]	1.695
Inclination_[deg]	71
RCS_[m2]	11.342
Launch_date	7/28/1998
Age_[year]	23.6
Source_or_ownership	cis

DISCOS	
Class	Rocket Body
Shape	Cyl
CS_min_[m2]	11.946
CS_max_[m2]	44.908
CS_ave_[m2]	39.973
Mass_[kg]	8226
Span_[m]	11.1
Height_[m]	11.1
Diameter_[m]	3.9
Depth_[m]	0
Width_[m]	0

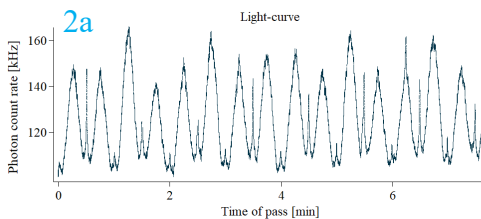
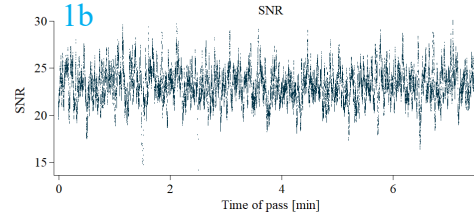
MMT	
Variability	N/A
Period_[s]	-1
StdMag_Clear	999

Results of the epoch methods

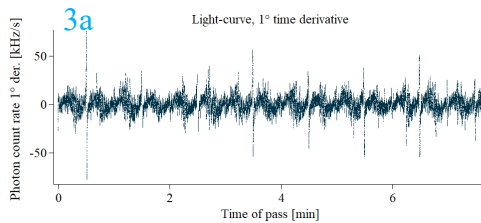
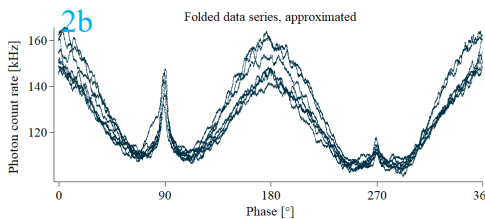
Object: CZ-3B R/B
 NORAD: 44867
 Cospar: 2019-090D
 Launch: 16 December, 2019
 Apogee alt.: 17557 km
 Perigee alt.: 267 km
 Inclination: 55.0°
 Orb. period: 5.2 h
 RCS: -
 Object age: 1.6 y
 Exp. spin status: periodic, 61.7 s
 Mag: 0.53 .. 6.77, 3.31, se 0.65
 datafile: a0420422_ct



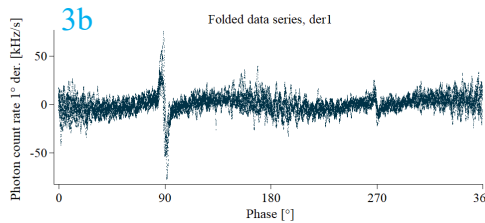
Pass information
 Duration: 7m:37s
 Data points: 45675
 Data rate: 100.0 Hz
 Closest approach
 Topo el: 33.0°
 Slant range: 19751 km
 Phase angle: 44.4°
 Sim. app. mag.: 11.6
 Solar Beta angle: 46.6°



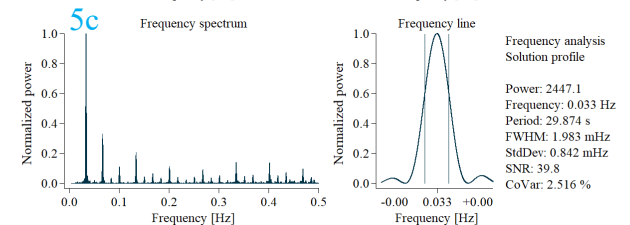
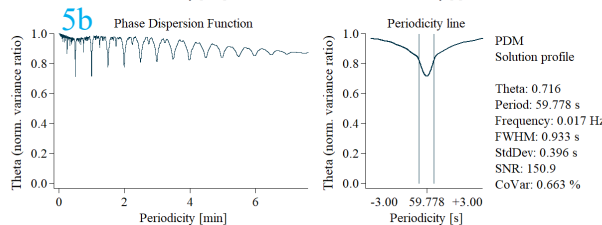
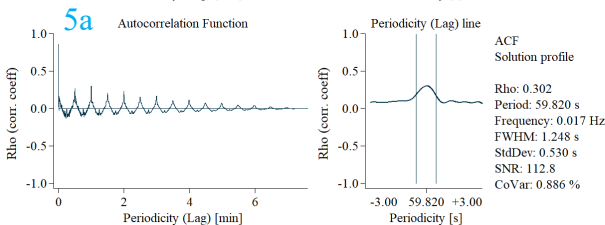
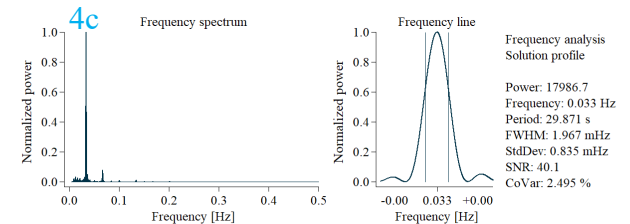
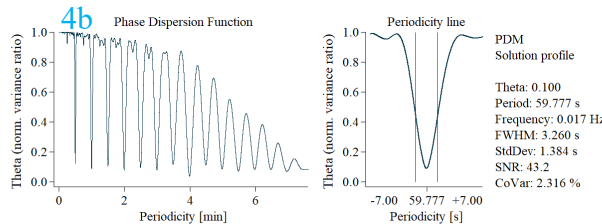
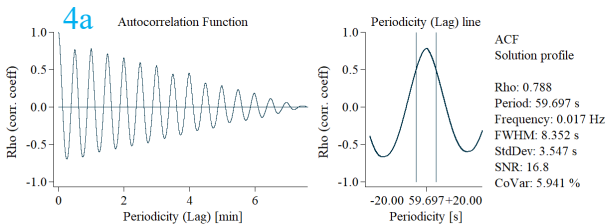
Data approximated
 RMS: 16012 Hz



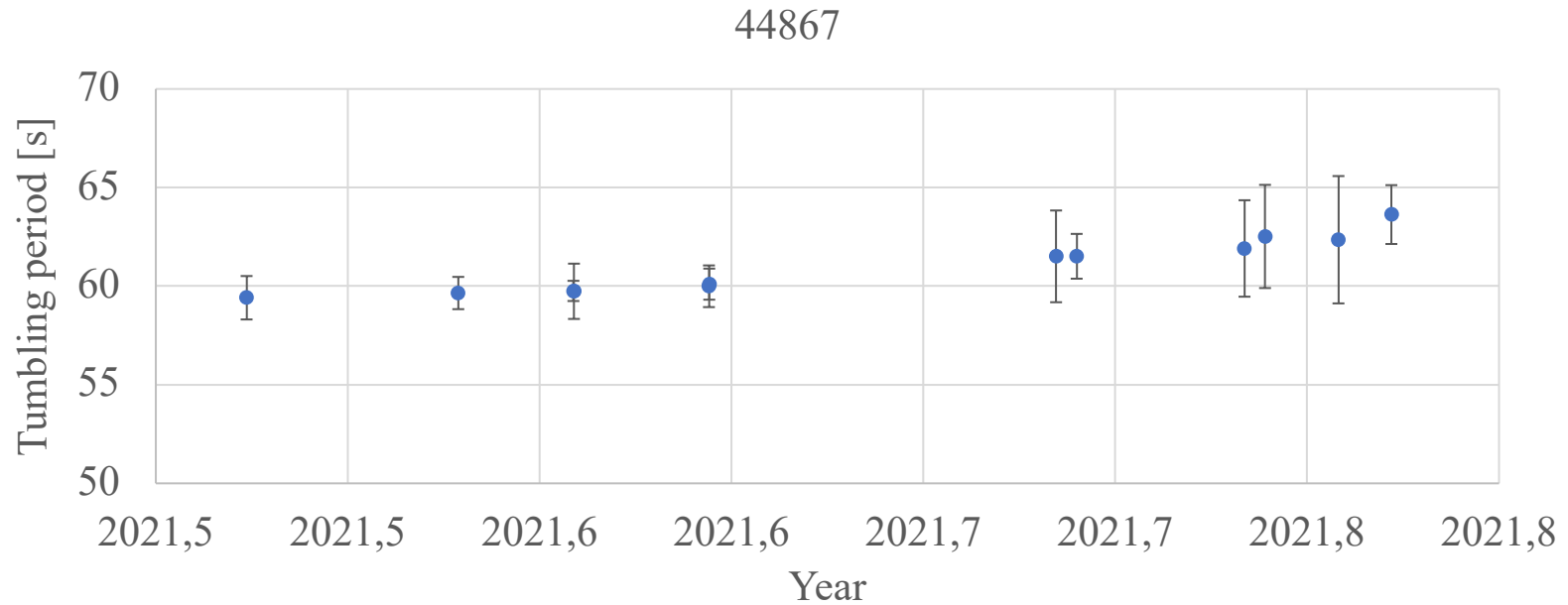
Time derivative, 1°
 RMS: 9028 Hz/s



Approximated data: 2-a,b, 4-a,b,c
 Time derivative: 3-a,b, 5-a,b,c



Results of the epoch methods

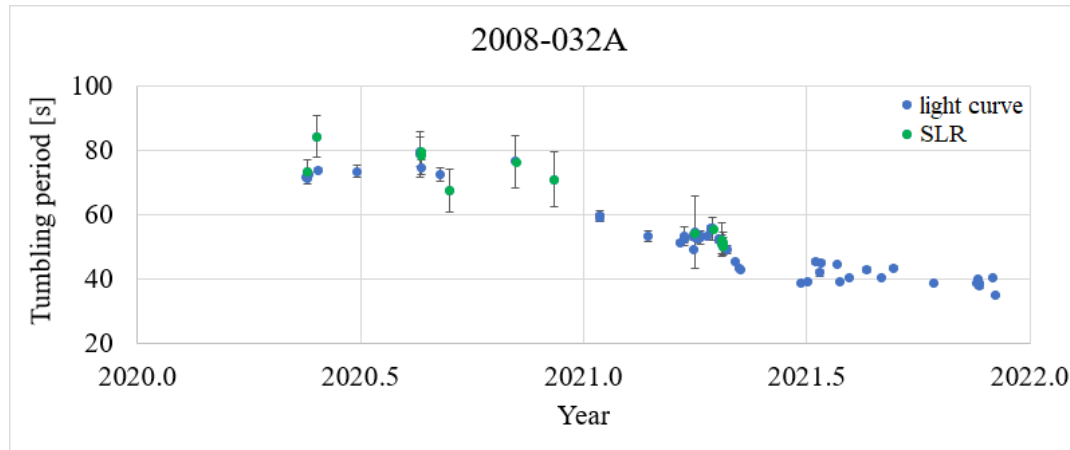


SatCat	
Name	CZ-3B
COSPAR	2019-090D
NORAD	44867
Apogee_altitude_[km]	17561
Perigee_altitude_[km]	268
Orbital_period_[h]	5.228
Inclination_[deg]	55
RCS_[m2]	-1
Launch_date	12/16/2019
Age_[year]	2.2
Source_or_ownership	prc

DISCOS	
Class	Rocket Body
Shape	Cyl + 1 Nozzle
CS_min_[m2]	7.069
CS_max_[m2]	39.134
CS_ave_[m2]	33.764
Mass_[kg]	2800
Span_[m]	3
Height_[m]	0
Diameter_[m]	0
Depth_[m]	0
Width_[m]	0

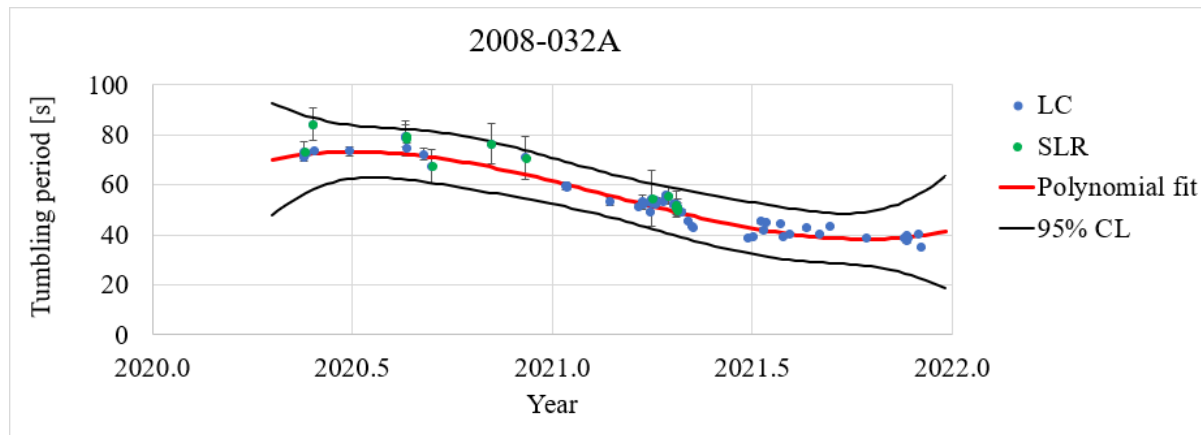
MMT	
Variability	periodic
Period_[s]	61.7
StdMag_Clear	3.3

Data fusion



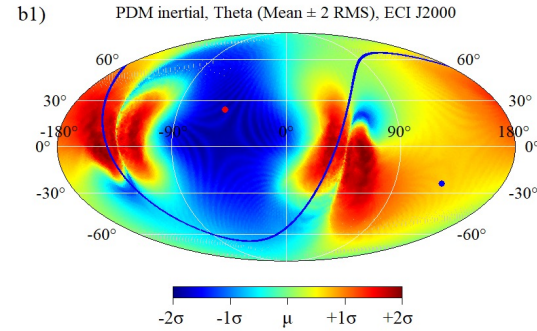
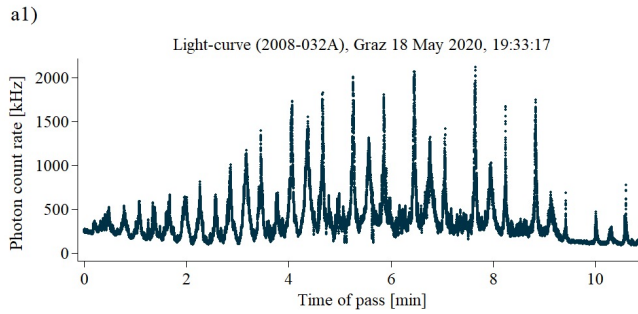
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Observed spin period of Jason-2 determined with epoch analysis of Graz single-photon light curves (blue dot) and SLR data (green dot). Appendix C presents example analysis.

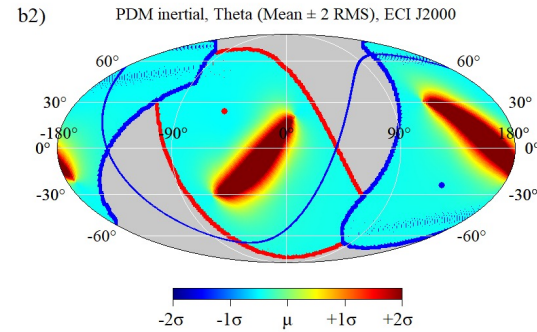
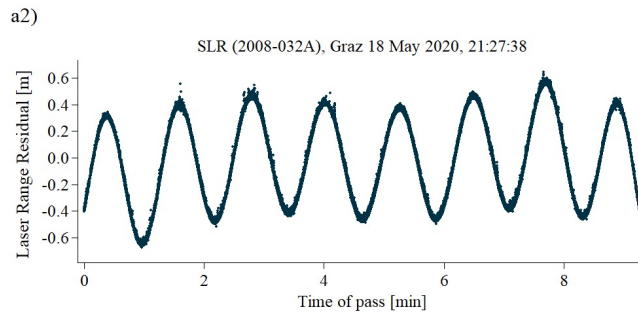


Fusion polynomial fit to light curve and SLR data with 95% confidence limits. A 3-degree weighted polynomial fit is constructed with fused spin period data from light curves and SLR observations of Jason-2.

Data fusion



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Jason-2 light curve (a1) and SLR (a2) data measured by Graz on 18 May, 2020.

Plots b1, b2 - present results of the inertial phase-folding applied to the respective time series.

Red and blue dots represent location of the positive and negative orbit normal vectors; a blue curve is a satellite orbital plane on the inertial sphere).

Conclusions

- The tumbling motion determination processes can be efficiently performed with a set of epoch methods that extract frequency and periodicity spectra from the observational data,
- this, however, requires an initial data treatment with Savitzky-Golay Filters (SGF) that have powerful denoising and detrending properties, but come at high computational cost.
- The spin vector orientation determination with the epoch methods can deliver only approximate results – a range of possible solutions in a given coordinate system.
- During this project we have identified several areas where more research could lead to a higher level of automation of the tumbling motion determination (multi-pass analysis, variable degree of SGF).

Danke für Ihre Aufmerksamkeit !

