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# Pointing Error Engineering Software Framework

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**Executive Summary** 





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# **Document Change Record**

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#### 1 Applicable and Reference Documents

#### 1.1 Applicable Documents

- [AD1] Cooperation on Space Standardization, Control Performance Standard ECSS-E-ST-60-10C, ESA-ESTEC Requirements & Standards Division, 2008.
- [AD2] Engineering Standardisation Board, Pointing Error Engineering Handbook ESSB-HB-E-003, ESA-ESTEC Requirements & Standards Division, 2011

#### 1.2 Reference Documents

- [RD1] Pittelkau, M.E., "Pointing Error Definitions, Metrics, and Algorithms", American Astronautical Society, AAS 03-559, p. 901, 2003.
- [RD2] Bendat J.S., Piersol A.G., Random Data Analysis and Measurement Procedures, Wiley, 3rd edition, 2000
- [RD3] IEEE Aerospace and Electronic Systems Society, IEEE Standard Specification Format Guide and Test Procedure for Linear Single-Axis, Nongyroscopic Accelerometers, IEEE Std 1253-1998 (R2008), IEEE, New York, NY, USA, 2008
- [RD4] IEEE Aerospace and Electronic Systems Society, IEEE Standard Specification Format Guide and Test Procedure for Single-Axis Interferometric Fiber Optic Gyros, IEEE Std 952-1997; IEEE, New York, NY, USA, 1998
- [RD5] Gentle, J.E, Computational Statistics, Springer, 2009
- [RD6] Gosh. S, Dependence in Stochastic Simulation Models, PhD thesis, Cornell University 2004
- [RD7] Simulating Dependent Random Variables Using Copulas, Mathworks Online Documentation, http://de.mathworks.com/help/, last accessed March 5, 2016



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# 2 Terms, Definitions and Abbreviated Terms

#### 2.1 Acronyms

The following abbreviations are used throughout this document.

Acronyms	
AD	Applicable Document
ECSS	European Cooperation for Space Standardization
ESA	European Space Agency
GUI	Graphical User Interface
PDF	Probability Density Function
PES	Pointing Error Source
PEC	Pointing Error Contributor
PEET	Pointing Error Engineering Tool
RD	Reference Document
sw	Software



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#### 3 Introduction

This document describes the work performed in the project *Pointing Error Engineering Software Framework* completed by Astos Solutions under contract of the European Space Agency with Airbus Defence and Space as subcontractor and the iFR (University Stuttgart) as consultant.

This activity is a follow-up and extension of the Pointing Error Engineering Tool prototyping phase, which developed a preliminary version of the error engineering tool with certain methodological restrictions and with basic GUI functionalities only. This GSTP activity complements the tool with advanced statistical methods and puts further focus on user-friendliness aspects.

The release version of PEET is a software tool which is dedicated to support systems and AOCS engineers in the setup and calculation of pointing and relative-positioning error budgets. It is based on standardized rules from the ECSS Control Performance Standard E-ST-60-10C [AD1]and the methodology described in the ESA Pointing Error Engineering Handbook [AD2].



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#### 4 Tool Overview

#### 4.1 Platforms and Requirements

PEET can be used on multiple platforms (Windows, Linux and Macintosh) on a standard desktop PC or laptop. The tool is designed as an extension to MATLAB and completely runs inside the MATLAB environment. Apart from a plain MATLAB installation (2011b and later), only the Control System Toolbox is required.

#### 4.2 Architecture and External Interfaces

The static architecture of PEET is shown in Figure 4-1. It mainly consists of two components: a dedicated graphical user interface (GUI) based on Java and the core computational routines implemented as MATLAB classes.

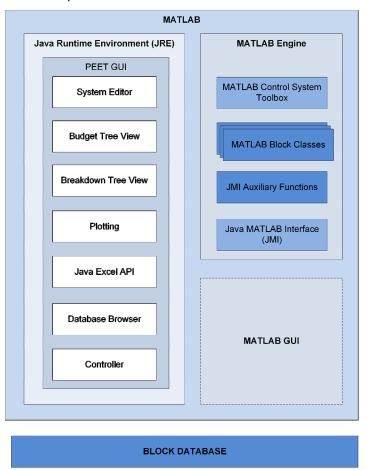


Figure 4-1: PEET static architecture

The GUI is used to define requirement specification parameters, values and identifiers and to set up the pointing system from error sources to the final error using blocks from a database. This can include system transfer models (as in Figure 4-2) or simply comprise a summation of errors on different requirement levels.



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Input data can directly be specified in tables or input fields or imported from MS Excel spreadsheets. In addition to numerical inputs, also MATLAB variable names and notation can be used to specify parameters. All relevant scenario data is stored in an XML file which serves as interface for the MATLAB core classes for the initialization and evaluation of the budget.

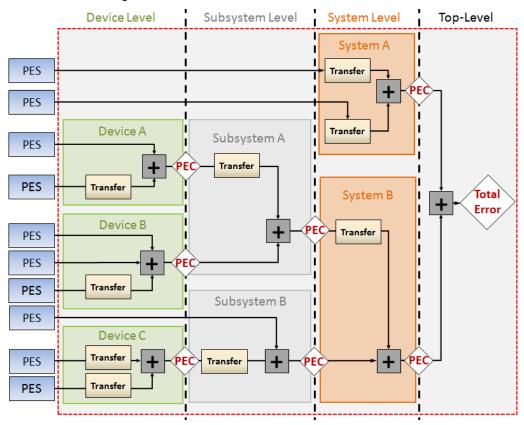


Figure 4-2: Routing from error source to the total error

Once a pointing system is created and saved using the GUI, two operational modes are possible. First, the GUI can be used directly to start an evaluation and to inspect the results. The communication between the GUI and the MATLAB classes is realized in this case via the Java MATLAB Interface (JMI). Second, a script-based execution of the tool via user-defined Matlab scripts is possible. Together with assigning MATLAB workspace variables to system parameters, this allows batch-mode operations without further use of the graphical user interface and an integration of PEET in a tool-chain with other analysis modules.

The budget results obtained with PEET can further be exported to MS Excel using a configurable report.



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#### 4.3 Graphical User Interface

This section gives a brief introduction to the features of the PEET GUI which consists of several dedicated windows:

#### **System Editor**

The editor panel in the System Editor (Figure 4-3) is the main tool to design the architecture of a pointing scenario. It can be populated with a selection of model blocks from the Block Database which then need to be connected to represent the error signal flow. The workflow for moving and connecting blocks is intentionally similar to the workflow with MATLAB Simulink. Also different levels (subsystems) are supported for a better overview in complex systems. Double-clicking a block opens a dialog where related parameters including signal & parameter units can be specified (supported by tooltips). The latter can be chosen from a predefined set of SI and non-SI units and also custom units can be created. When connecting blocks, the compliance of units is automatically checked by the tool.

The menus present in the System Editor window serve for file management, requirement definition (multiple requirements sets can be specified in a single scenario) and definition of error source dependencies (correlation, coherence and phase relations).

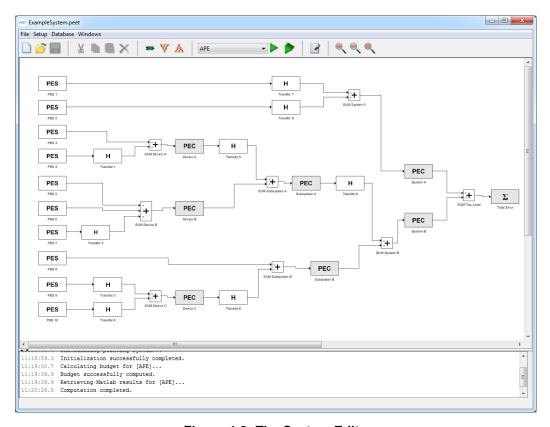


Figure 4-3: The System Editor

An execution log serves as scope to track the progress of the evaluation and issues occurring meanwhile (e.g. invalid user parameter inputs).



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#### **Block Database**

The Block Database (Figure 4-4) - similar to the Library Browser in MATLAB Simulink – contains all building blocks which can be used to populate a pointing scenario. The blocks are categorized in groups (errors sources, static/dynamic systems, etc.).

It contains both generic blocks and parametric models of sensors, actuators and transfer systems which are based on standardized models wherever possible (e.g. [RD3], [RD4]). For each model, the block database shows a "quick-view" help with important information about the model (purpose, input/output dimensions, background).

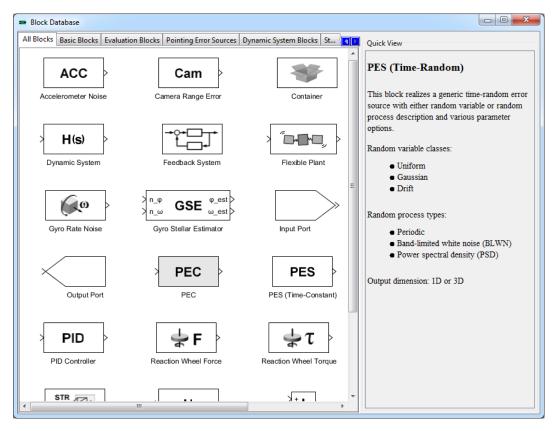


Figure 4-4: The Block Database

#### **Budget Tree View**

The Budget Tree View (Figure 4-5) serves to analyze error contributions and the signal content (from different error signal classes) of the entire pointing system. Selecting a block in the tree-like representation of the pointing system shows the related signal content in the information panel on the right. For each signal class, statistical information is provided in terms of mean and standard deviation together with a plot preview of the PDF (or PSD respectively for a random process error signal). In case of spectral requirements, only random process contributions are displayed.



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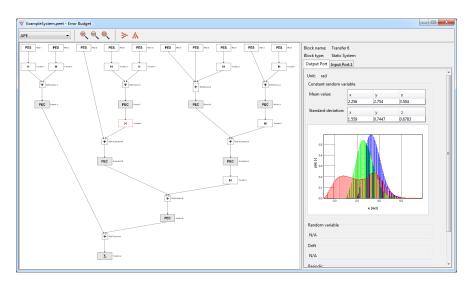


Figure 4-5: The Budget Tree View

#### **Breakdown Tree View**

The Breakdown Tree View (Figure 4-6) is used to check the compliance of the budget. It shows only those blocks where requirements have been associated with (by value and optionally an ID). For statistical requirements, the information panel displays a comparison of budget and requirement values for the time-constant, time-random and total error contributions on all axes together with a plot preview of the underlying CDF. For spectral requirements, the budget spectrum is plotted versus the specified requirement function.

In addition, colour-coding is used in the tree view for an easy determination of budget violations or proximity to margins.

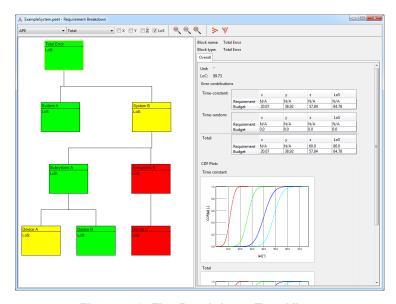


Figure 4-6: The Breakdown Tree View



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#### **Plot Window**

The Plot window can be accessed by double-clicks on any preview plot in the Budget Tree View or the Breakdown Tree View. Here typical user interactions such as data picking, axes selection, zooming or exporting plots are available. By default, the window just shows an enlarged version of the preview, however also further information can be displayed, such as scatter plots indicating the correlation or cross-spectral density plots.

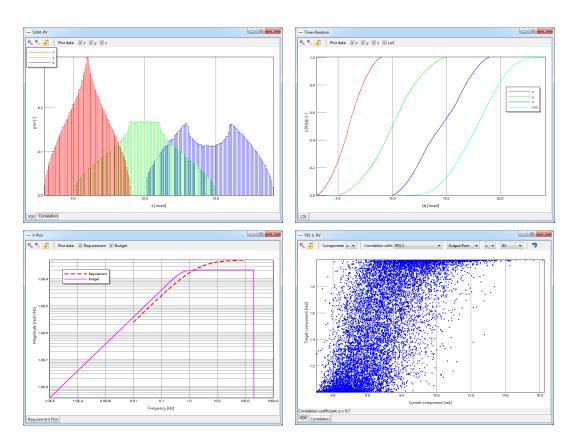


Figure 4-7: Selected plot types: PDF plots (top-left), CDF plots (top-right), PSD vs. spectral requirement function (bottom-left), scatter-plot indicating correlation (bottom-right)

#### 4.4 Software Limitations

PEET needs to be considered as a systems engineering tool for budgeting with a focus on steady-state solutions rather than a replacement for a non-linear simulator. Functionalities concerning the treatment of non-linearities, transients, system uncertainties and non-stationary random processes cannot be explicitly accounted for in the tool. There is a lack of standards for these topics which are partially at the interface of research and tool development where dedicated algorithms and methods are not mature enough.

Furthermore, although the tool takes over the entire computation chain of an error budget, still a certain amount of user expertise on (pointing) error engineering is required.



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In particular, for a proper definition of requirements or modelling of error sources, no universal guidelines or automated rules can be provided. Finally, PEET is not intended to be a tool for control/estimator design & tuning, as e.g. the no stability checks of closed-loops in defined feedback systems are carried out. The software intentionally covers only the perimeter of the PEEH (i.e. the pointing error engineering itself) and provides interfaces such that users can plug PEET into own tools for design/tuning.



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#### 5 Translating the PEEH Methodology to the Tool

The methodology described in [AD2] defines explicit guidelines on how to define requirements and how to define and evaluate pointing errors from source to the final error contribution using dedicated analysis steps (ASTs). This section summarizes the reflection of these steps in the software tool.

#### 5.1 Requirement Specification Parameters ("AST-0")

Dependent on its nature (statistical or spectral), [AD2] provides sets of specification parameters to unambiguously define pointing requirements which are completely reflected in the tool. For statistical requirements, the following parameters are used:

- Required maximum error value (defined as deviation per axis or half-cone error)
- A related level of confidence that above-mentioned error values are not exceeded
- Metric for time-windowed errors in terms of performance/knowledge indices as defined in [AD1] and related parameters (e.g. window time)
- The statistical interpretation that applies to the evaluation

The statistical interpretation as defined in [AD1] describes how the resulting errors shall be treated in the evaluation concerning the domains "time" and "ensemble" (e.g. different satellites, observations, etc.). As depicted in Figure 5-1, "ensemble" interpretation accounts for the statistics of the worst-case errors in each realization no matter when they occur in time. "Temporal" interpretation accounts for the temporal statistics of the realization that contains the worst-case value and "mixed" interpretation finally accounts for the entire statistics over both time and ensemble realizations.

This concept has been further generalised during the PEET project to obtain a more flexible definition of requirements (see also section 6.2).

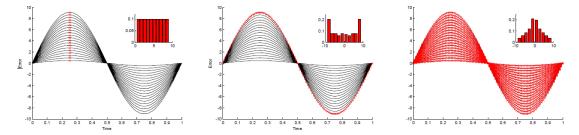


Figure 5-1: Statistical interpretations [AD1]: "ensemble" (left), "temporal"(centre), "mixed" (right)

For spectral requirements, only the metric for time-windowed errors is used. All other specification parameters above are replaced by a spectral requirement function that describes the upper bound of the power spectral density of the error over frequency.



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#### 5.2 Error Source Characterization (AST-1)

This analysis step aims for identifying all potential error sources that affect the budget and classifying these sources on the basis of their characteristic properties (e.g. time-constant vs. time-random, random vs. deterministic, random variable vs. random process).

The first part obviously remains an engineering task as it differs for each application case and the tool cannot provide explicit support. For the classification however, PEET covers all the error signal classes listed in [AD2] which can be modelled as:

- Time-constant random variables, i.e. biases which can be further subject to an ensemble-randomness (e.g. alignment within a certain bound)
- Time-random variables, i.e. errors that are random over time; the parameters of such sources can further be subject to an ensemble-randomness (e.g. a noise whose upper bound is dependent on temperature conditions that are considered constant within an observation but may differ between observations)
- Random processes, i.e. (stationary, ergodic) noise sources that are completely defined by their power spectral density or by a given variance and bandwidth
- Periodic errors, in particular sinusoidal disturbances whose amplitudes may again be subject to an ensemble-randomness.
- Drift errors, i.e. linearly increasing errors which can be reset after a certain amount of time (e.g. due to calibration); the linear slope may again be distributed over the ensemble of realizations

All ensemble-random properties of the error source classes can either be set as discrete or be selected from common statistical distributions: uniform, (truncated) Gaussian, arcsine, Rayleigh. Furthermore, also arbitrary probability density functions (PDFs) can be defined by the user.

For time-random variables, the temporal distributions are restricted to be uniform and Gaussian, as only for these distributions rules are provided in [AD1] on how the pointing error metrics in AST-3 need to be applied.

#### 5.3 Transfer analysis (AST-2)

The transfer analysis determines how the initial error sources (e.g. a noise or misalignment of a sensor in its mechanical frame) affects the figure of merit defined by the requirement (e.g. the pointing error of a payload in the pointing reference frame). The "route" to this final pointing error contribution may underlie various transfers such as coordinate transformations or closed-loop transfers. The situation for a general transfer analysis setup over multiple subsystem levels in sketched in Figure 4-1.

As for the error source classification, no distinct rules can be provided for the identification of relevant transfer models for a certain application case. However, the tool provides a selection of generic static and dynamic system models as well as dedicated parametric models for certain sensor and actuator error characteristics.

The dynamic system models available are based on linear time-invariant models with multiple inputs/outputs to account for cross-couplings between different axes. Feedback loops can be realized by either directly specifying the closed-loop transfer function or by setting up the loop explicitly in a dedicated model block.



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The transfer rules for the dynamic system transfer depend on the class of the error signal. Periodic and random process errors are accurately transferred using the system gain at the relevant frequency or the entire frequency response respectively [RD2]. Time-constant random variables are scaled using the system's DC gain. For time-random variables, where no frequency domain information is present by definition, the system's  $H_{\infty}$  is used as conservative upper bound.

#### 5.4 Error Index Contribution (AST-3)

This step analyses the impact of the time-windowed pointing error metric as one part of the requirement specification parameters. The definition of all available metrics is depicted in Figure 5-2 with the typical abbreviations as defined in [AD1] and [AD2].

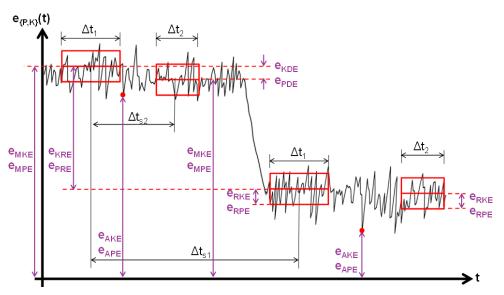


Figure 5-2. Time-windowed error contributions for different pointing metrics

As an example: the relative performance error (RPE) describes the variation of an error with respect to the mean error in a given time window  $\Delta t$ . If this metric is applied to a periodic signal with a very large period compared to the window time, the variation within the window is much smaller than the signal amplitude. Conversely, for a high frequency periodic signal, the entire amplitude variation shows up within one time window.

For random variables, [AD1] provides explicit tables with rules on how the various metrics affect the effective errors which are completely integrated in the tool. For random process and periodic error signals, an equivalent representation of the metrics in terms of rational transfer functions exists [AD2], [RD1]. This allows an accurate frequency domain contribution analysis with the tool.

#### 5.5 Error Evaluation (AST-4)

The final analysis step accounts for the evaluation of the total pointing error or the current error at any stage where a (sublevel-) requirement is defined. According to [AD2], three



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contributions are assessed for statistical requirements: the time-constant, the time-random and the total error contribution where in the end the absolute value of the error for a given level of confidence is of interest.

The summation of the error contribution depends on the statistical method applied [AD2]. For the "simplified statistical method" (realized in the PEET prototype), the error contributions are fully described by their equivalent mean and standard deviation (i.e. the underlying distribution is discarded at this point). Mean values are summed linearly. Standard deviations are summed either in an RSS sense (assuming no correlation) or linearly (assuming full correlation). Then, a confidence factor  $n_p = 1,2,3$  is used to scale the overall standard deviation (i.e. to represent a  $1\sigma$ ,  $2\sigma$  or  $3\sigma$  confidence level).

The "advanced statistical method" relies on a characterisation of the error properties in terms of probability density functions (PDFs). The total error is generally obtained via convolution of the joint PDF of all error contributions. For the resulting PDF, the respective cumulative distribution function is computed and evaluated for the level of confidence specified in the requirement.

The major restrictions of the simplified statistical approach is that is assumes applicability of the central limit theorem. For that reason, one of the key extensions of the PEET release version is an update to the advanced statistical method (see section 4.1).

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#### 6 New Features

#### 6.1 Advanced Statistical Method

#### **Background**

The PEET prototype was restricted to the simplified statistical method described in [AD1] and [AD2], i.e. the error evaluation is based on the assumption that the central limit theorem is applicable and the total error follows a (nearly) Gaussian distribution (on all axes). While this assumption holds for a "sufficient" number of contributors which have approximately the same magnitude, a significant systematic error is present when dominating non-Gaussian contributions exist.

Figure 6-1 depicts such situation for two zero-mean distributions (for simplicity & without loss of generality): Assume that the absolute value of the error shall remain below a given value with 99.7% probability ( $p_c$  = 0.997), i.e. with a confidence factor  $n_p$  = 3 as mentioned in section 2.5. For an ideal Gaussian, this obviously results in the exact error value of e =  $3\sigma$  "by definition". For a uniform distribution with bounds  $\pm b$ , the standard deviation is given by  $\sigma_u$  =  $b/\sqrt{3}$ . Applying the same confidence factor under the same assumptions would then result in error value  $e = \sqrt{3}b$  while the correct result would be e = 0.997b. Thus, a systematic error of more than 80% is present which (already for a " $2\sigma$  error") exceeds the explicit bounds of the underlying distribution.

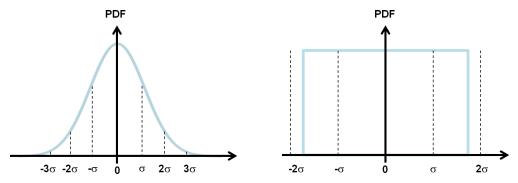


Figure 6-1. Location of typical confidence for a Gaussian distribution (left) and for a uniform distribution (right) "assuming" a Gaussian shape

Another issue with the simplified statistical method is related to the evaluation of line of sight errors (LoS) - even if the errors on individual axes are ideally Gaussian. Assume the z-axis is the desired LoS axis and rotational errors around the 3 target axes are given by  $[e_x, e_y, e_z]$ . Then the proper statistical evaluation of the error is represented by:

$$e_{LOS} = \int_{0}^{LoC} p \left( \sqrt{e_{x}^{2} + e_{y}^{2}} \right) de$$
 (1)

[AD2] provides a derived expression which is explicitly valid for instantaneous (or deterministic) LoS errors, i.e. it is not applicable to "statistical" axes errors evaluated from  $|\mu_i| + n_p \sigma_i$ .

$$e_{LOS} = \sqrt{e_x^2 + e_y^2}$$
 (2)



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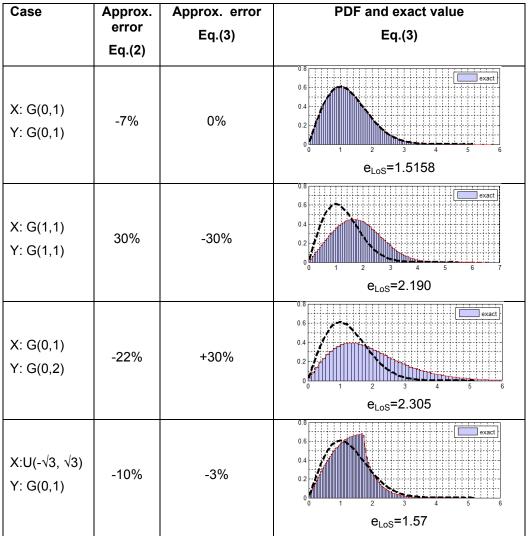
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For "statistical" errors, [AD1] provides an approximate solution of the form

$$e_{LOS} = \max(\sigma_x, \sigma_y) \sqrt{-2\log(1 - p_c)}$$
 (3)

which is valid for zero-mean Gaussian distributions with closely equal standard deviations on the relevant axis. A "careless" application of equations (2) or (3) to non-matching conditions (e.g. non-Gaussian, non-zero means, significantly different standard deviations) may generally lead to significant systematic errors as depicted in Table 1.

Table 1. Exemplary 68.3% line-of-sight errors



One way to treat these issues is to investigate if the central limit theorem or abovementioned LoS conditions are applicable and assess the impact case-by-case. The other way is to fully discard the evaluation based on mean on standard deviation only and systematically account for the underlying probabilities using the advanced statistical method.



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#### **Implementation Details**

An analytical treatment of the advanced statistical method implies that the "summation" of moments is replaced by the "summation" of distributions (i.e. convolution). This step as well the subsequent evaluation of errors with a given level of confidence from the cumulative distribution functions (CDFs) requires integration of PDFs. If also correlation between different contributions needs to be considered, this further implies to knowledge of the joint PDF all error sources for the convolution.

These preconditions are however in conflict with several constraints for the implementation:

- Apart from the Control System Toolbox, no further MATLAB toolboxes shall be used (e.g. the Symbolic Toolbox)
- Even with a toolbox for symbolic integration/ convolution, closed-form solutions for an arbitrarily complex system cannot be guaranteed
- Even with a numerical approach for the convolution, the required joint PDF is usually
  not known by the user and cannot simply be determined; as a maximum, the marginal
  distributions of the sources and correlation in terms of a correlation coefficients are
  expected to be available to users

For these reasons, an entirely numerical (sample-based) approach is realized in PEET. The intrinsic drawback of this approach is a loss of accuracy with respect to the analytical computation. The induced error is however expected to be smaller than 1% in the overall computation chain using a sufficiently large sample number (around 1e6 per error source) and a sufficiently large bin number (around 10000) for retrieving the PDF and CDF by numerical integration.

The cost of this numerical error can however safely be neglected compared to the potentially large systematic errors related to the simplified statistical method.

The numerical approach for random variable error sources and distributed parameters requires the generation of random samples that represent both the random variable with the desired distribution and the desired correlation between the axes and error sources.

Several methods exist to create samples of correlated standard normal distributed random variables. The method used in PEET is based on [RD5]. It first requires the covariance matrix  $\Sigma$  of size  $m \times m$  for the m error signals to be realized. As standard normals are used (i.e. having unity variance), the covariance matrix is equal to the matrix of correlation coefficients. Then a Cholesky decomposition is applied to the matrix to obtain an upper triangular matrix A:

$$\mathbf{\Sigma} = \mathbf{A}^{\mathrm{T}} \mathbf{A} \tag{4}$$

In a next step, a vector  $\mathbf{n}$  of length  $n_s$  is drawn from of standard normal distribution. Both operations are easily feasible using standard MATLAB functions. Then a matrix  $\mathbf{N}$  of size  $n_s$  x m is computed:

$$\mathbf{N} = \begin{bmatrix} \mathbf{n}_1 & \dots & \mathbf{n}_m \end{bmatrix} = \mathbf{A} \mathbf{n} \tag{5}$$

This matrix contains m vectors of standard normal samples with the desired correlation between each vector. The next step is to transform these to the desired target



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distribution. This is realized using the so called NORTA (NORmal To Anything) algorithm [RD6] based on an inverse transform sampling method.

In an intermediate step, the CDF of the normal distribution (hard-coded in the tool) is applied to each column in the matrix N.

$$\mathbf{U} = \begin{bmatrix} \mathbf{u}_1 & \dots & \mathbf{u}_m \end{bmatrix} \quad \mathbf{u}_i = \mathbf{\Phi}(\mathbf{n}_i) \tag{6}$$

In this specific case, this results in a set of vectors  $\mathbf{u}_i$  whose samples represent a uniform distribution between 0 and 1. Then, similarly the inverse CDF (ICDF) of the target distribution is applied to each value of the vectors  $\mathbf{u}_i$ .

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_1 & \dots & \mathbf{x}_m \end{bmatrix}, \quad \mathbf{x}_i = ICDF_{\text{target}}(\mathbf{u}_i)$$
 (7)

The ICDFs of the target distributions are also represented numerically in Eq. (7) and each  $x_i$  is interpolated for the current  $u_i$  value.

This finally gives vectors of random samples  $\mathbf{x}_i$  for each error signal which describe random variables with the desired PDF. And, more important, the transformation method preserves the correlation.

Preserving the correlation under any monotonic transformation is only valid assuming rank correlation (i.e. Spearman coefficients), but not for linear correlation as e.g. represented by Pearson product-moment coefficients [RD6], [RD7]. As the matrix  $\Sigma$  itself requires Pearson product-moment coefficients  $\rho_p$  for the setup, first a conversion from Spearman ( $\rho_s$ ) to Pearson ( $\rho_p$ ) coefficients is internally realized [RD7]:

$$\rho_p = 2\sin\left(\rho_s \frac{\pi}{6}\right) \tag{8}$$

This also implies that correlation coefficients specified in PEET represent Spearman rank correlation coefficients, not Pearson product-moment coefficients.

#### 6.2 Concept of Statistical Domains

The concept of statistical domains is a compliant generalisation of the statistical interpretation concept in [AD1] and [AD2] which allows a more flexible definition of requirements.

Basically, the statistical interpretation is a rule for how the "pair" of temporal and ensemble properties of each error source shall be evaluated, i.e. either taking into account the worst-case or the statistical distribution of the domains "time" and "ensemble".

While the domain "time" is unique for all error sources, different "ensemble" domains can exist in a pointing scenario (indicated in Figure 6-2) dependent on the "background" of the randomness, e.g. random misalignments due to the manufacturing, different sensor noise levels due to different orientation in space or temperature conditions.

The statistical interpretation inseparably treats contributions from these domains and there is no possibility to adapt their statistical evaluation individually apart from manually "remodelling" error source descriptions. With the domain concept, the statistical treatment (worst-case or statistical) of the domain "time" and each "ensemble" domain can be configured individually according to the evaluation needs of a requirement.



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This allows for instance, the evaluation of a budget for the worst case satellite ("temporal interpretation" for manufacturing errors), but statistical evaluation of errors varying over an ensemble of observations ("mixed interpretation").

Additionally, also different levels of confidence for contributions from different domains can be defined.

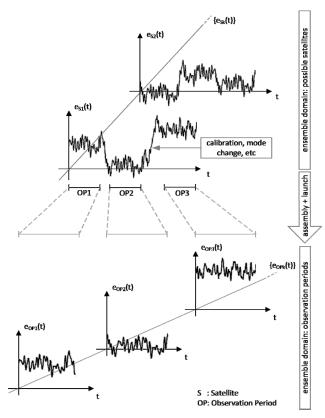


Figure 6-2. Different ensemble domains "satellite" (e.g. misalignments) and "observation period"

#### 6.3 Error Source Dependencies

In the PEET prototype, the dependencies between different error sources could be defined by specifying either full correlation or no correlation between all axes of a single error source or entirely between different error sources.

In the release version of PEET, the user is able to specify correlation individually (between axes of a single error source or axes of different sources) by specifying correlation coefficients. The specified correlation matrix is automatically checked for validity by the tool and a valid "close" realization is provided in case of an infeasible setup [RD6].

Furthermore, the tool distinguishes between temporal and ensemble correlation, i.e. timerandom sources can be correlated in time while their specification parameters can be



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correlated over an ensemble in the given domain and valid setup combinations are provided automatically.

The dependencies for random process error sources described as power spectral densities, can either be defined by coherence factors or an explicit definition of cross-spectra.

As (different to the simplified statistical method) also the phase relation between periodic signals affect the error PDF, phase properties can also explicitly be defined.



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#### 7 Summary

The release version of PEET is a tool to accurately compute statistical and spectral error budgets. Although developed for pointing applications, there is no intrinsic restriction for an application in other engineering fields. With the Excel and MATLAB interfaces, it is further well-suited for integration in analysis tool chains. PEET files can directly be exchanged among users, divisions or contractors and provide a much better transparency in the assumptions and models used to evaluate a budget than available from a purely "tabular" result.

It is suitable for the use throughout different project phases as models can easily be exchanged or added when new or more detailed information of the system behaviour is available. The remaining - but intended - restriction is that the tool cannot explicitly account for non-linearities or transient system behaviour, i.e. it shall not be understood as a replacement for E2E simulators.

The experience gained within the project provides a thorough understanding of the methodology of the PEEH and its application to the pointing error engineering process. With this experience and the tool being ahead of the described methodology at certain points, valuable inputs could be provided for supporting an update or revision of the PEEH for an even better useability.