



# Mapping the Spacetime Metric with a Global Navigation Satellite System

## Executive Summary

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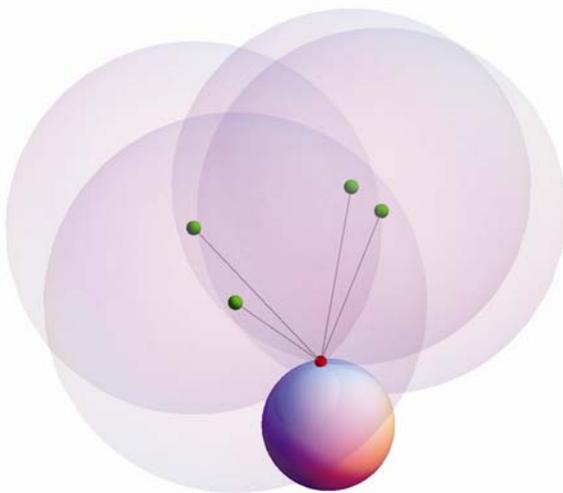
**Ariadna ID:** 09/1301  
**Study Type:** Standard  
**Contract Number:** 22709/09/NL/CBI

**Research Category:**  
*Fundamental Physics*

**Code and Title of the study:**  
*09/1301 Mapping the Space-time Metric with a Global Navigation Satellite System*

**Contract characteristics:**  
University: *University of Ljubljana*  
Academic researchers: *Uroš Kostič, Andrej Čadež*  
ACT researcher: *Pacôme Delva*  
Type of study: *Standard*

**Picture:**



**Motivation:**

*The scientific goal of the study was to introduce general-relativistic null coordinates in the global positioning system, to understand in what respect they can improve the accuracy and reliability of the system and to find efficient numerical routines to be used in such a system.*

**Methodology:**

*Present global positioning systems use basically Newtonian trigonometric methods to determine positions on the Earth, using reference ground stations on the Earth as base points. The necessary relativistic corrections are taken into account only in terms of post-Newtonian corrections. Following the ESA proposal, we applied analytic methods of general relativity to the problem of defining global positioning system. In this part of the project we defined a local Schwarzschild coordinate frame in which null coordinates were introduced (as part of ESA requirement). Furthermore we used numerical methods to implement analytic formulae and to test the consistency*

*of the solution. During a literature search we identified the relevant non-gravitational noise sources .*

### **Results:**

*The most significant results obtained during this study are:*

- 1) The global navigation system was modelled in a fully relativistic framework, based on null coordinates originating from proper time of GNSS satellites; algorithms to read local Schwarzschild coordinates from null coordinates were designed, implemented and tested.*
- 2) A brief analysis of non-gravitational perturbations has shown that clock errors are by far the largest noise source. This led us to propose that the dynamics of the constellation of satellites be used as a platform to define a self consistent local inertial frame, superior on longer term to that defined by atomic clocks.*

### **Publications:**

*Delva, Kostić, Čadež: Numerical Modeling of a global Navigation Satellite System in a General Relativistic Framework, submitted to Advances in Space Research, Special Issue on Galileo, arXiv:1003.5836*

### **Highlights:**

*The most relevant result of this study is the realization that a local Schwarzschild coordinate frame can be self-consistently defined in fully relativistic terms and thus provides a transparent and complete description of the global positioning system. Since the relation of the local Schwarzschild frame with respect to the global inertial reference is fully describable by gravitational perturbations due to mass-moments of the Earth, and gravitational perturbations of all the bodies in the solar system, the implementation of such a system can provide very precise data on these perturbations and thus give us a tool to better understand the distribution and study changes of the distribution of mass beneath the Earth.*