



# **Trajectory Optimisation under Uncertainties**

## **Executive Summary**

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Available on the ACT  
website

<http://www.esa.int/act>

**Ariadna ID:** 10/4101

**Ariadna study type:** Standard

**Contract Number:** 4000103161

**Theme:** Mission Analysis.

**Code and Title of the study:** AO/1-6448/10/NL/CBi Trajectory Optimisation under Uncertainties

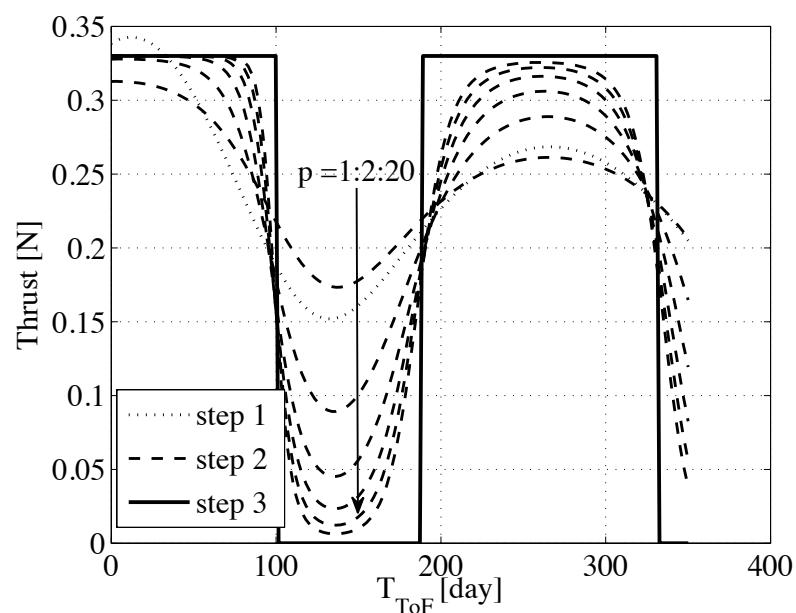
**Contract characteristics:**

University/Department: Politecnico di Milano/Department of Aerospace Engineering

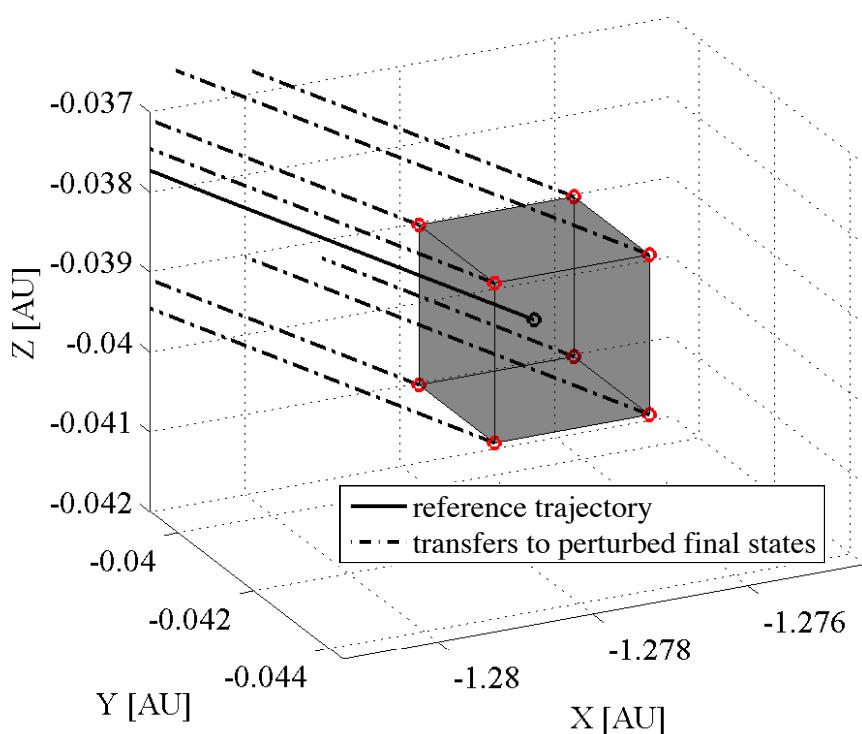
ACT researcher: Dario Izzo

Duration of the study: 4 months

**Picture:**



**Figure 1 - 2-bang Earth-1996 FG3 transfer. Three-step procedure for the numerical solution of the fuel optimal transfer problem: minimum energy problem (step 1); analytic approximation of the control (step 2); bang-bang solution of the fuel optimal problem (step 3).**



**Figure 2 - 2-bang Earth-1996 FG3 transfer. Optimal trajectories from the reference initial state to  $10^{-3}$  AU perturbed final target positions: detail at arrival.**

### Methodology:

This work focuses on the development of optimal nonlinear control strategies to manage uncertain boundary conditions in transfer problems for continuously propelled spacecraft with saturating actuators. A high order optimal control strategy is proposed, based on the use of differential algebraic techniques. Differential algebra (DA) is used to represent, by high order Taylor polynomials, the dependency of the spacecraft state on boundary conditions. The resulting polynomials are manipulated to obtain the high order expansion of the solution of fuel-optimal control problems about reference trajectories, including constraints on control saturation. Whenever perturbations in the reference conditions occur, new optimal control laws for perturbed initial and final states are obtained by the fast evaluation of polynomials.

### Results:

The computation of the reference transfer trajectories for continuously propelled spacecraft has been first investigated, including control saturation constraints. Within this frame, a three-step numerical procedure based on the use of a smoothing method has been developed and tested on a transfer to asteroid 1996 FG<sub>3</sub>. Then, two algorithms have been developed for the high order expansion of the solution of the fuel-optimal problem with respect to boundary conditions: the first one takes advantage of a smooth approximation of the control profile, whereas the second is able to deal with the discontinuous nature of the exact bang-bang solution. Both methods have shown good performances for a large uncertainty set on the boundary conditions. For the Earth-1996 FG<sub>3</sub> transfer, the methods are able to compensate for a

maximum error of 1E-3 AU, which is well above the errors to be managed in practical applications.

### **Publications:**

- 1) P. Di Lizia, R. Armellin, A. Morselli, F. Bernelli Zazzera, High order optimal feedback control of low-thrust orbital transfers with saturating actuators, 1st IAA Conference on Dynamics and Control of Space Systems, Porto, Portugal, 19-21 Mar. 2012.
- 2) P. Di Lizia, R. Armellin, A. Morselli, F. Bernelli Zazzera, High order optimal feedback control of space trajectories with bounded control, Submitted to Acta Astronautica, 2012.

Two additional articles are scheduled for submission to international journals: one article on the numerical procedure for the solution of the fuel-optimal transfer problem; one article on the high order expansion of the fuel-optimal solution with respect to boundary conditions for increasing transfer time and number of bangs

### **Highlights:**

The study shows that differential algebra can be profitably used to develop fast nonlinear optimal control strategies in the frame of orbital mechanics. It is demonstrated that the discontinuity of the exact solution of fuel-optimal transfer problems, which usually poses significant challenges to optimal control, can be overcome by explicitly relating the switching times to the uncertainties using high order Taylor polynomials. This enables the development of innovative high order strategies to optimally manage uncertain initial and final conditions. In addition, taking advantage of the availability of polynomial maps for the consumed propellant, polynomial bounders are used for the quick assessment of their ranges over the admissible uncertainty sets and the computation of mission margins.