Assessment of Collision Avoidance Manoeuvre Planning for Low-Thrust Missions

ELECTROCAM

Final presentation

July 27th 2023

ELECTROCAM team

POLITECNICO MILANO 1863

uc3m Universidad Carlos III de Madrid

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Agenda

- **Welcome and introductions** (10 mins)
- **Management aspects** (5 mins)
  - Team
  - Tasks
  - Schedule
- **Technical aspects** (120 mins)
  - Task 1: Critical review of available systems (15 mins)
  - Task 2: Approach to uncertainty evolution (40 mins)
  - Task 3: Operational concepts for CAM for low-thrust missions (55 mins)
  - Task 4: Update of the ESA DRAMA ARES tool (10 mins)
- **Other aspects** (5 mins)
  - Participation in congresses and conferences (5 mins)
- **AOB** (10 mins)

Welcome and introductions

Welcome and Introductions

- **ESA**
  - Klaus Merz
  - Francesca Letizia
  - ?

- **GMV+PoliMi+UC3M**
  - Diego Escobar
  - Ángel Gallego
  - Pau Gago
  - Marc Torras
  - Jorge Rubio
  - F. Javier Atapuerca
  - Pierluigi Di Lizia
  - Camilla Colombo
  - Juan Luis Gonzalo
  - Michele Maestrini
  - Andrea De Vittori
  - Joaquín Miguez
  - Manuel Sanjurjo-Rivo
  - Javier Lopez

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Management aspects

Team

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Tasks

- **Task 1**: Critical review of available systems → GMV
  - **Task 1.1**: Overview of low-thrust adoption
  - **Task 1.2**: Overview of performance
  - **Task 1.3**: Overview of typical operational profiles
  - **Task 1.4**: Overview of autonomy approaches
  - **Task 1.5**: Integration in DISCOS

- **Task 2**: Approach to uncertainty evolution → PoliMi & UC3M & GMV
  - **Task 2.1**: Overview of theory on uncertainty propagation
  - **Task 2.2**: Approach to uncertainty propagation in presence of continuous manoeuvring
  - **Task 2.3**: Assessment of suitability of the selected approach

- **Task 3**: Operational concepts for CAM for low-thrust missions → GMV & PoliMi
  - **Task 3.1**: Constraints analysis
  - **Task 3.2**: Approach for conjunction screening
  - **Task 3.3**: Approach for collision avoidance
  - **Task 3.4**: Simulations
  - **Task 3.5**: Sensitivity analysis

- **Task 4**: Update of the ESA DRAMA ARES tool → GMV
  - **Task 4.1**: Technical specification and documentation
  - **Task 4.2**: Software
  - **Task 4.3**: Software test plan
  - **Task 4.4**: Successful software tests

Schedule

Planned schedule

- **Task 1**: T0 → T0 + 6m
- **Task 2**: T0 → T0 + 6m
- **Task 3**: T0 → T0 + 12m
- **Task 4**: T0 + 6m → T0 + 15m (27m)

Executed schedule

- **Task 1**: T0 → T0 + 8m
- **Task 2**: T0 → T0 + 8m
- **Task 3**: T0 → T0 + 15m
- **Task 4**: T0+14m → T0 + 23m (35m)

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Technical aspects

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Task 1: Critical review of available systems

Task 1: Critical review of available systems

- **Task 1.1**: Overview of low-thrust adoption
- **Task 1.2**: Overview of performance
- **Task 1.3**: Overview of typical operational profiles
- **Task 1.4**: Overview of autonomy approaches
- **Task 1.5**: Integration in DISCOS

Key aspects of the proposed solution

- **GMV:** Literature review
- **GMV:** Experience of the team
- **GMV:** Involvement of satellite/thruster manufacturers

Task 1.1: Overview of low-thrust adoption

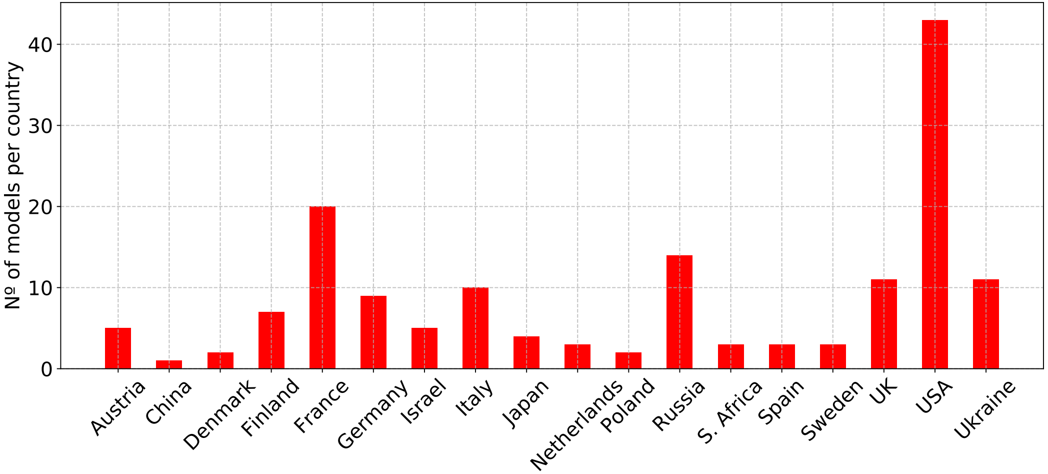
- Low-thrust propulsion solutions have been used **since 1964** (with NASA's SERT-1 mission).
- **Different technologies** fall into this category:

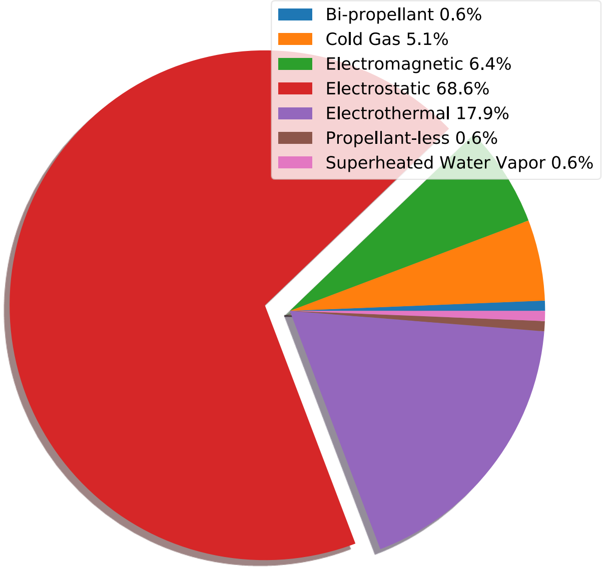
Electric propulsion *(EP)*	Resistojets, arcjets, ion thrusters, FEEP
Chemical propulsion	Cold gas, bi-propellant
Propellant-less systems	Solar sails, electrodynamic tethers

- Popularity of EP is rapidly **increasing:**
  1. *OneWeb* and *Starlink* constellations in **LEO** employ electric propulsion.
  2. The *Artemis* mission, as well as *telecom satellites*, demonstrated feasibility of station keeping and orbit raising in **GEO.**
  3. The future *Galileo Second Generation* in **MEO** will make use of electric propulsion.

Task 1.1: Overview of low-thrust adoption

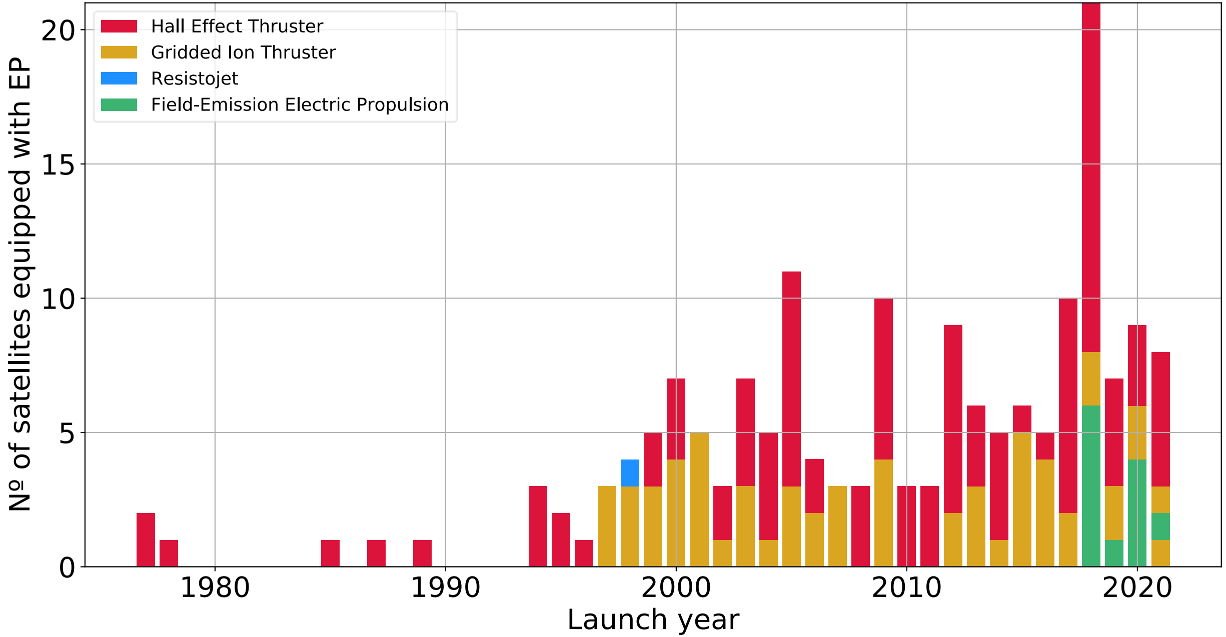
- To get awareness of the current and future low-thrust solutions, a **database** containing information about thrusters and their performances is built up (in ***ELECTROCAM Propulsion Database.xlsx***) → total of **156** thrusters.
- **Sources** for information are:
  1. Manufacturers' websites.
  2. *Gunter's Space Page* (https://space.skyrocket.de/, info obtained via web scraping with Python).
  3. *Epic-SRC* (https://www.epic-src.eu/, H2020 funded research).





Task 1.1: Overview of low-thrust adoption

- Using the same sources of information, a **second database** which lists satellite that have flown or are flying with **EP** is built-up (***Satellite_Thruster_Mapping***).
- By now, **28 different** EP thrusters have flown in a total of **602** satellite-thruster pairings.



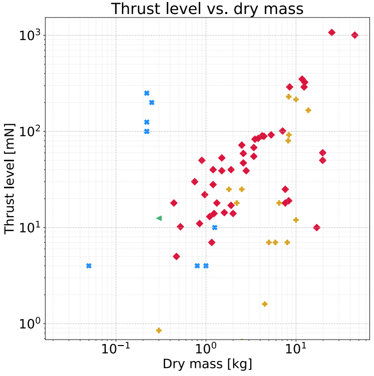
(does not include OneWeb sats (428 by April 2022) nor Starlink (2494 by April 2022), for which no specifics about thruster are available)

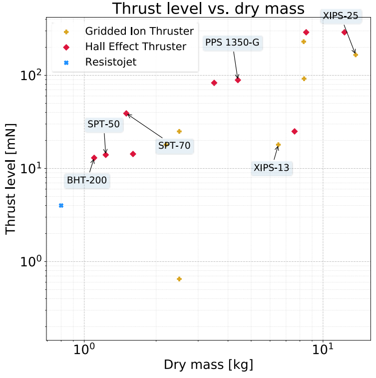
Task 1.2: Overview of performance

- Some of the **electric propulsion** low-thrust solutions:

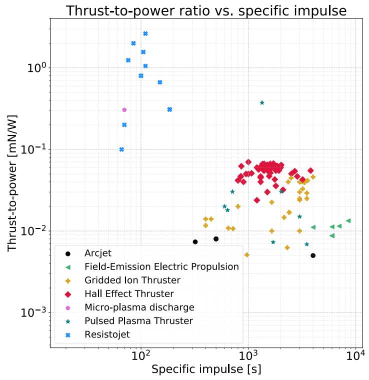
Low-thrust subclass	Low-thrust class	Description	Flown thrusters	Performances
Gridded Ion Thrusters	Electrostatic	Generation of plasma via an emitter and acceleration through a grid.	BHT-200, XIPS-13, XIPS-25	*Thrust:* 0.01–750 mN *Isp:* 800-900 s
Hall Effect Thrusters	Electrostatic	Generation of plasma via a Hall-effect electric field.	PPS 1350-G, PPS-5000, SPT-50, SPT-100	*Thrust:* 0.01-2000 mN *Isp:* 600-3000 s
Resistojets	Electrothermal	Gas heated by electric resistance and expanded in nozzle.	AQUARIUS 1-U	*Thrust:* 0.5-6000 mN *Isp:* 150-850 s
FEEPs	Electrostatic	Acceleration of liquid metal ions extracted from surface instabilities.	Enpulsion NANO, Enpulsion MICRO R3	*Thrust:* 0.001-1 mN *Isp:* 4000-12000 s

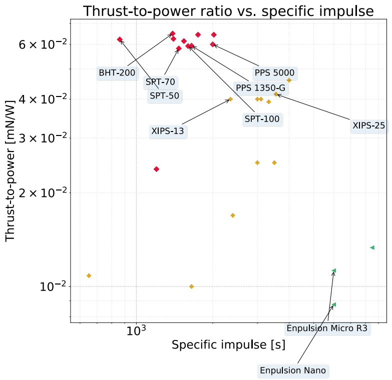
Task 1.2: Overview of performance

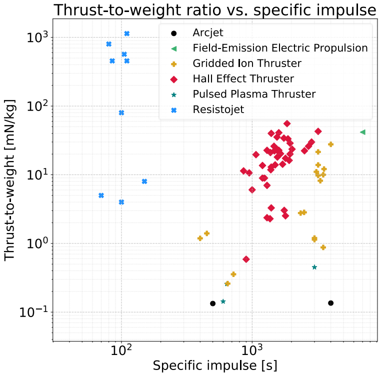


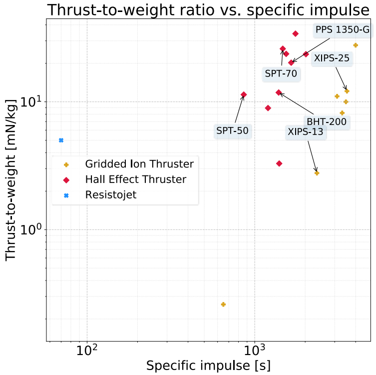


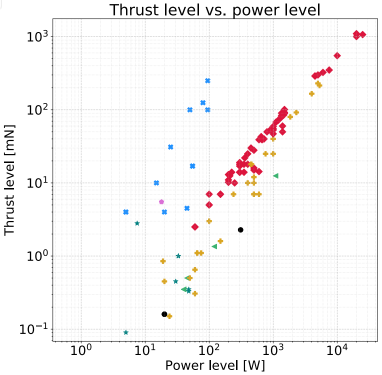
- ● Arcjet
- ◀ Field-Emission Electric Propulsion
- ✦ Gridded Ion Thruster
- ◆ Hall Effect Thruster
- ● Micro-plasma discharge
- ★ Pulsed Plasma Thruster
- ✖ Resistojet

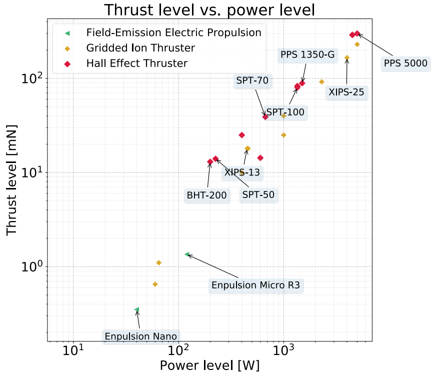












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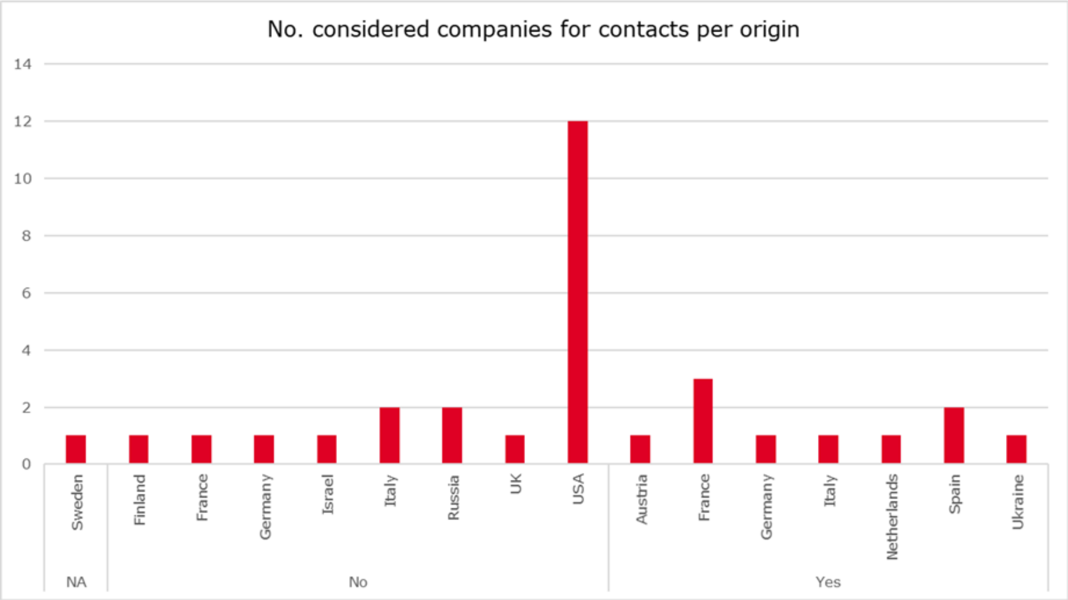
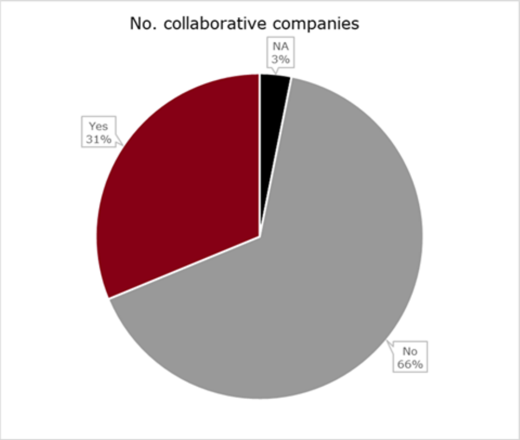
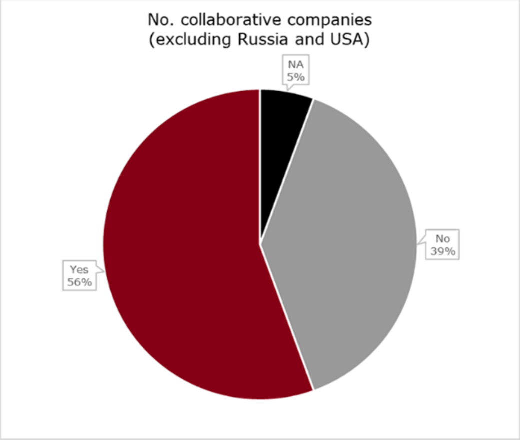
Task 1.2: Overview of performance

- **Advanced performance** and **uncertainty characterization**
  of thrusters:
  → direct contact with **manufacturers.**


No. considered companies for contacts per origin


No. collaborative companies


No. collaborative companies
(excluding Russia and USA)

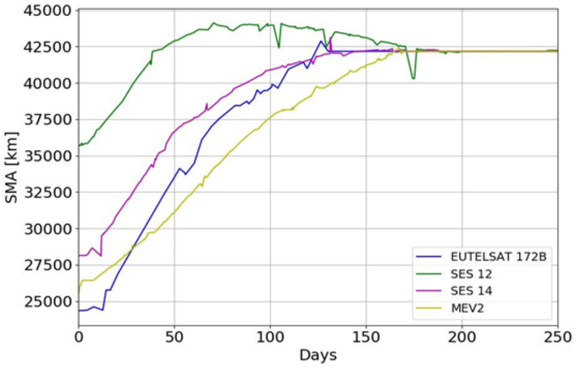
Task 1.2: Overview of performance

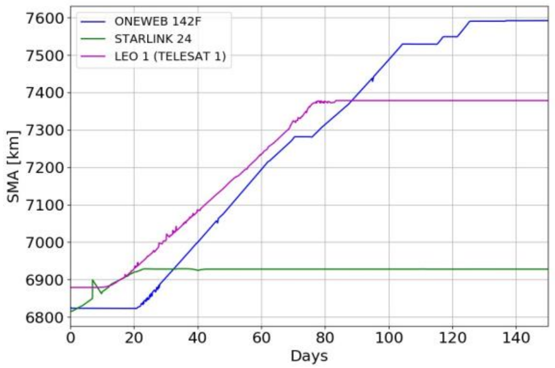
- **Information** obtained for **16** thrusters from **9** different companies.
- Information about **5** thrusters was given under **NDA.**

Model	Manufacturer	Duration of continuous thrust	Total maximum duration in operation	Thrust error	Reliability depending on failure type	Thrust error time correlation	Pointing error	Attitude error time correlation	Degradation over time of the previous parameters
ExoMG nano	Exotrail	Only limited by the platform energy constraints	800 hrs	< 5% / < 10%	Not available	Not evaluated	<+/- 5°	< +/- 1° / < +/- 0.75° / < +/- 0.5°	Applicable
RIT µX	Ariane Group	Continuous	10-30 kHour	~1-2%	System aspect	n/a	<0.5°/ 1°	N/A	Depend on mission scenario
PJP	Comat	30min	400Ns	+/- 5 - 10%	N/A	N/A	N/A	N/A	N/A
NPT30-I2	ThrustMe		< 5500-9500 Ns	-	-	-	< 1°	-	-
ST-40	SETS	Not limited	3000 hrs	N/A	N/A	N/A	N/A	N/A	< -10% at EoL
Enpulsion Micro R3	Enpulsion	>1500h	Propellant limited	< 5% (design objective)	7 year	-	< 2°	N/A	Expected negligible

Task 1.3: Overview of typical operational profiles

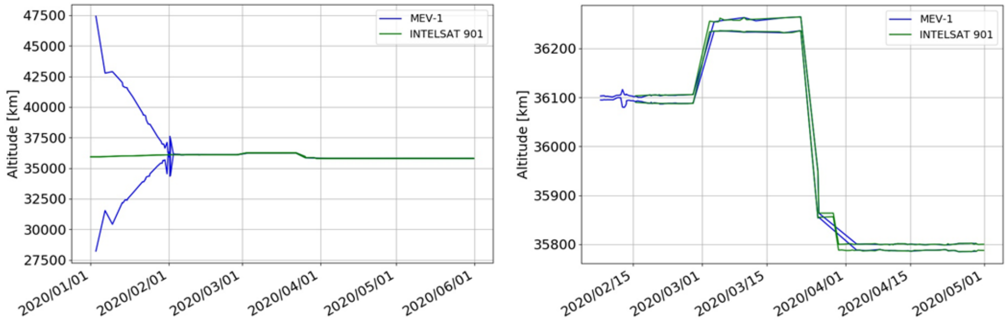
- Literature review + analysis of TLE data/publicly available ephemeris + **relevant team's experience**. **EOR**:
  - **GTO to GEO:**
    - **Robust** and more **compact** designs → EP often the choice.
    - Optimal control problem extensively applied.
    - EOR **strategy** can target multiple **constraints** (sma & eccentricity evolution, ToF)
  - **LEO to high LEO:**
    - EP has enhanced capabilities of LEO S/C in the recent years.
    - Small S/C become manoeuvreable → disposal, extended lifetimes, CAM…
    - Large-constellations: **OneWeb**, Starlink, Telesat


GTO2GEO

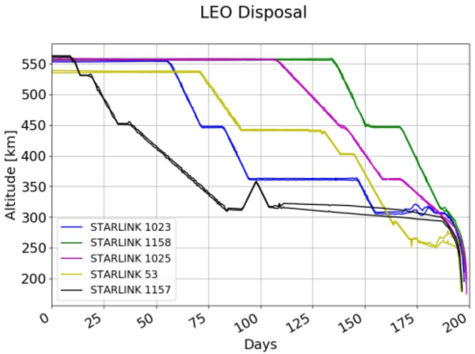

LEO2LEO

Task 1.3: Overview of typical operational profiles

- Literature review + analysis of TLE data/publicly available ephemeris + **relevant team's experience**. **EOL/Disposal**:
  - **GEO graveyarding:**
    - EoL GEO-ring clearance of +250km
    - Reinsertion MEV-1/Intelsat 901 is taken as reference for strategy/timeline



- **LEO disposal:**
  - Large-constellations life-cycle
  - Strategies: **forced** re-entry (**Starlink**) or position on **decaying** orbit (**OneWeb**)

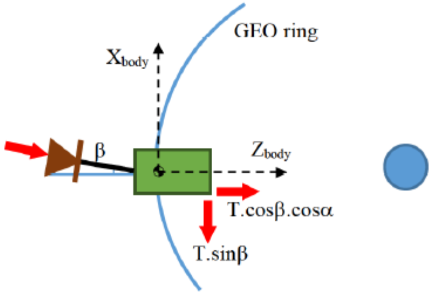


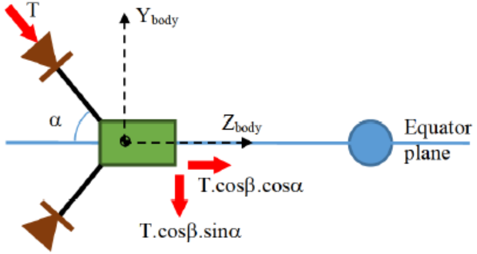
Task 1.3: Overview of typical operational profiles

- Literature review + analysis of TLE data/publicly available ephemeris + **relevant team's experience**. **Orbit maintenance**:

  - **GEO SK:**
    - Zero net tangential drift, momentum damping, North-South (~90%).
    - Frequent firings (e.g. 40 mins on a daily basis)
    - Thrusters operate at a lower power setting wrt EOR

  - **LEO SK:**
    - Tailored to mission, tighter tolerances. Absolute/relative SK
    - Groundtrack drift, constellation geometry, tube control, drag compensation





Task 1.4: Overview of autonomy approaches

- Main considerations for **autonomous station-keeping** on-board
  - Level of autonomy
    - A key factor is if the orbit determination can be performed on-board
    - GNSS receiver needed on-board
  - Processing power
- **Parameters** that will characterise the autonomy of a SC:
  - Accuracy
  - Complexity
  - Robustness
  - Availability
  - Computational load (to run dynamic models)

Main mission with autonomous control for station keeping: **GOCE**

Drag-free-control with electric propulsion to provide the necessary measurements, and the system on-board defined the thrust level to be applied

Such orbit maintenance manoeuvres were significantly **less frequent** than originally expected (very good performance of the Drag-Free and Attitude Control System DFACS)

Task 1.4: Overview of autonomy approaches

Focusing on the **CAMs**, the only known case is **Starlink** (Space Exploration Technologies Corp, or SpaceX)

Their approach if not fully public

The orbit of the satellites is determined on board

→ transmitted to the 18th SPCS, LeoLabs and other operators

→ there they are screened for possible conjunctions

→ Warnings are uploaded to the satellite

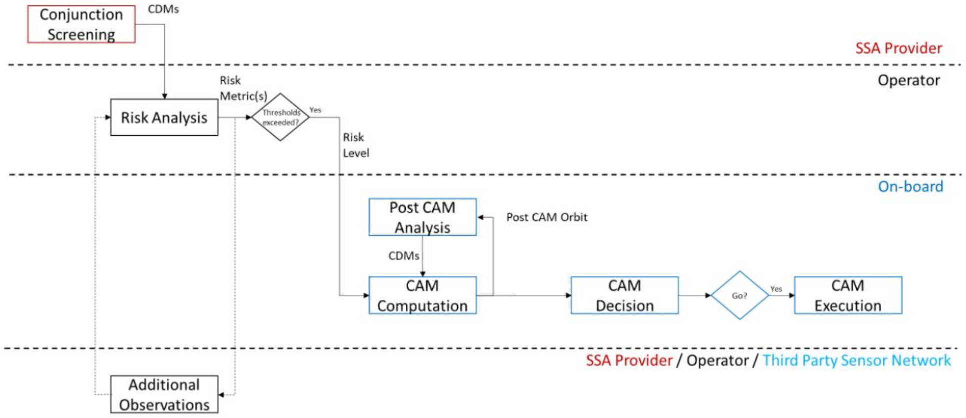
→ it will compute an appropriate CAM and execute it (on-board also analyses screening for secondary conjunctions)

Future ephemerides updated three times per day on Space-Track.org

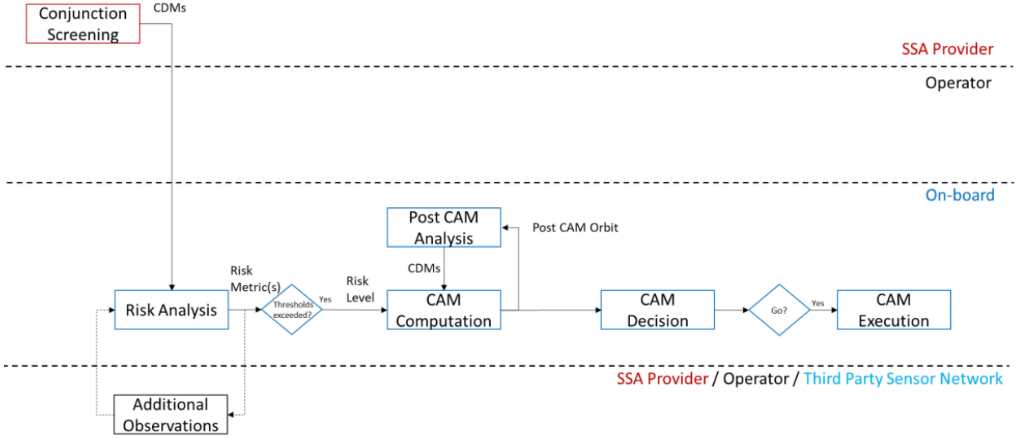
Some consequences of the Starlink strategy:

- Places significant demand on the **communications** system (good for constellations with ISL)
- The **OB processing** seems to have advanced to make this possible
  - Possible that they do not optimize the manoeuvre, and have **pre-loaded** strategies
- Possible reduction of the **workload** on ground, as decisions are autonomous
- Deciding on-board means decisions can be **closer to the event**
- However, they should be evaluated **periodically**
- And when **two operational satellites** may collide, there must be human intervention from both operators

Task 1.4: Overview of autonomy approaches



Conjunction Screening
CDMs
SSA Provider
Operator
Risk Analysis
Risk Metric(s)
Thresholds exceeded?
Yes
Risk Level
On-board
Post CAM Analysis
Post CAM Orbit
CDMs
CAM Computation
CAM Decision
Go?
Yes
CAM Execution
SSA Provider / Operator / Third Party Sensor Network
Additional Observations



Conjunction Screening
CDMs
SSA Provider
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Risk Metric(s)
Risk Level
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Yes
CAM Computation
CAM Decision
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CAM Execution
SSA Provider / Operator / Third Party Sensor Network
Additional Observations

Task 1.5: Integration in DISCOS

- Databases about **EP,** built up in Tasks *1.1* and *1.2*, are converted into **CSV files** to be imported to **DISCOS.**
  - ✓ Knowledge about low-thrust propulsion solutions and their **performances**.
  - ✓ Acquire knowledge of **satellites** equipped with EP, benefiting **CAM design** and **planning**.

- **Three** tables originally planned:
  - ***Table 1:*** database for low-thrust propulsion solutions.
  - ***Table 2:*** database for advanced performance and uncertainty characterization (thrust error, pointing error, degradation etc.).
  - ***Table 3:*** mapping between in-flight/flown satellites with the equipped EP solutions.

Task 1.5: Integration in DISCOS

- **Table 1**: database for low-thrust propulsion solutions.

Thruster ID	Thruster name	Company name	Country	Low-thrust type	Low-thrust subclass	Propellant	In-orbit or attitude control	Dry mass	Wet mass

Min. power	Nominal power	Max. power	Min. thrust	Nominal thrust	Min. specific impulse	Max. specific impulse	Min. total impulse	Max. total impulse	Source

Mass, Thrust, Specific Impulse and Total Impulse have been divided in 2 or 3 columns each in order to keep **single numeric values** while representing the **range**.

- **Table 2:** database for advanced performance and uncertainty characterization.
  - Information for **just 16** thrusters (5 of them being under **NDA**).
  - Data is **uneven** (each company provides different metrics) and **sparse**.
  - **Main data** is **already** in Table 1.

→ **Not integrated** in DISCOS.

Task 1.5: Integration in DISCOS

- **Table 3**: mapping between in-flight/flown satellites with the equipped EP solutions.

DISCOS ID	Platform	Power to EP	Thruster ID	Nº of thrusters	EOR capabilities	SK capabilities	Source

Multiple thruster models: **multiple rows** (with the same DISCOS ID).

Mapping → **DISCOS ID** ↔ **Thruster ID**

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Task 2: Approach to uncertainty evolution

Task 2: Approach to uncertainty evolution

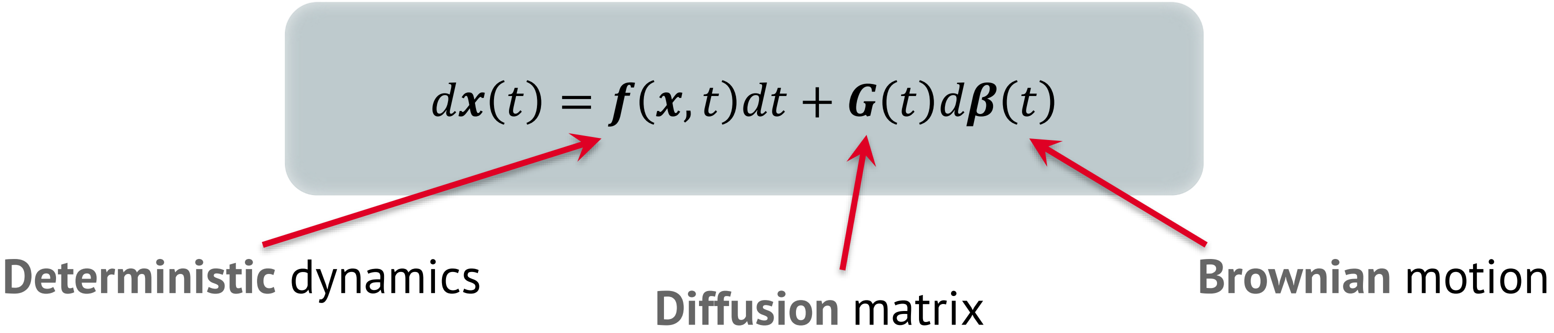
- **Task 2.1**: Overview of theory on uncertainty propagation
- **Task 2.2**: Approach to uncertainty propagation in presence of continuous manoeuvring
- **Task 2.3**: Assessment of suitability of the selected approach

Key aspects of the proposed solution

- **PoliMi**: Dynamics-based methods
- **UC3M**: Probabilistic-based methods
- **GMV**: Definition of benchmark scenarios

Task 2.1: Overview of theory on uncertainty propagation. Dynamics-based methods

- Orbital dynamic problem entailing uncertainty can be expressed by the **Itô stochastic differential equation**



$$d\boldsymbol{x}(t) = \boldsymbol{f}(\boldsymbol{x}, t)dt + \boldsymbol{G}(t)d\boldsymbol{\beta}(t)$$

Deterministic dynamics

Diffusion matrix

Brownian motion

Different dynamics-based methods have been developed to solve this problem **(summary in the next slide)**:

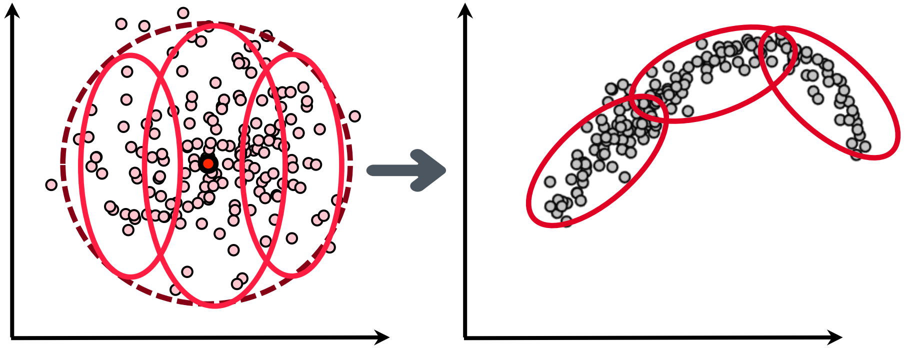
- **Full SDE solution**
- **Linear Methods**: Local Linearization of dynamics, Statistical Linearization…
- **Nonlinear Methods**: Polynomial Chaos, **Gaussian Mixture Models (GMM)**, State Transition Tensors, **Differential Algebra (DA)**

Task 2.1: Overview of theory on uncertainty propagation. Dynamics-based methods

	Methods	Advantages	Drawbacks
Linear	**LinCov [BI308]**	Simple, high computation efficiency	Differentiable assumption on dynamics, inaccurate for nonlinear systems
	CADET [BI309]	Without differentiable assumption on dynamics	Inaccurate for nonlinear systems
Nonlinear	**UT [BI310]**	Existing dynamics solvers are usable, high computation efficiency	No knowledge on higher-order moments and non-Gaussian PDF
	CB [BI406]	Existing dynamics solvers are usable, high computation efficiency	No knowledge on higher-order moments
	PC [BI311]	Existing dynamics solvers are usable, up to exponential convergence	Curse of dimensionality
	STT [BI312]	Semi-analytical, high computational efficiency	Complex, differentiable assumption on dynamics
	DA [BI313]	Efficient numerical computation of higher-order derivatives	Differentiable assumption on dynamics
	GMM [BI317]	Only the first two moments require propagating	Curse of dimensionality
	FPE [BI305]	A direct numerical solution of FPE, true evolution of PDF	Curse of dimensionality, heavy computation

Task 2.1: Overview of theory on uncertainty propagation. Dynamics-based methods

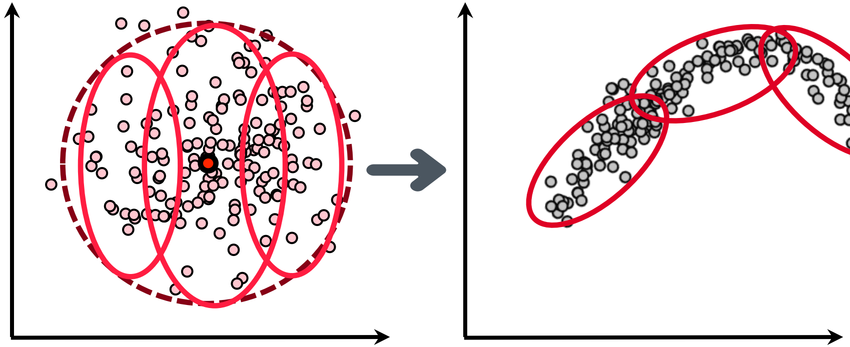
When **one Gaussian is not sufficient** to accurately represent the propagated statistics: **N weighted Gaussian kernels.**



- Optimization needed to retrieve **weights, means, and covariances** of the GMM
- The **number of kernels cannot be determined a priori**

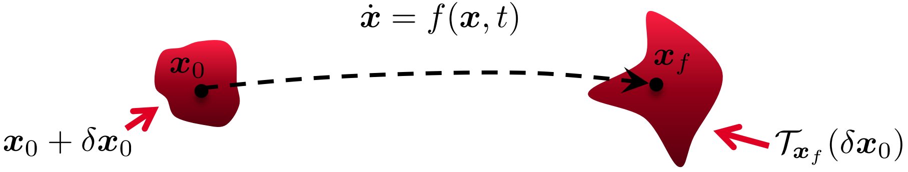
Task 2.1: Overview of theory on uncertainty propagation. Dynamics-based methods

When **one Gaussian is not sufficient** to accurately represent the propagated statistics: **N weighted Gaussian kernels.**



- Optimization needed to retrieve **weights, means, and covariances** of the GMM
- The **number of kernels cannot be determined a priori**

Differential Algebra (DA) is used to substitute algebra of real numbers with an **algebra of Taylor Polynomials**



Initial conditions and **integration scheme as DA** gives **k-th order Taylor expansion of the solution**

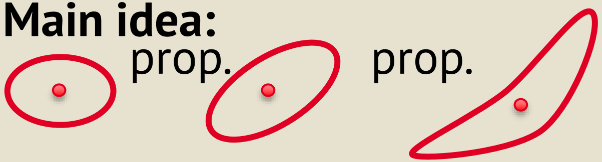
- **Linear covariance propagation**
- **DA-based Monte Carlo**
- **High-order propagation of statistical moments**

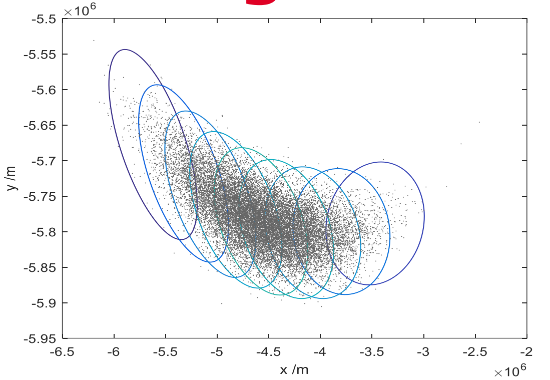
Task 2.2: Approach to uncertainty propagation in presence of continuous manoeuvring.

Dynamics-based methods

- Adaptive DA-GMM (A-DAGMM)

Main idea:



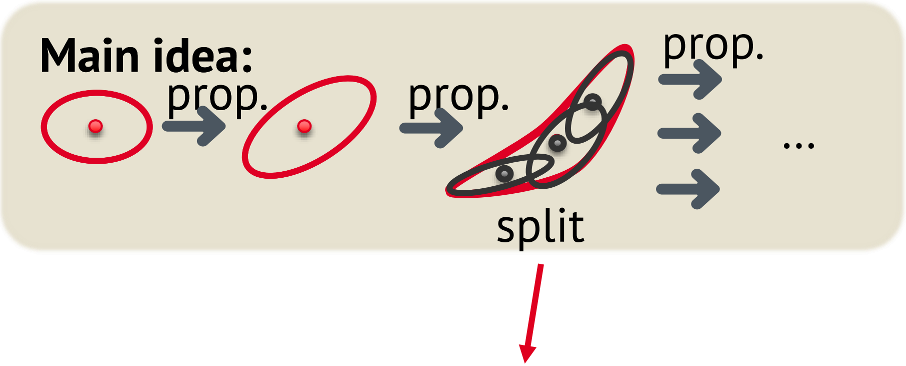


0.8 orbits, **9 mixands**

Task 2.2: Approach to uncertainty propagation in presence of continuous manoeuvring.

Dynamics-based methods

- Adaptive DA-GMM (A-DAGMM)



Main idea:

prop.

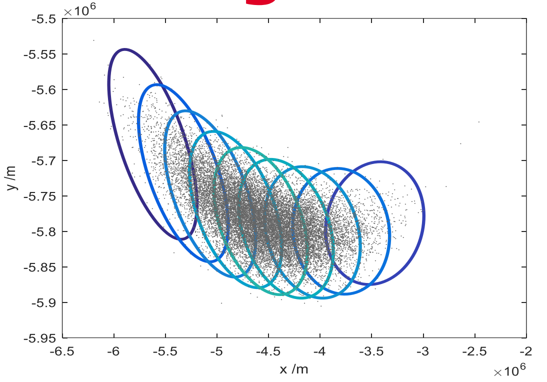
prop.

split

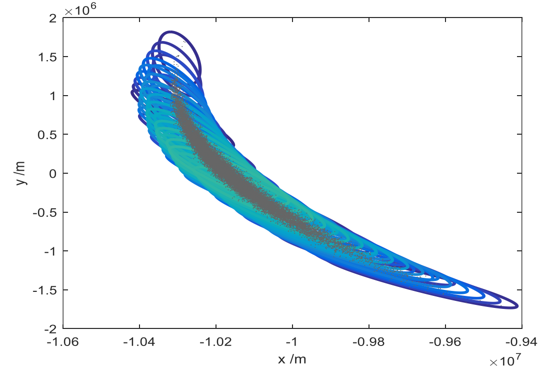
prop.

…

Each GME is propagated at **1st and 2nd order thanks to DA**: **Hellinger Distance** to detect onset of nonlinearity



0.8 orbits, **9 mixands**



1.5 orbits, **81 mixands**

Task 2.2: Approach to uncertainty propagation in presence of continuous manoeuvring.

Dynamics-based methods

- Stochastic Taylor Model (STM)

DA-based integration of orbital dynamics provides the analytical map:

$$[x(t_f)] = \mathcal{T}_x(\delta x_0)$$

A **covariance inflation term** is computed to include process noise by the DA integration:

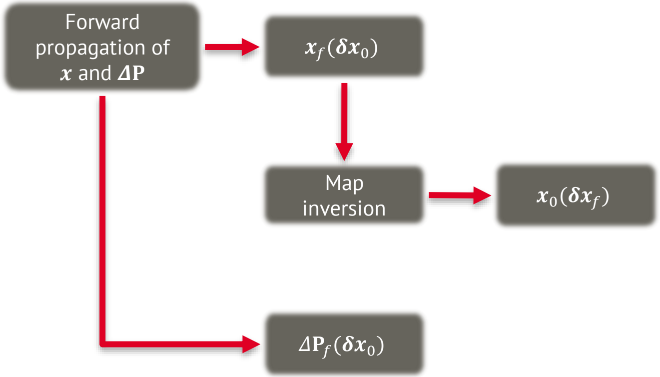
$$[\Delta P(t_f)] = \mathcal{T}_{\Delta P}(\delta x_0)$$

Stochastic Taylor Model

$$\boldsymbol{\Delta\dot{P}} = \boldsymbol{F}(t,\boldsymbol{x})\boldsymbol{\Delta P} + \boldsymbol{\Delta P F}(t,\boldsymbol{x})^T + \boldsymbol{G}(t,\boldsymbol{x})\boldsymbol{Q}(t)\boldsymbol{G}(t,\boldsymbol{x})^T$$

Task 2.2: Approach to uncertainty propagation in presence of continuous manoeuvring.

Dynamics-based methods



Forward propagation of $\boldsymbol{x}$ and $\Delta\mathbf{P}$

$\boldsymbol{x}_f(\boldsymbol{\delta x}_0)$

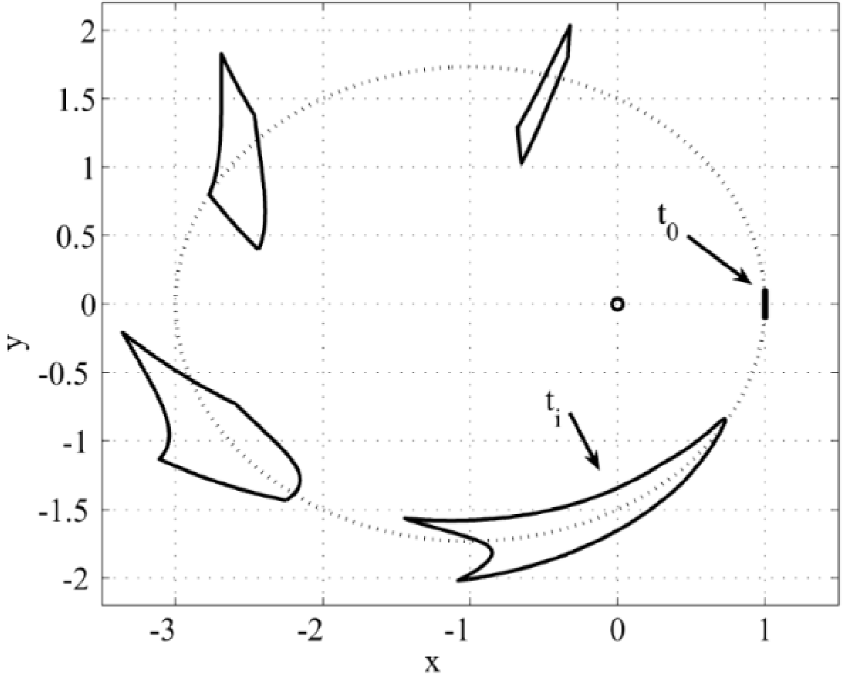
Map inversion

$\boldsymbol{x}_0(\boldsymbol{\delta x}_f)$

$\Delta\mathbf{P}_f(\boldsymbol{\delta x}_0)$

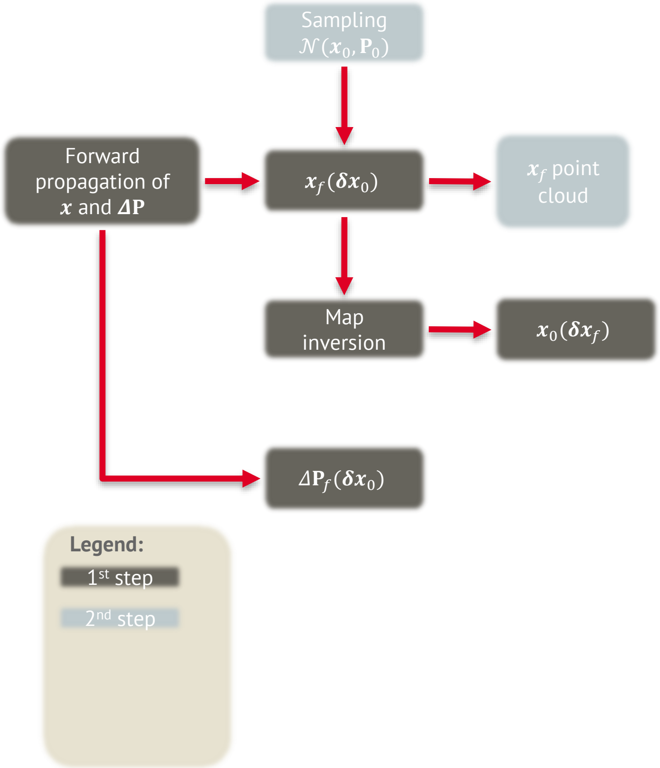
Legend:

1st step



Task 2.2: Approach to uncertainty propagation in presence of continuous manoeuvring.

Dynamics-based methods



Sampling $\mathcal{N}(\boldsymbol{x}_0, \mathbf{P}_0)$

Forward propagation of $\boldsymbol{x}$ and $\boldsymbol{\Delta}\mathbf{P}$

$\boldsymbol{x}_f(\boldsymbol{\delta x}_0)$

$\boldsymbol{x}_f$ point cloud

Map inversion

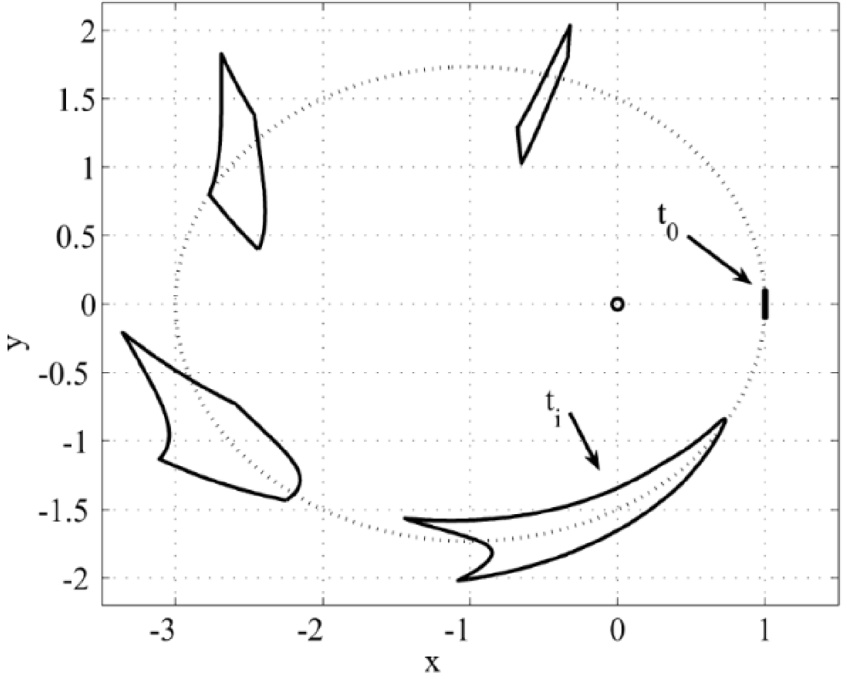
$\boldsymbol{x}_0(\boldsymbol{\delta x}_f)$

$\Delta\mathbf{P}_f(\boldsymbol{\delta x}_0)$

Legend:

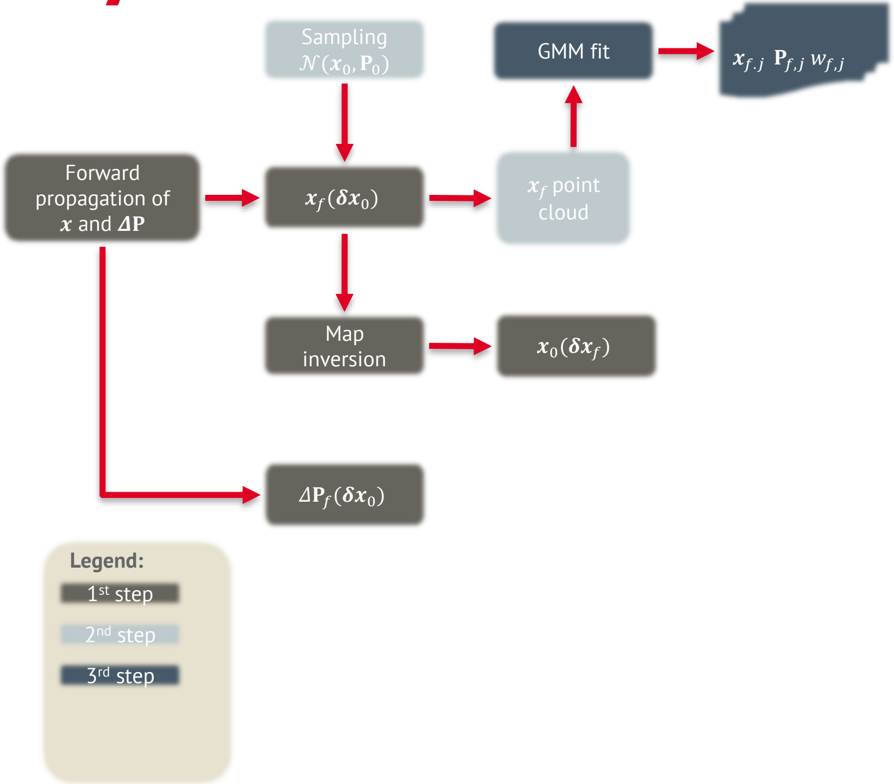
1st step

2nd step



Task 2.2: Approach to uncertainty propagation in presence of continuous manoeuvring.

Dynamics-based methods



Sampling $\mathcal{N}(\boldsymbol{x}_0, \mathbf{P}_0)$

GMM fit

$\boldsymbol{x}_{f,j}$ $\mathbf{P}_{f,j}$ $w_{f,j}$

Forward propagation of $\boldsymbol{x}$ and $\Delta\mathbf{P}$

$\boldsymbol{x}_f(\boldsymbol{\delta x}_0)$

$\boldsymbol{x}_f$ point cloud

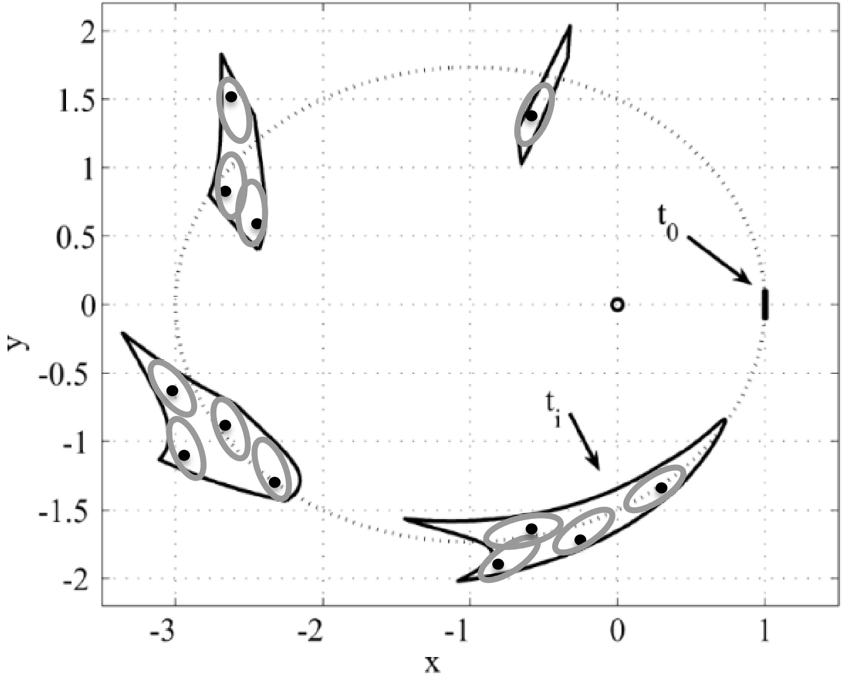
Map inversion

$\boldsymbol{x}_0(\boldsymbol{\delta x}_f)$

$\Delta\mathbf{P}_f(\boldsymbol{\delta x}_0)$

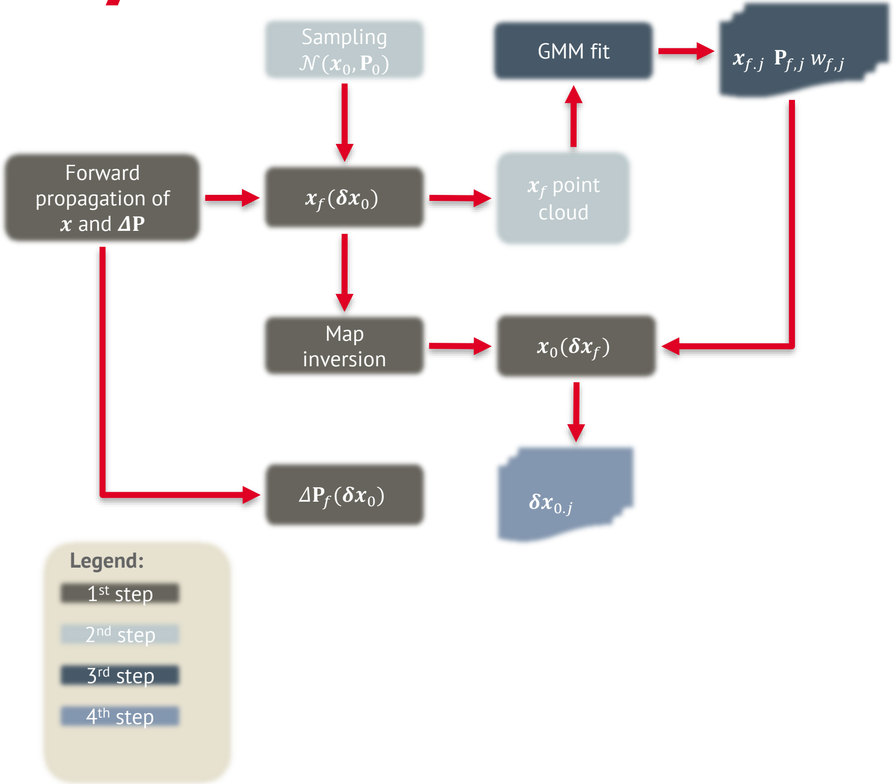
Legend:

- 1st step
- 2nd step
- 3rd step



Task 2.2: Approach to uncertainty propagation in presence of continuous manoeuvring.

Dynamics-based methods



Sampling $\mathcal{N}(\boldsymbol{x}_0, \mathbf{P}_0)$

GMM fit

$\boldsymbol{x}_{f,j}$ $\mathbf{P}_{f,j}$ $w_{f,j}$

Forward propagation of $\boldsymbol{x}$ and $\boldsymbol{\Delta}\mathbf{P}$

$\boldsymbol{x}_f(\boldsymbol{\delta x}_0)$

$\boldsymbol{x}_f$ point cloud

Map inversion

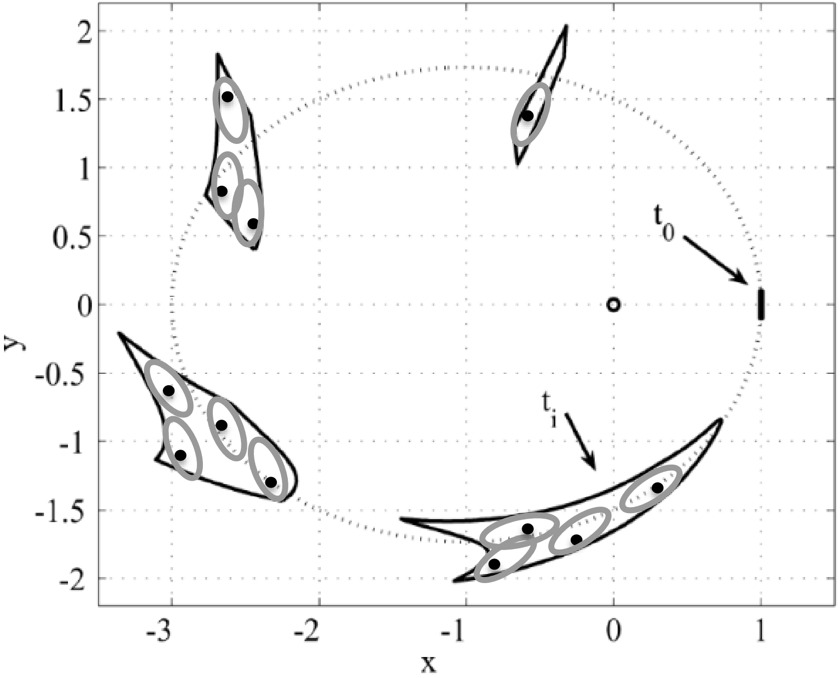
$\boldsymbol{x}_0(\boldsymbol{\delta x}_f)$

$\Delta\mathbf{P}_f(\boldsymbol{\delta x}_0)$

$\boldsymbol{\delta x}_{0,j}$

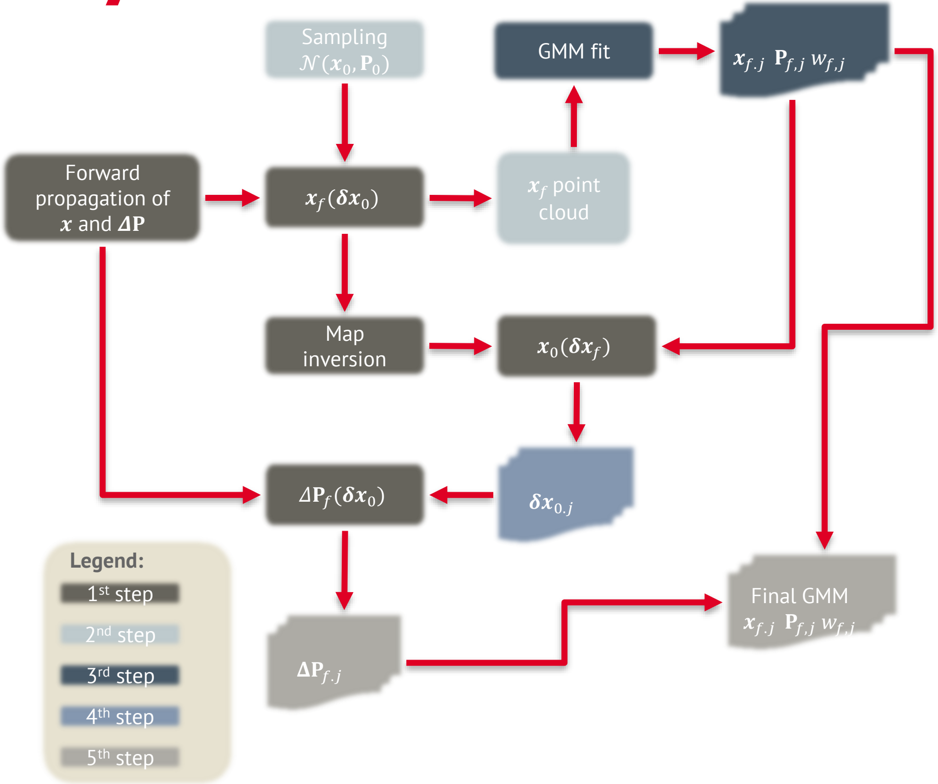
Legend:

- 1st step
- 2nd step
- 3rd step
- 4th step



Task 2.2: Approach to uncertainty propagation in presence of continuous manoeuvring.

Dynamics-based methods



Sampling $\mathcal{N}(\boldsymbol{x}_0, \mathbf{P}_0)$

GMM fit

$\boldsymbol{x}_{f.j}$ $\mathbf{P}_{f,j}$ $w_{f,j}$

Forward propagation of $\boldsymbol{x}$ and $\boldsymbol{\Delta}\mathbf{P}$

$\boldsymbol{x}_f(\boldsymbol{\delta x}_0)$

$\boldsymbol{x}_f$ point cloud

Map inversion

$\boldsymbol{x}_0(\boldsymbol{\delta x}_f)$

$\boldsymbol{\Delta}\mathbf{P}_f(\boldsymbol{\delta x}_0)$

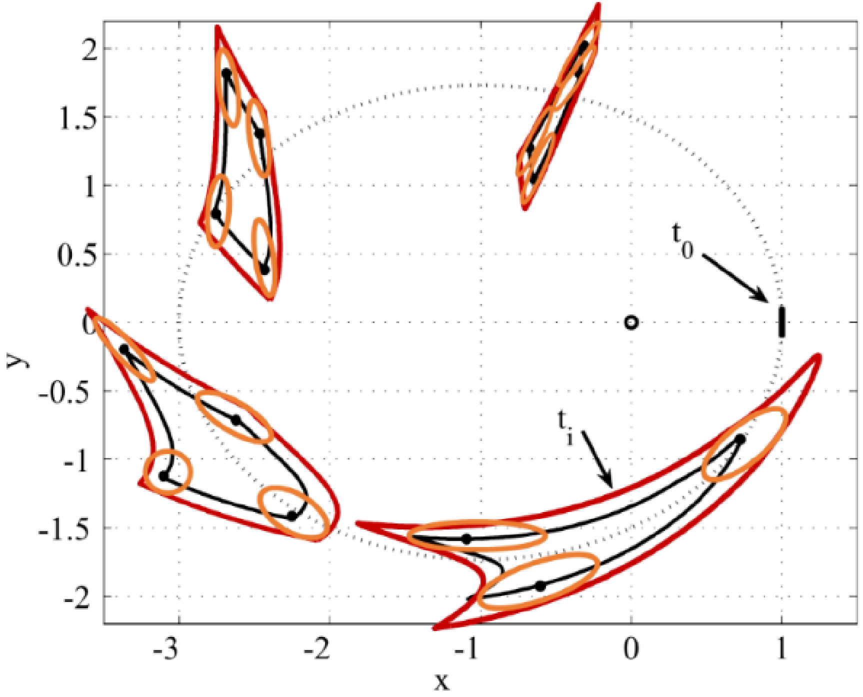
$\boldsymbol{\delta x}_{0.j}$

$\Delta \mathbf{P}_{f.j}$

Final GMM $\boldsymbol{x}_{f.j}$ $\mathbf{P}_{f,j}$ $w_{f,j}$

Legend:

- 1st step
- 2nd step
- 3rd step
- 4th step
- 5th step



Task 2.1: Overview of theory on uncertainty propagation. Probabilistic-based methods

- **Modelling framework**
  - Initial uncertainty: mean & covariance of the spacecraft state

$$\mathbf{X}(t) = \left[\begin{array}{c} \mathbf{r}(t) \\ \mathbf{v}(t) \end{array} \right], \quad \mathbf{X}(t_0) \sim \mathcal{N}(\bar{\mathbf{x}}_0, \mathbf{C}_0)$$

  - Stochastic dynamical model
    - Itô SDE: $d\mathbf{X} = f(\mathbf{X}, t)dt + G(\mathbf{X}, t)d\mathbf{W}$
    - Reduces to an ODE when there is no process noise: $G(\mathbf{X}, t) = \mathbf{0} \Rightarrow \dot{\mathbf{X}} = f(\mathbf{X}, t)$
- **Numerical schemes** for computer models
  - Different schemes available for SDEs with distinct theoretical properties
    - Strong vs. weak convergence
  - A general representation: $\mathbf{X}_n = \Phi_n(\mathbf{X}_{n-1}, \mathbf{Z}_n)$ with $\mathbf{X}_0 = \mathbf{X}(t_0)$

numerical approx. scheme

process noise

Task 2.1: Overview of theory on uncertainty propagation. Probabilistic-based methods

- **Goal**: to approximate the pdf of $\mathbf{X}_n$
- We seek simple **black-box methods** that map $p(\mathbf{x}_0)$ into $p(\mathbf{x}_n)$
- Black-box?
  - we only assume the ability to run the numerical scheme

- Two classes of methods
  - Gaussian approximations using reference points & weights
  - (Fixed) Gaussian mixture approximations

Task 2.1: Overview of theory on uncertainty propagation. Probabilistic-based methods

- **Gaussian approximations**:
  - Represent $p(\mathbf{x}_0) = \mathcal{N}(\bar{\mathbf{x}}_0, \mathbf{C}_0)$ as $\bar{\mathbf{x}}_0 = \sum_i \mathbf{x}_0(i) w_0(i)$ and $\mathbf{C}_0 = \sum_i \left(\mathbf{x}_0(i) - \bar{\mathbf{x}}_0\right)\left(\mathbf{x}_0(i) - \bar{\mathbf{x}}_0\right)^\top w_0(i)$
  - Propagate the reference points

$$\mathbf{x}_n(i) = \Phi_n(\mathbf{x}_{n-1}(i), \mathbf{Z}_n), \quad n = 1, 2, \ldots$$

  - to obtain $p(\mathbf{x}_n) \approx \mathcal{N}(\bar{\mathbf{x}}_n, \mathbf{C}_n)$ with $\bar{\mathbf{x}}_n = \sum_i \mathbf{x}_n(i) w_0(i)$ and $\mathbf{C}_n = \sum_i \left(\mathbf{x}_n(i) - \bar{\mathbf{x}}_n\right)\left(\mathbf{x}_n(i) - \bar{\mathbf{x}}_n\right)^\top w_0(i$
  - Several versions of both UT and cubature representations.

- Propagation of kernel density estimators (**KDEs**)
  - Fixed Gaussian mixture $p(\mathbf{x}_0) = \frac{1}{N} \sum_{j=1}^{N} \mathcal{N}(\mathbf{x}_0 | \bar{\mathbf{x}}_0^j, \mathbf{C}_0^j)$
  - Propagate component-wise to obtain $p(\mathbf{x}_n) \approx \frac{1}{N} \sum_{j=1}^{N} \mathcal{N}(\mathbf{x}_n | \bar{\mathbf{x}}_n^j, \mathbf{C}_n^j)$

Task 2.1: Overview of theory on uncertainty propagation. Probabilistic-based methods

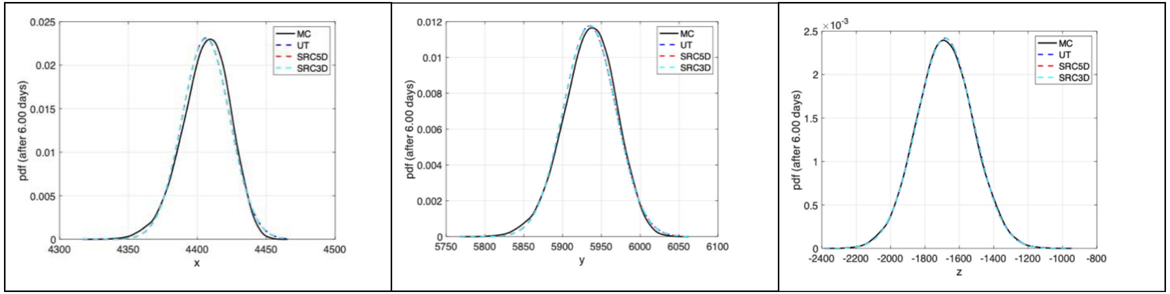
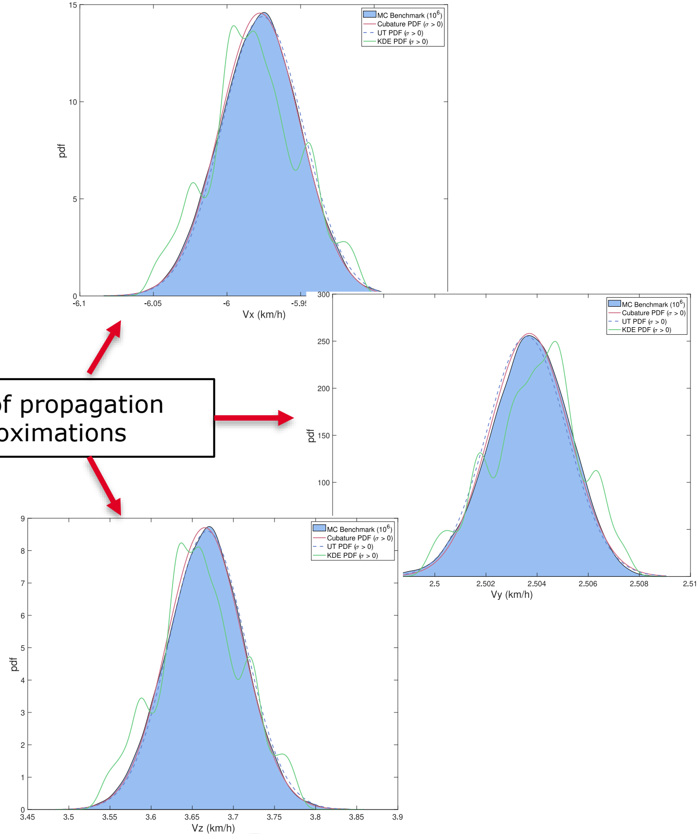


Figure 6-1: PDF of the position coordinates (*x*, *y*, *z*) of OneWeb 0012. Propagation over 6 days using Monte Carlo, UT and cubature (SRC3D and SRC5D).



PDF of velocity coordinates for Sentinel 3A after 3 days of propagation using Monte Carlo (solid blue), UT, cubature & KDE approximations

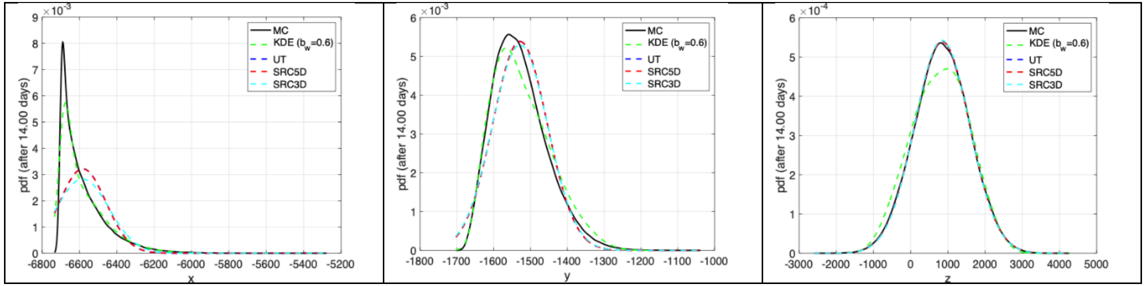


Figure 6-2: PDF of the position coordinates (*x*, *y*, *z*) of Flock 3k3. Propagation over 14 days using Monte Carlo, KDE, UT and cubature (SRC3D and SRC5D).

Task 2.2: Approach to uncertainty propagation in presence of continuous manoeuvring. Probabilistic-based methods

- We have addressed 4 problems

1. Modelling of thruster uncertainty by stochastic differential equations (SDEs)
2. Numerical schemes for SDE integration
3. UP methods
4. *Model inference*

Task 2.2: Approach to uncertainty propagation in presence of continuous manoeuvring. Probabilistic-based methods.

Uncertainty modelling via SDEs

- We start from a standard ODE: $\dot{\mathbf{v}} = -\mu\frac{\mathbf{r}}{r^3} + \frac{1}{2}\rho_{\mathbf{r}} v_{\text{rel}} B \mathbf{v}_{\text{rel}} + \sum_l \mathbf{p}_{J_l} + \mathbf{a}_{\mathsf{T}},$
- We obtain an Itô SDE by introducing a diffusion term

Nominal thruster acceleration

$$\begin{cases} d\mathbf{r} = \mathbf{v}dt, \\ d\mathbf{v} = \underbrace{f(\mathbf{X},t)dt}_{\text{drift}} + \underbrace{\mathbf{G}(\mathbf{a}_{\mathsf{T}})d\mathbf{W}}_{\text{Diffusion (‘noise’)}}, \end{cases}$$

$$\text{where } \mathbf{X}(t) = [\mathbf{r}(t)^\top, \mathbf{v}(t)^\top]^\top, \qquad f(\mathbf{X},t) = -\mu\frac{\mathbf{r}}{r^3} + \frac{1}{2}\rho_{\mathbf{r}} v_{\text{rel}} B \mathbf{v}_{\text{rel}} + \sum_l \mathbf{p}_{J_l} + \mathbf{a}_{\mathsf{T}}$$

- More compactly

$$d\mathbf{X} = \tilde{f}(\mathbf{X},t)dt + \tilde{\mathbf{G}}(\mathbf{a}_{\mathsf{T}})d\mathbf{W},$$

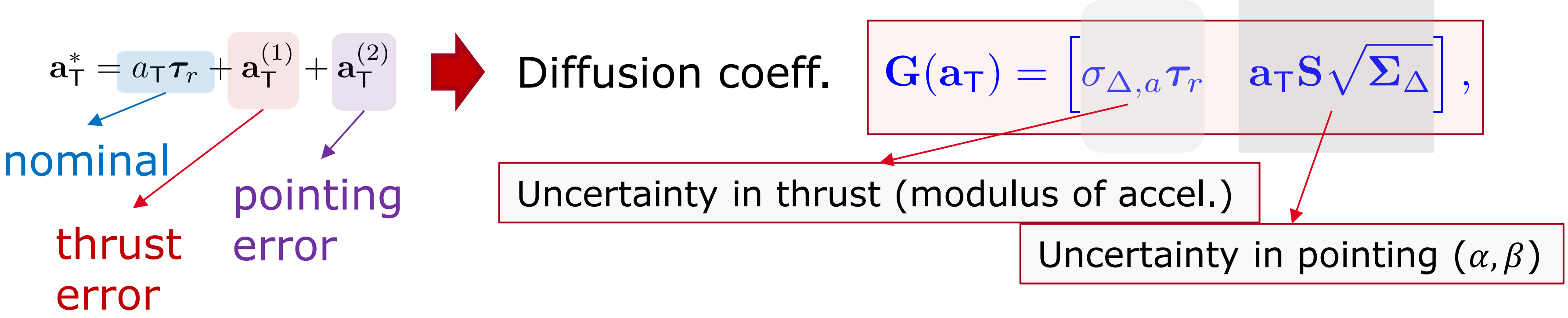
$$\tilde{f}(\mathbf{X},t) = \begin{bmatrix} \mathbf{v}(t) \\ f(\mathbf{X},t) \end{bmatrix} \quad \text{and} \quad \tilde{\mathbf{G}}(\mathbf{a}_{\mathsf{T}}) = \begin{bmatrix} \mathbf{0}_{3\times 3} \\ \mathbf{G}(\mathbf{a}_{\mathsf{T}}) \end{bmatrix}.$$

- ○ **W**(t) is a 3x1 Wiener process
- ○ **X**(t) is a 6x1 state (position + velocity)
- ○ How do we choose the diffusion term?

Task 2.2: Approach to uncertainty propagation in presence of continuous manoeuvring. Probabilistic-based methods.

Uncertainty modelling via SDEs

- Choice of diffusion term
  - Simple models: diffusion coefficients constant or proportional to thrust
  - A more realistic model: uncertainty in thrust & pointing



$$\mathbf{a}_\mathsf{T}^* = a_\mathsf{T}\boldsymbol{\tau}_r + \mathbf{a}_\mathsf{T}^{(1)} + \mathbf{a}_\mathsf{T}^{(2)}$$

Diffusion coeff. $\mathbf{G}(\mathbf{a}_\mathsf{T}) = \left[\sigma_{\Delta,a}\boldsymbol{\tau}_r \quad \mathbf{a}_\mathsf{T}\mathbf{S}\sqrt{\boldsymbol{\Sigma}_\Delta}\right]$,

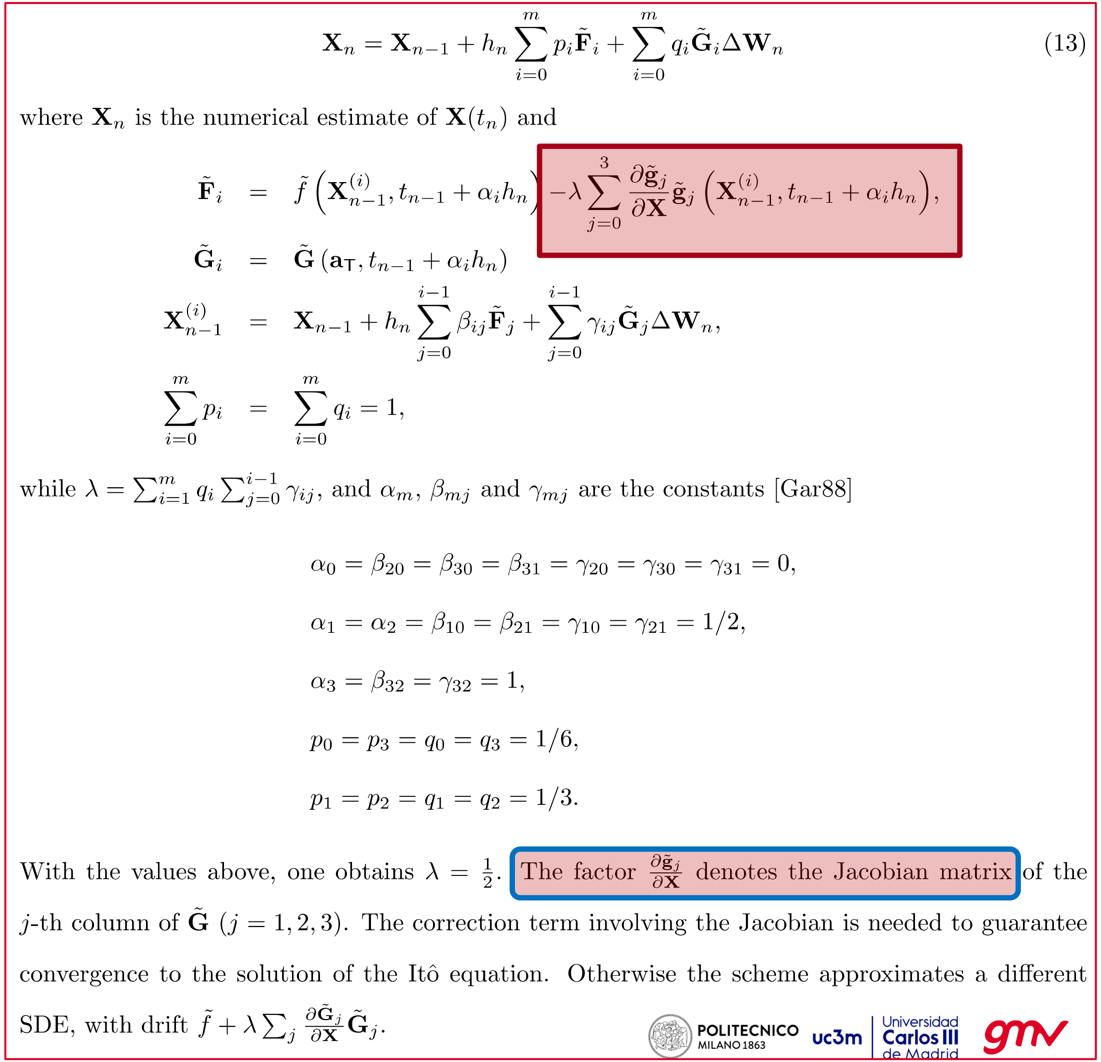
Uncertainty in thrust (modulus of accel.)

Uncertainty in pointing (α, β)

Task 2.2: Approach to uncertainty propagation in presence of continuous manoeuvring. Probabilistic-based methods

Numerical schemes for SDEs

- Strong order 1.0 stochastic Runge-Kutta [Rümelin, 1982]
- $\mathbf{G} \equiv \mathbf{0}$ no diffusion when there is no thrust
- Computation of the Jacobians of the columns of $\tilde{\mathbf{G}}$



$$\mathbf{X}_n = \mathbf{X}_{n-1} + h_n \sum_{i=0}^{m} p_i \tilde{\mathbf{F}}_i + \sum_{i=0}^{m} q_i \tilde{\mathbf{G}}_i \Delta \mathbf{W}_n \tag{13}$$

where $\mathbf{X}_n$ is the numerical estimate of $\mathbf{X}(t_n)$ and

$$\begin{aligned}
\tilde{\mathbf{F}}_i &= \tilde{f}\left(\mathbf{X}_{n-1}^{(i)}, t_{n-1} + \alpha_i h_n\right) \boxed{-\lambda \sum_{j=0}^{3} \frac{\partial \tilde{\mathbf{g}}_j}{\partial \mathbf{X}} \tilde{\mathbf{g}}_j \left(\mathbf{X}_{n-1}^{(i)}, t_{n-1} + \alpha_i h_n\right),} \\
\tilde{\mathbf{G}}_i &= \tilde{\mathbf{G}}\left(\mathbf{a}_{\mathsf{T}}, t_{n-1} + \alpha_i h_n\right) \\
\mathbf{X}_{n-1}^{(i)} &= \mathbf{X}_{n-1} + h_n \sum_{j=0}^{i-1} \beta_{ij} \tilde{\mathbf{F}}_j + \sum_{j=0}^{i-1} \gamma_{ij} \tilde{\mathbf{G}}_j \Delta \mathbf{W}_n, \\
\sum_{i=0}^{m} p_i &= \sum_{i=0}^{m} q_i = 1,
\end{aligned}$$

while $\lambda = \sum_{i=1}^{m} q_i \sum_{j=0}^{i-1} \gamma_{ij}$, and α_m, β_{mj} and γ_{mj} are the constants [Gar88]

$$\alpha_0 = \beta_{20} = \beta_{30} = \beta_{31} = \gamma_{20} = \gamma_{30} = \gamma_{31} = 0,$$

$$\alpha_1 = \alpha_2 = \beta_{10} = \beta_{21} = \gamma_{10} = \gamma_{21} = 1/2,$$

$$\alpha_3 = \beta_{32} = \gamma_{32} = 1,$$

$$p_0 = p_3 = q_0 = q_3 = 1/6,$$

$$p_1 = p_2 = q_1 = q_2 = 1/3.$$

With the values above, one obtains $\lambda = \frac{1}{2}$. The factor $\frac{\partial \tilde{\mathbf{g}}_j}{\partial \mathbf{X}}$ denotes the Jacobian matrix of the j-th column of $\tilde{\mathbf{G}}$ ($j = 1, 2, 3$). The correction term involving the Jacobian is needed to guarantee convergence to the solution of the Itô equation. Otherwise the scheme approximates a different SDE, with drift $\tilde{f} + \lambda \sum_j \frac{\partial \tilde{\mathbf{G}}_j}{\partial \mathbf{X}} \tilde{\mathbf{G}}_j$.

Task 2.2: Approach to uncertainty propagation in presence of continuous manoeuvring. Probabilistic-based methods

UP algorithms

UP algorithm	**Cost***
Monte Carlo (time reference)	*N*=500 trajectories
Monte Carlo (accuracy reference)	*N*=50,000 trajectories
Symmetric UT	$2d+1$
Reduced UT	$d+1$
Spherical-radial cubature degree 3	$2d$
Spherical-radial cubature degree 5	$2d^2+1$
KDE (fine bandwidth)	$2d\cdot N$, $N=20$ (240 cubature points)
KDE (coarse bandwidth)	$2d\cdot N$, $N=20$ (240 cubature points)

*Computational cost given by the number of initial points to be propagated through the numerical scheme!

- Initial KDE bandwidth (Crisan & Miguez, 2014)

$$p(\mathbf{x}_0) = \frac{1}{N}\sum_{i=1}^{N}\mathcal{N}(\mathbf{x}_0^i, b_N\mathbf{C}_0), \quad b_N = CN^{-\frac{1}{2(2d+1)}}, \quad C \in [0.2, 1]$$

- "Coarse" is $C=0.7$, "fine" is $C=0.3$

Task 2.3: Assessment of suitability of the selected approach

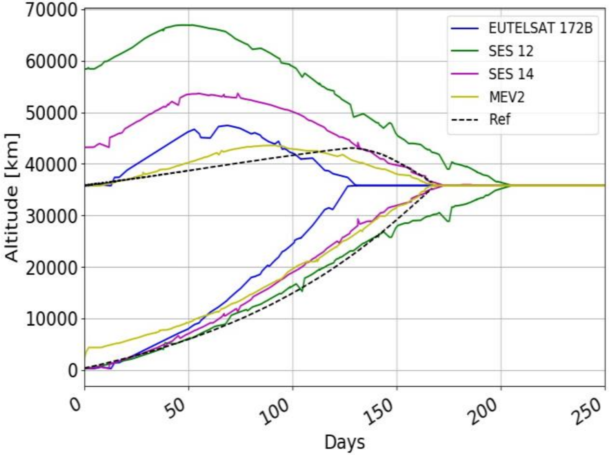
Operational scenarios

- EOR transfers derived by means of GMV’s optimization tool OPEPOR.
- SK scenarios derived from TLE analysis and typical yearly budgets.

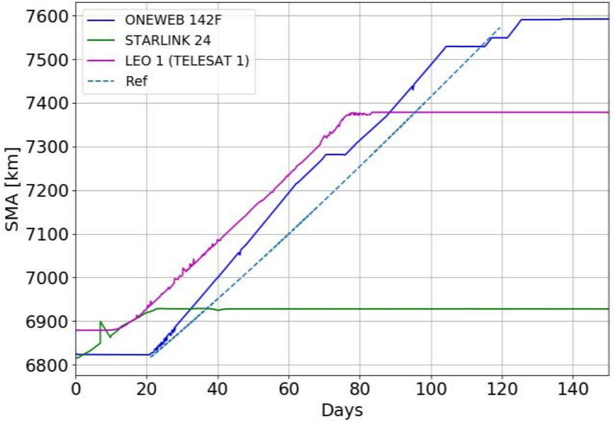
Scenario	**Transfer phase**	**Orbit description**	**Manouvre Description**
LEO DISPOSAL	End	Low LEO (Starlink)	LEO Disposal like Starlink, end of the transfer
GEO GRAVEYARD	Start	GEO	GEO Disposal into graveyard orbit
GTO2GEO	Start	GTO (0.7)	GTO-to-GEO EOR, first interval (near GTO)
GTO2GEO	End	near-GEO	GTO-to-GEO EOR, last interval (near GEO)
LEO2LEO	Start	Low LEO (Starlink)	LEO-to-LEO EOR, first interval (low LEO)
GEO SK	N/A	GEO	GEO SK with EP. Daily maneuver with lower thrust
LEO Constellation	N/A	Low LEO (Starlink)	Several small along-track manoeuvres, one every day

Task 2.3: Assessment of suitability of the selected approach

Operational scenarios



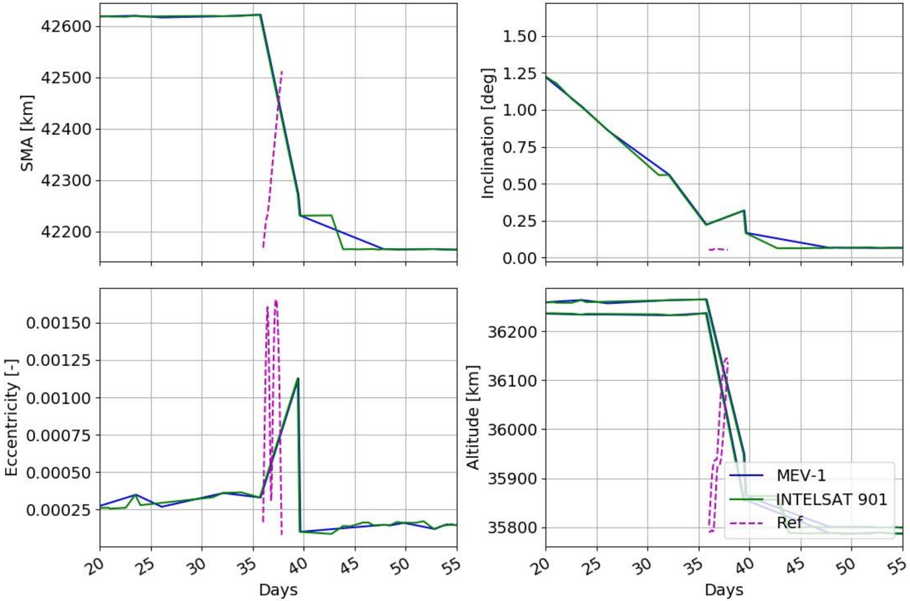
GTO to GEO



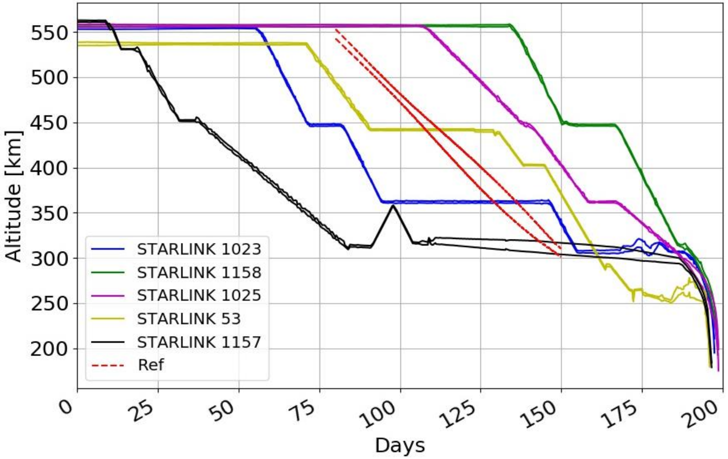
LEO to LEO

Task 2.3: Assessment of suitability of the selected approach

Operational scenarios



Graveyard



LEO Disposal

Task 2.3: Assessment of suitability of the selected approach

Results with dynamics-based methods

- Comparison metrics **(Benchmark given by 50’000 samples of MC)**
- **L_2 norm of mean position and velocity**
- **L_2 norm of position and velocity covariances**
- Additional **non-Gaussianity** metrics:
  - **Maximum Total Variation Distance (TVD)** computed state by state
  - **Maximum Mean Integrated Square Error** computed state by state
- **Computational time** (Benchmark given by 500 samples of MC to reduce time)

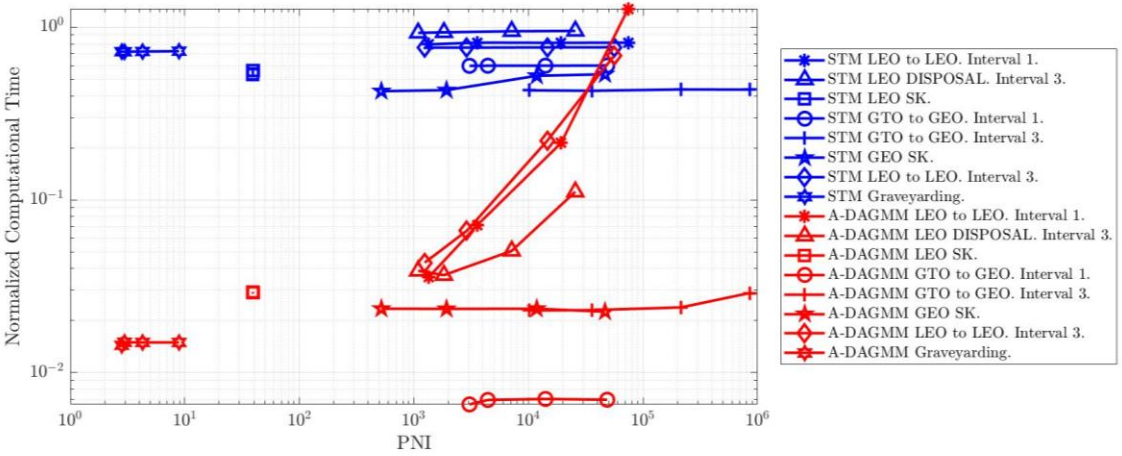
All scenarios can be compared through the **Process Noise Index (PNI)**. Represents a cumulative index of nonlinearity:

$$\boxed{PNI = \frac{\boldsymbol{P}_f}{\boldsymbol{P}_0}}$$

Task 2.3: Assessment of suitability of the selected approach

Results with dynamics-based methods

- Computational time
  - **STM** has **stable performance**, duration depends on **duration of propagation** and **length of maneuvers:** averages **~0.7**.
  - **A-DAGMM has dependency on thruster accuracy** (PNI increases for varying uncertainty if scenario is fixed): averages **~0.1**.
  - **PNI lumps all sources of nonlinearity into one parameter**: clear from LEO SK that has short propagation and small maneuvers

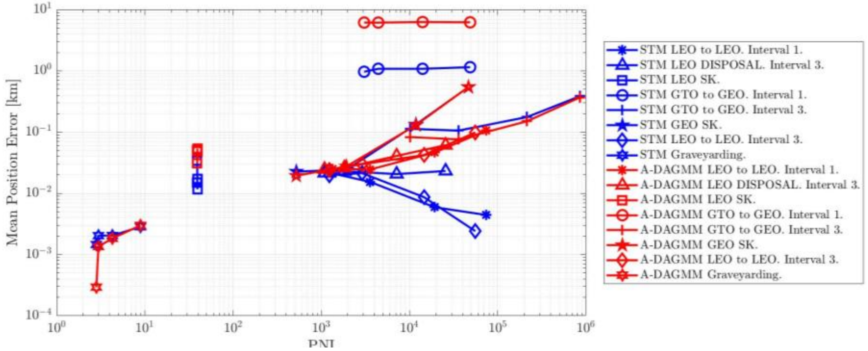


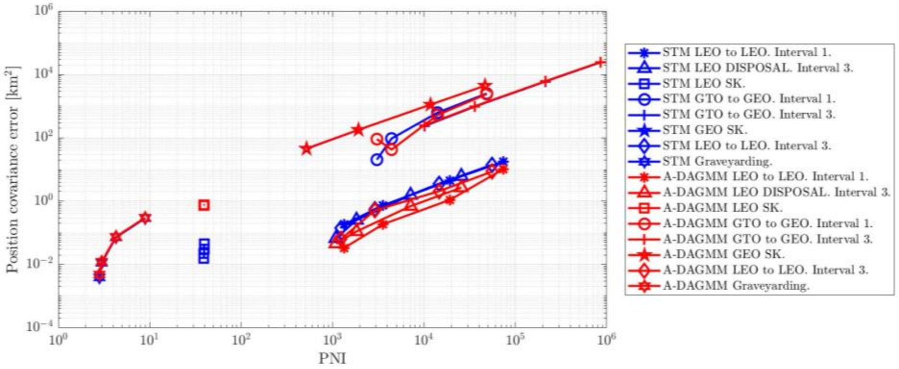
Task 2.3: Assessment of suitability of the selected approach

Results with dynamics-based methods

- Gaussian Estimation

- **Neither methods significantly impacted on mean estimation**
- Both methods have **dependency in covariance** estimation
- **STM** achieves overall **better estimated mean** whereas **A-DAGMMo** has **better estimated covariance**



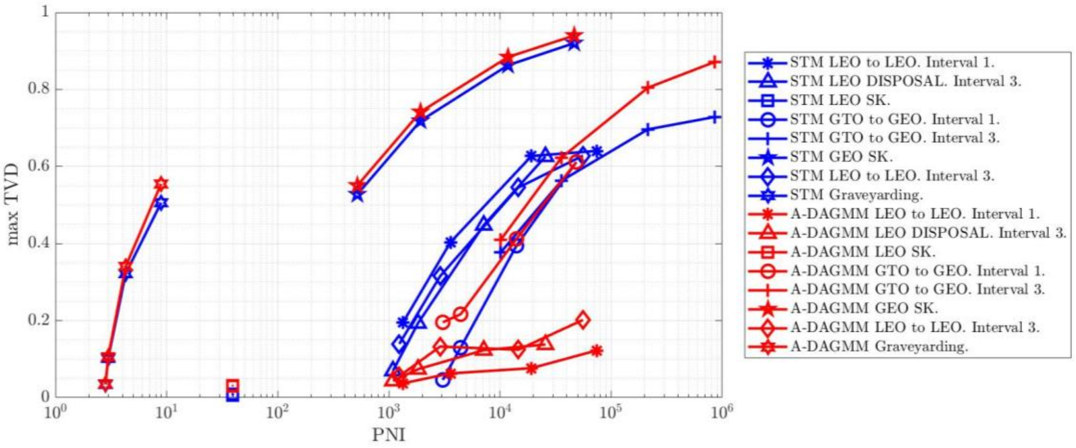


Task 2.3: Assessment of suitability of the selected approach

Results with dynamics-based methods

- Non-Gaussian Error

- **Similar trends as in covariance**
- **A-DAGMM can adapt number of GMEs** hence performs better
- STM is tuned for lower levels of error and kept constant
- **hyperparameters of both methods can be fixed for similar scenarios**: they may vary if the propagation is particularly different (SK vs EOR)



Task 2.3: Assessment of suitability of the selected approach

Results with probabilistic-based methods

- Assessment of 6 algorithms for UP (described in Task 2.2)
  - Two UT methods : complete UT (2 x dim + 1 sigma points) & reduced (dim + 1 sigma points)
  - Two SRC methods : SRC 3D & SRC 5D
    - SRC 3D equivalent to UT with 2 x dim sigma points
  - Two KDE methods : “fine” bandwidth 0.3 & “coarse” bandwidth 0.7
- Assessment based on analysis of (described before) :
  - Six scenarios.
  - Four noise levels (from 1, lowest, to 4, highest).
  - MC 5 x 10^4 samples as reference solution.
- Performance metrics used for the assessment:
  - Relative computational time with respect to MC with 500 samples
  - Max. MISE (Mean integrated squared error)
    - Normalized (by the integral of the square of the density)
  - Max. TVD (Total variation distance)
    - 0 when equal densities but for isolated points, 1 when probability densities are disjoint.
  - Mean position / velocity errors

Task 2.3: Assessment of suitability of the selected approach.

Results with probabilistic-based methods

- Total number of experiments: 6 scenarios x 4 noise models x 6 algorithms x repetitions
  - Large amount of data

- Analysis based on “Process Noise Index” (PNI)
  - Ratio between traces of covariance matrices provided by reference MC at final time and initial covariance.

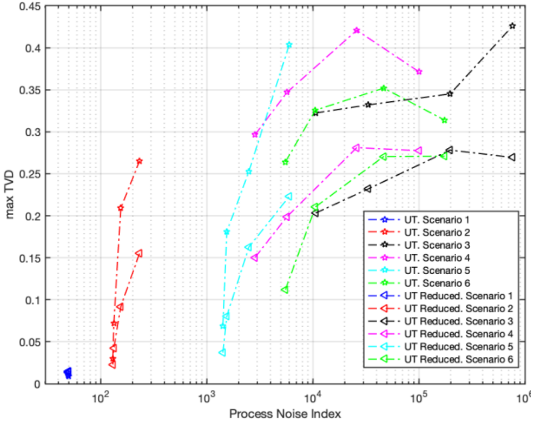
$$PNI = \frac{tr(C_{\mathbf{x}}(t_f))}{tr(C_{\mathbf{x}}(t_0))}$$

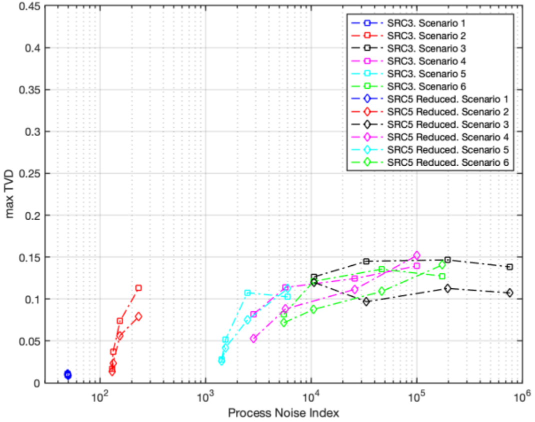
  - Extreme cases: Starlink 03 (noise model 1) and LEO 2 LEO (noise model 4)

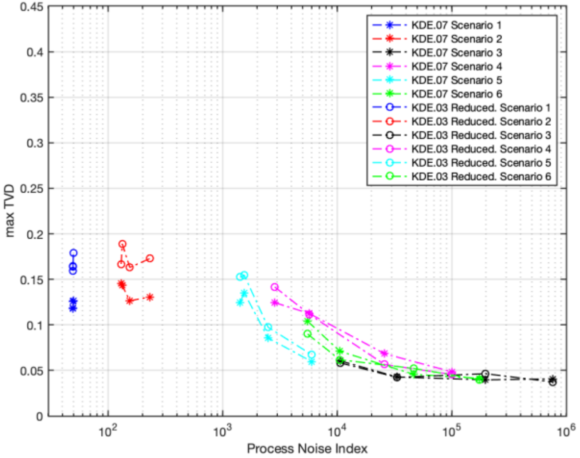
Task 2.3: Assessment of suitability of the selected approach

Results with probabilistic-based methods

- Results for Assessment
  - UT:
    - Performance degrades fast with increasing values of PNI
    - Best computational cost
    - Reduced better results than complete
      - In accordance with previous results in UNCPROP



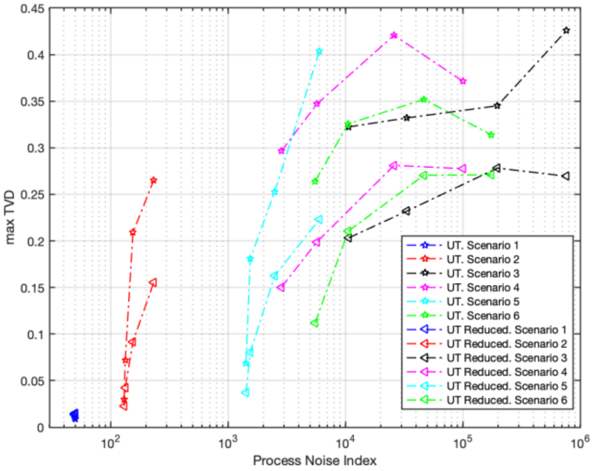


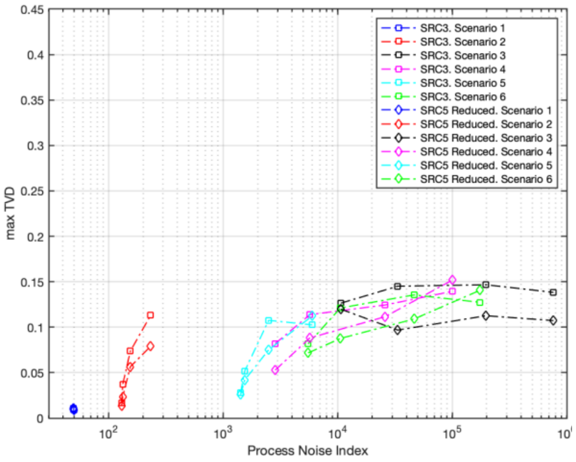


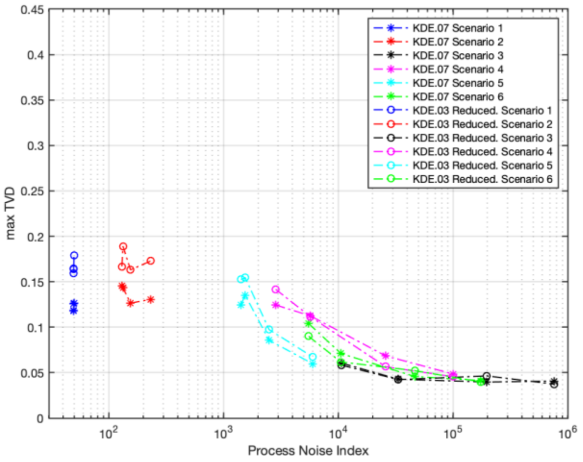
Task 2.3: Assessment of suitability of the selected approach

Results with probabilistic-based methods

- Results for Assessment
  - SRC:
    - Performance better with low PNI, but stable (SRC 5D better than SRC 3D)
    - Intermediate computational cost
    - Robust approach



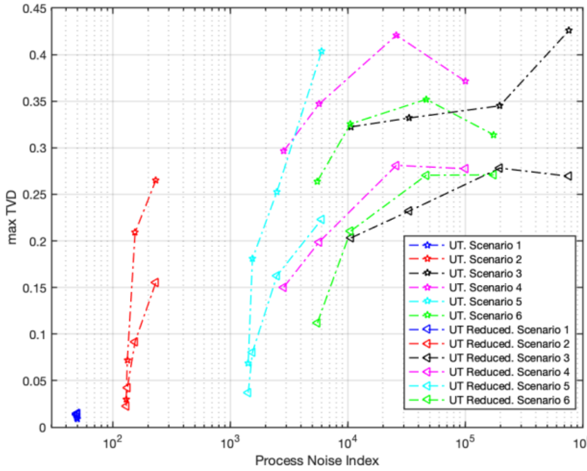


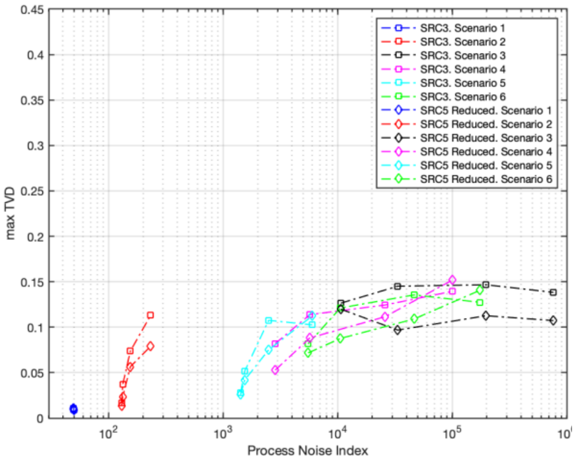


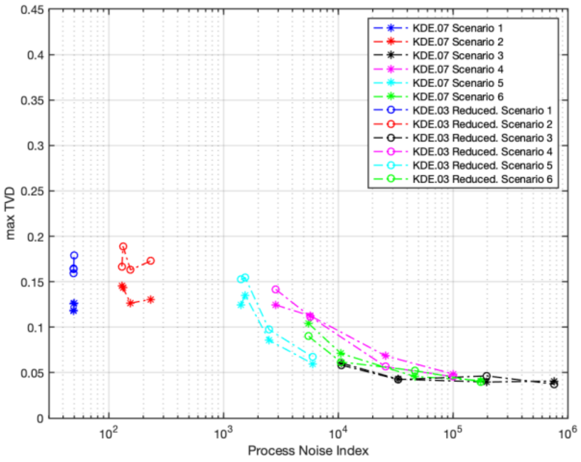
Task 2.3: Assessment of suitability of the selected approach

Results with probabilistic-based methods

- Results for Assessment
  - KDE:
    - Performance better with high PNI.
    - Largest computational cost
    - “Coarse” bandwidth KDE better performance but for large PNI





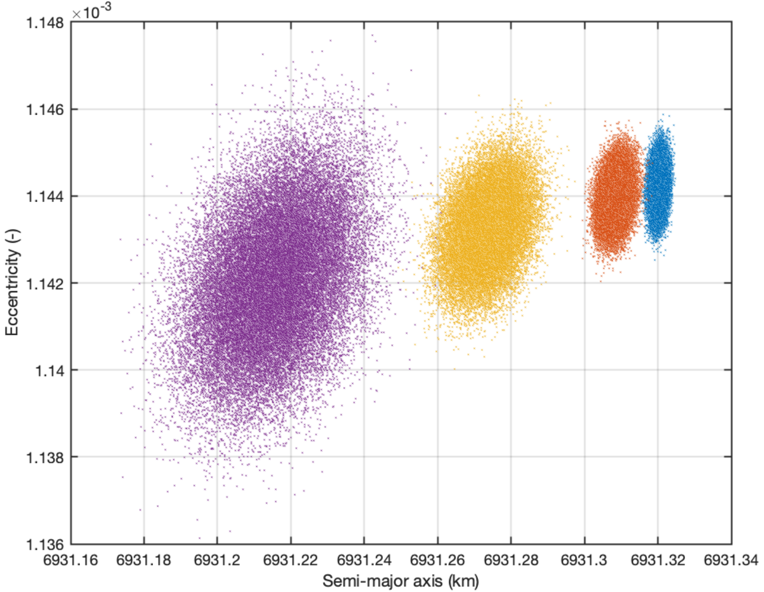


Task 2.3: Assessment of suitability of the selected approach.

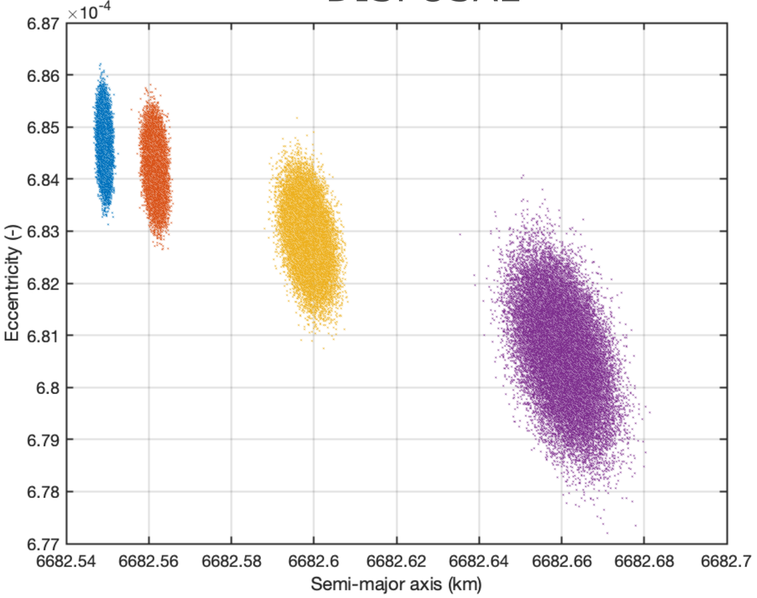
Results with probabilistic-based methods

- Assessment of effect of thrust acceleration noise in state uncertainty at final time.
  - Relevance of correct noise characterization

LEO 2 LEO

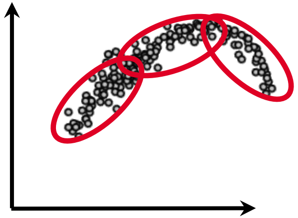


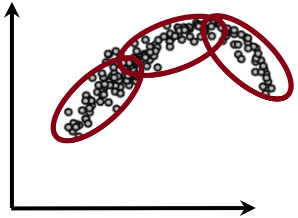
DISPOSAL



Task 2.3: Assessment of suitability of the selected approach for PoC computation

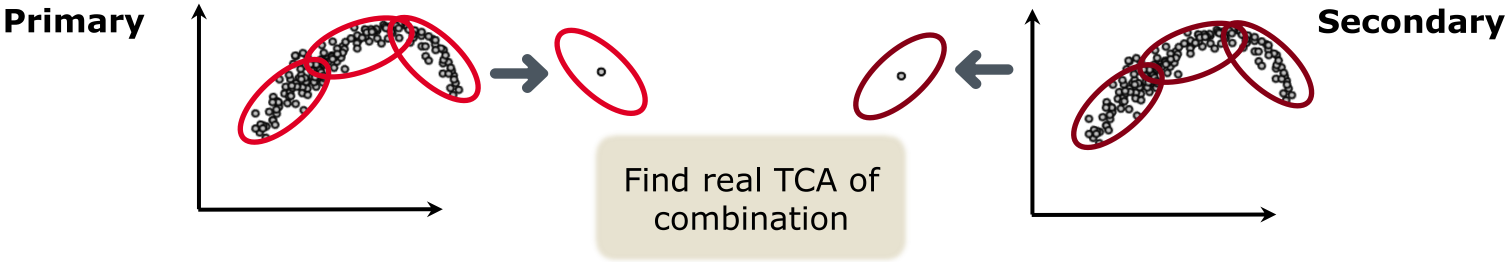
Primary





Secondary

Task 2.3: Assessment of suitability of the selected approach for PoC computation

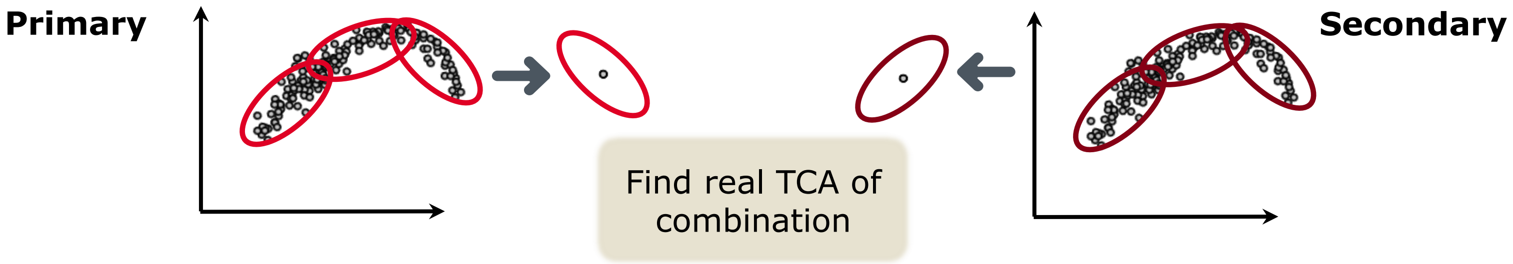


Primary

Secondary

Find real TCA of combination

Task 2.3: Assessment of suitability of the selected approach for PoC computation



Find real TCA of combination

Combine contributions

$$\mathrm{PoC}_{\mathrm{GMM}} = \sum_{i=1}^{n_p}\sum_{j=1}^{n_s} w_i^p w_j^s \mathrm{PoC}_{\mathrm{Chan}}\left(\mathbf{x}_i^p\left(\mathrm{TCA}_{i,j}\right), \mathbf{x}_j^s\left(\mathrm{TCA}_{i,j}\right), \mathbf{P}_i^p\left(\mathrm{TCA}_{i,j}\right), \mathbf{P}_j^s\left(\mathrm{TCA}_{i,j}\right)\right)$$

Task 2.3: Assessment of suitability of the selected approach for PoC computation

- PoC is **always close to the linear propagation**
- **Probabilistic** methods propagation **changes conjunction geometry** (some cases)
- All scenarios are almost linear

	LEO to LEO. Interval 1	LEO DISPOSAL. Interval 3	LEO SK	GTO to GEO. Interval 1 (perigee)	GTO to GEO. Interval 1 (apogee)	GTO to GEO. Interval 3	GEO SK
	PoC / σ PoC	PoC / σ PoC	PoC / σ PoC	PoC / σ PoC	PoC / σ PoC	PoC / σ PoC	PoC / σ PoC
Reference Linear Propagation	7.8984E-05 / n.a.	6.1197E-04 / n.a.	3.7338E-04 / n.a.	1.9472E-04 / n.a.	1.4464E-03 / n.a.	6.6233E-04 / n.a.	5.5866E-03 / n.a.
Stochastic Taylor Model	7.8235E-05 / n.a.	5.9741E-04 / n.a.	3.6334E-04 / n.a.	1.6375E-04 / n.a.	1.4029E-03 / n.a.	8.9033E-04 / n.a.	5.4691E-03 / n.a.
A-DAGMM	7.9897E-05 / n.a.	6.0673E-04 / n.a.	3.7008E-04 / n.a.	1.8967E-04 / n.a.	1.4439E-03 / n.a.	8.0935E-04 / n.a.	5.5866E-03 / n.a.
UT (2d+1)	1.0715E-04 / 3.6444E-06	6.5057E-04 / 4.7394E-06	3.6722E-04 / 4.4230E-06	0.0000E-00 / 0.0000E-00	0.0000E-00 / 0.0000E-00	2.4182E-10 / 1.0483E-09	2.6645E-30 / 5.5395E-30
SRC5	1.0782E-04 / 8.6900E-07	6.5056E-04 / 1.2619E-06	3.7017E-04 / 1.6712E-06	0.0000E-00 / 0.0000E-00	0.0000E-00 / 0.0000E-00	1.0210E-12 / 1.7123E-12	6.8515E-31 / 3.4705E-31
KDE	7.6399E-05 / 2.2771E-05	6.4632E-04 / 3.4697E-04	3.8714E-04 / 1.0282E-04	0.0000E-00 / 0.0000E-00	0.0000E-00 / 0.0000E-00	1.8271E-12 / 7.7383E-12	5.2736E-32 / 4.2648E-31
Reference MC	n.a. / n.a.	n.a. / n.a.	n.a. / n.a.	n.a. / n.a.	n.a. / n.a.	7.6273E-04	5.0727E-03 / n.a.

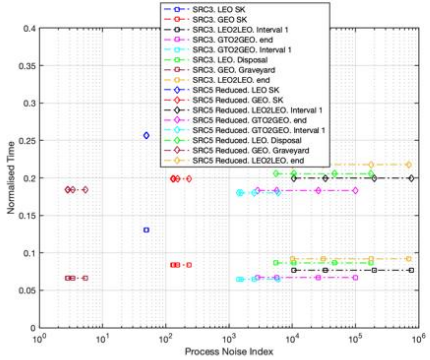
Task 2.3: Assessment of suitability of the selected approach

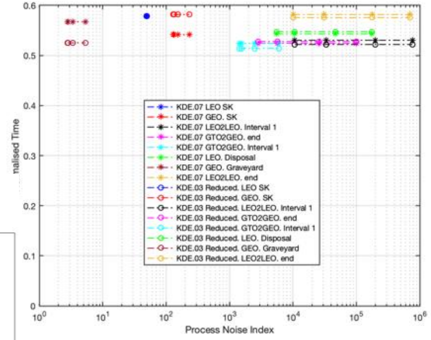
Comparison

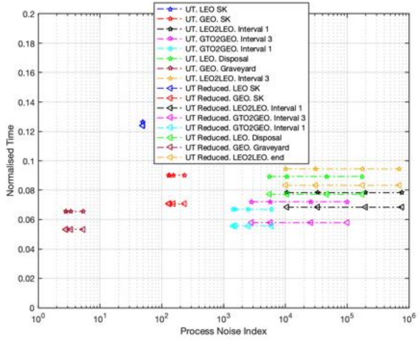
- The comparison is performed in terms of two performance metrics: computational cost and maximum TVD, taken as a proxy of uncertainty realism.

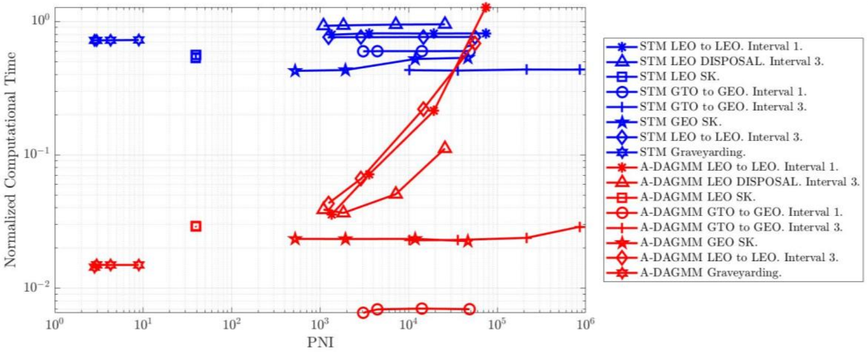
In terms of computational time:

  - **A-DAGMM** offer the best performance (on average) in line with **UT** methods.
  - **SRC** approaches provide a good middle ground
  - **KDEs** and **STM** methods are the most computationally expensive







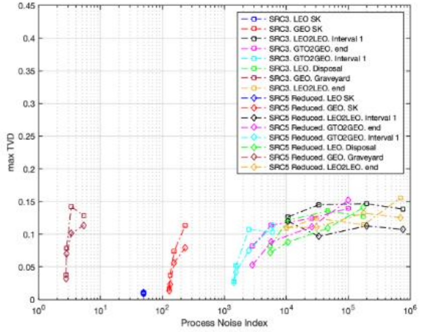


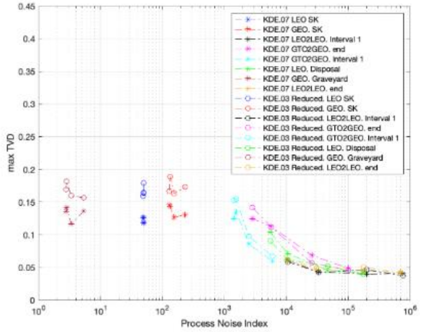
Task 2.3: Assessment of suitability of the selected approach

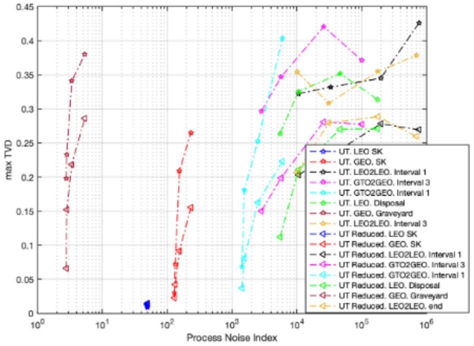
Comparison

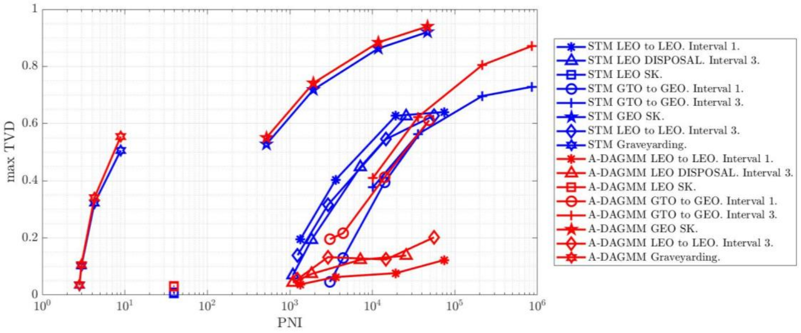
When comparing uncertainty realism:

- **UT** methods work best for low PNI
- **KDEs** are better suited for propagations where higher uncertainties are involved
- **SRC** methods provide a middle ground between the two.
- The trend of **A-DAGMM** and **STM** is closer to that of **UT:** discrepancies between dynamics-based methods are related to the different scenario of propagation rather than the level of noise involved.









ELECTROCAM

Task 3: Operational concepts for collision avoidance for low-thrust missions

Task 3: Operational concepts for collision avoidance for low-thrust missions

- **Task 3.1**: Constraints analysis
- **Task 3.2**: Approach for conjunction screening
- **Task 3.3**: Approach for collision avoidance
- **Task 3.4**: Simulations
- **Task 3.5**: Sensitivity analysis

Key aspects of the proposed solution

- **GMV**: Constrains from literature review and experience
- **GMV**: Ops concepts for conjunction screening and collision avoidance
- **PoliMi&GMV**: analytical, semi-analytical & numerical methods for CAM
- **PoliMi&GMV**: simulations of conjunction screening & collision avoidance
- **PoliMi**: Sensitivity analysis from results of simulations

Task 3.1: Constraints analysis

- Constraint identification and alternative **grouping** to reduce ambiguity of traditional grouping into platform/operational constraints:
  - **CA constraints**: CA service provider response time, type of secondary object (active or debris), detection time, interaction delays…
  - **Mission constraints**: operational overheads, OCMs, acceptable times for manoeuvre execution, attitude restrictions…
  - **Propulsive and power constraints**: propulsion capabilities (max thrust, maximum/minimum firing duration, ATOX, thrust pointing…), power subsystem (battery capacity, radiation during Van Allen Belt crossing…)…

Task 3.2: Approach for conjunction screening
Operational concepts

- **Conjunction screening. Key points:**
  - Operational cycle
    - Satellite tracking
    - Orbit determination
    - Conjunction monitoring
  - CA service provider
    - Space debris catalogue
    - Interaction delays
  - Level of autonomy
    - On-board processes
  - Screening volume selection (typical approach and EOR approach)
  - Risk assessment (Geometry, PoC, Mahalanobis distance, Time to TCA)

Task 3.3: Approach for collision avoidance Operational concepts

- **Collision avoidance. Key points:**
  - Thruster considerations:
    - Acceleration level
    - Thrust limitations
    - Thrust uncertainty
  - CAM design methods:
    - Use planned manoeuvres for CAM
    - CAM during EOR
    - Multiple event CAM
    - Return manoeuvre
  - CAM communication paths

Task 3.3: Approach for collision avoidance
Analytical and semi-analytical methods for CAM

- **Outline:**
  - Review of the Energy-Optimal (EO) CAM design with Chan's PoC method
  - Review of the Fuel-Optimal (FO) CAM design with Chan's PoC method
  - Review of the EOR CAM design with Chan's PoC method
  - Brief introduction to the EO/FO/EOR CAM design with Chan's GMM PoC method
  - Single-averaged CAM models

Task 3.3: Approach for collision avoidance
Energy-Optimal Chan-based CAM

- Problem dynamics

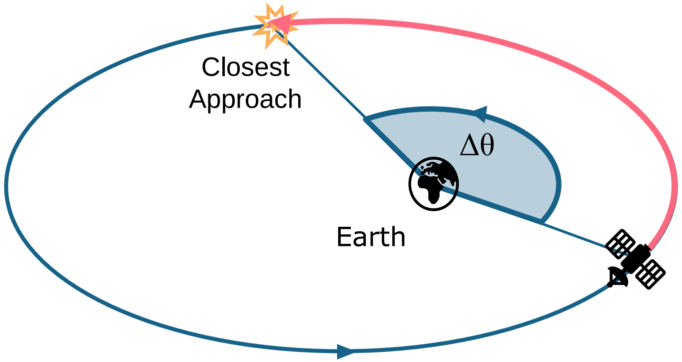
$$\begin{cases} \dot{\mathbf{r}} = \mathbf{v} \\ \dot{\mathbf{v}} = -\dfrac{\mu}{r^3}\mathbf{r} + \mathbf{a}_c \end{cases} \qquad \text{ICs}: \begin{cases} \mathbf{r}(t_0) = \mathbf{r}_0 \\ \mathbf{v}(t_0) = \mathbf{v}_0 \end{cases}$$

- Optimal control problem

$$\begin{cases} \bar{J} = \nu\Psi(\mathbf{r}_{p,f}) + \displaystyle\int_{t_0}^{t_f} \left\{ \frac{1}{2}\epsilon^2 + \right. \\ \qquad \left. + \boldsymbol{\lambda}^\top(t)[\dot{\mathbf{x}}_p(t) - \mathbf{f}_p(t, \mathbf{x}_p, \epsilon)] \right\} \mathrm{d}t \\ t_0,\ t_f\ fixed \\ \boxed{\Psi(\mathbf{r}_{p,f}) = (\boldsymbol{r}_f - \boldsymbol{r}_s)^\top \boldsymbol{R}_{b,\text{2D}}^\top \boldsymbol{C}^{-1} \boldsymbol{R}_{b,\text{2D}} (\boldsymbol{r}_f - \boldsymbol{r}_s) - \bar{d}_M^2 = 0} \\ \dot{\mathbf{x}}_p(t) = \mathbf{f}_p(t, \mathbf{x}_p, \epsilon). \end{cases}$$

SMD at conjunction

Chan PoC constraint



SOLUTION

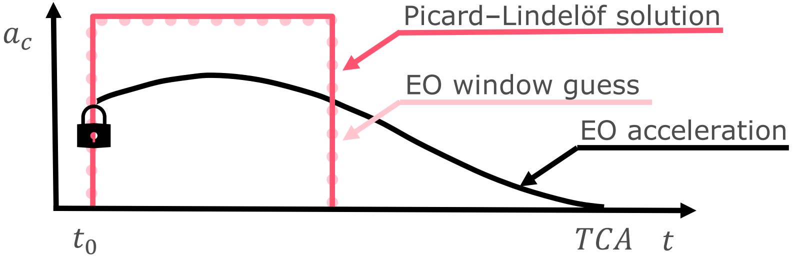
Compute the **State Transition Matrix** of the **Hamiltonian system**, rearrange the **boundary conditions**, and **solve an analytic formula** for Lagrange multipliers (quartic equation in ν).

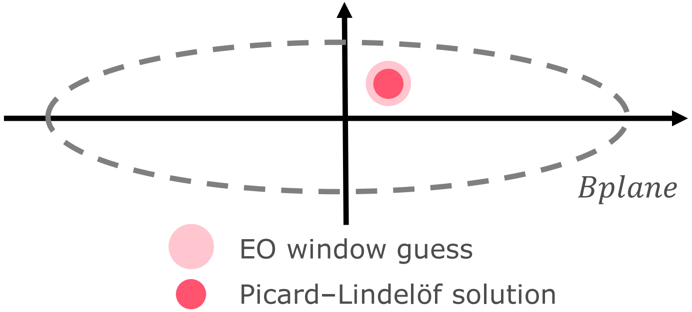
all the results are validated with the fully non-linear dynamics (keplerian + a_c)*

Task 3.3: Approach for collision avoidance

Fuel-Optimal Chan-based CAM

- With **DA-Picard Lindelöf** iterations, expand the **firing window guess** and then propagate **ballistically** to meet the constraint on PoC. The **EO window guess** has the same area as the **EO acceleration** one (**same** Δv). Technique suitable for **Just-in-time CAMS.**





Task 3.3: Approach for collision avoidance
Fuel-Optimal Chan-based CAM

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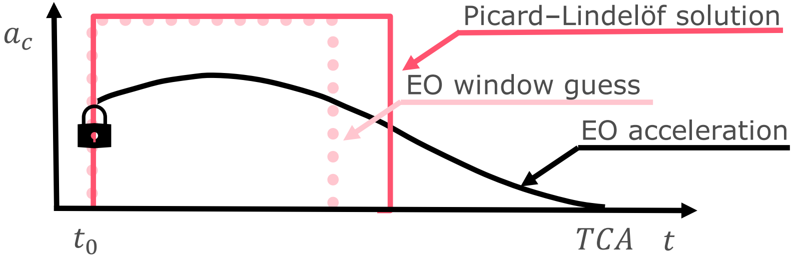

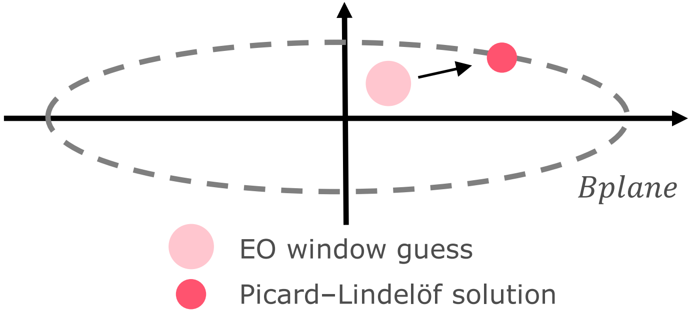

Task 3.3: Approach for collision avoidance Fuel-Optimal Chan-based CAM

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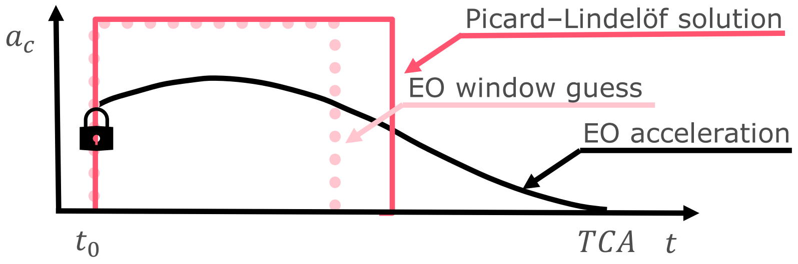
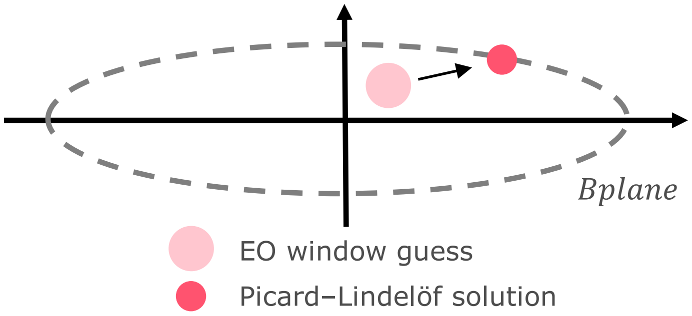
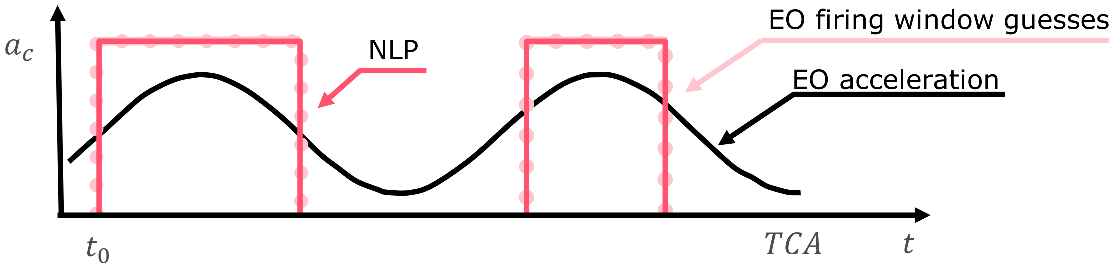
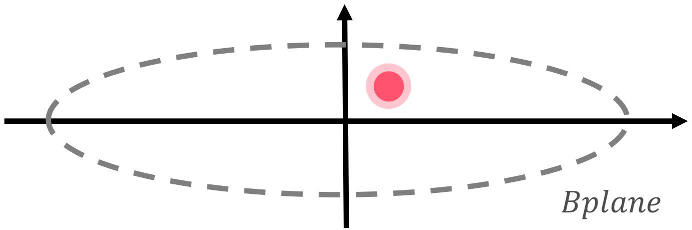




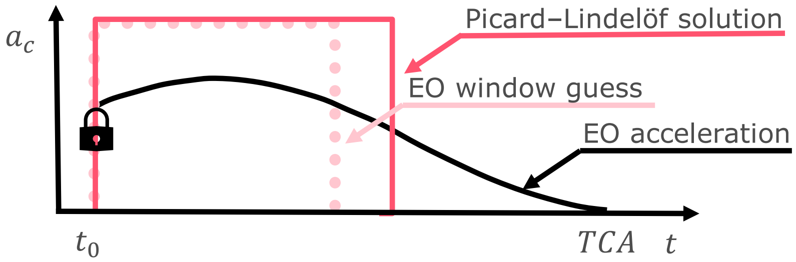
- **Single arc firing selection** with an **NLP** leveraging **analytical propagators** for tangentially thrusted (Bombardelli et al.) and ballistic arcs. The **EO window guesses** have the same area as the **EO acceleration** one (**same** Δv). Technique suitable for **Advance notification CAMS.**

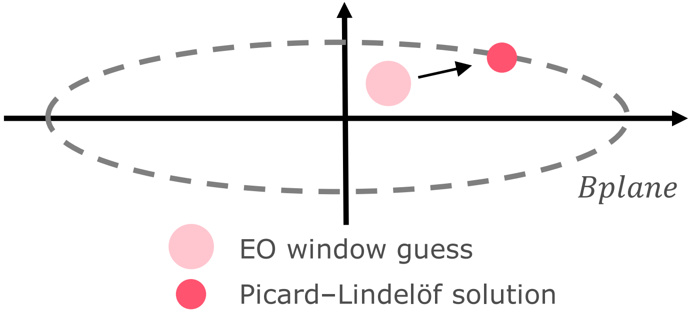




Task 3.3: Approach for collision avoidance
Fuel-Optimal Chan-based CAM

- With **DA-Picard Lindelöf** iterations, expand the **firing window guess** and then propagate **ballistically** to meet the constraint on PoC. The **EO window guess** has the same area as the **EO acceleration** one (**same** Δv). Technique suitable for **Just-in-time CAMS.**





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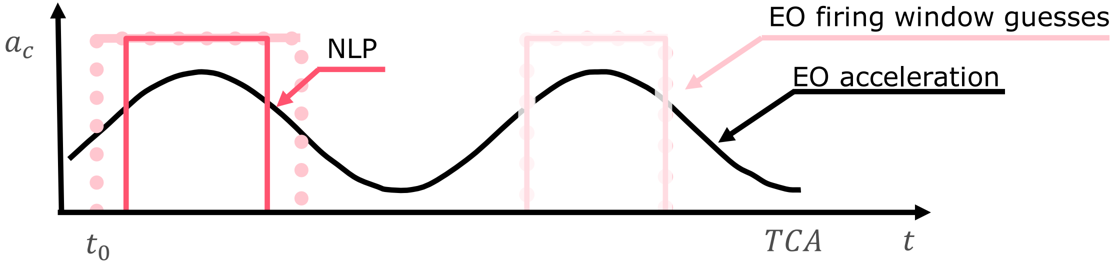

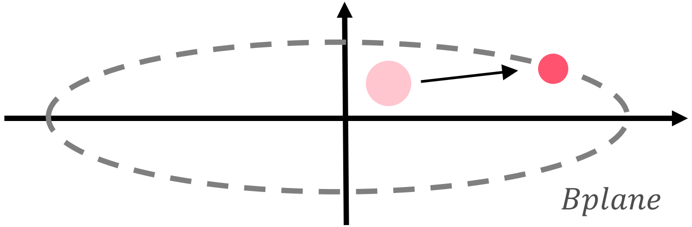

Task 3.3: Approach for collision avoidance
Fuel-Optimal Chan-based CAM

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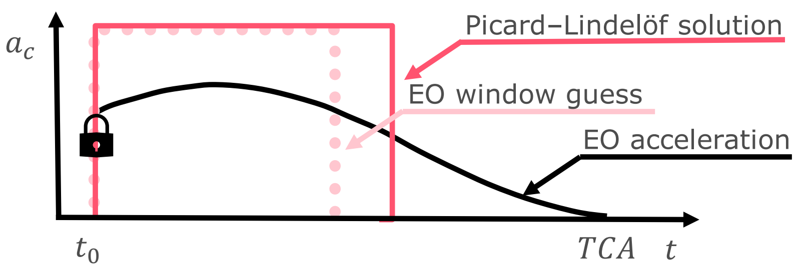

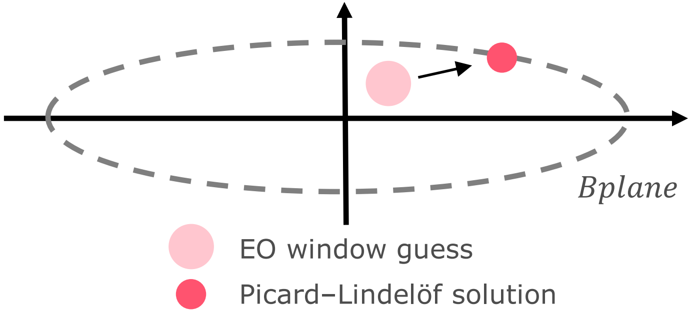

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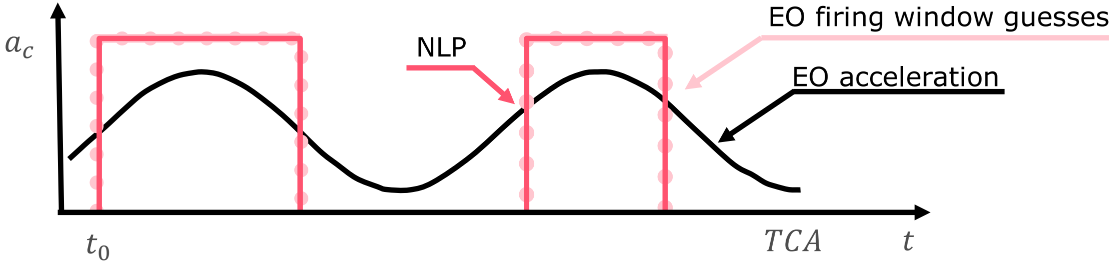

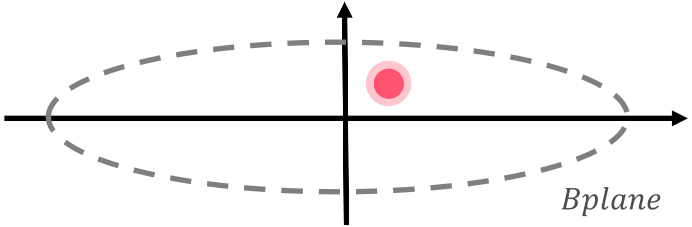

Task 3.3: Approach for collision avoidance
Fuel-Optimal Chan-based CAM

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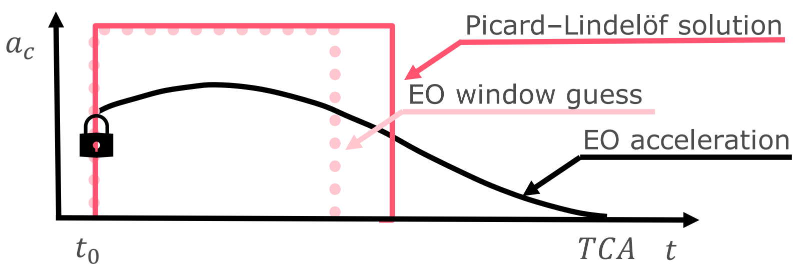
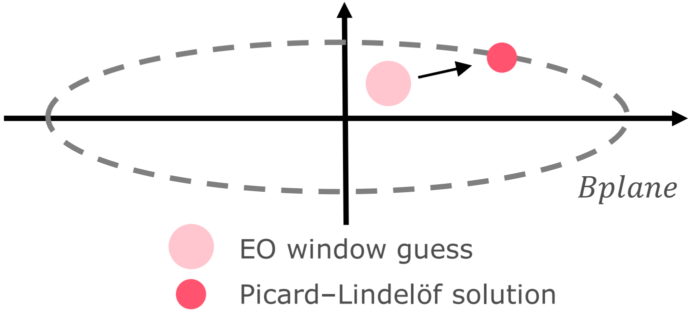
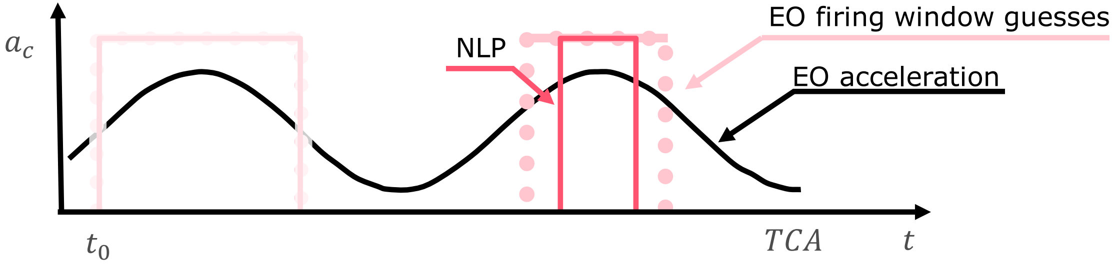
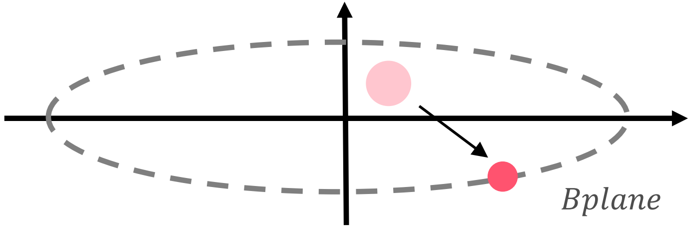




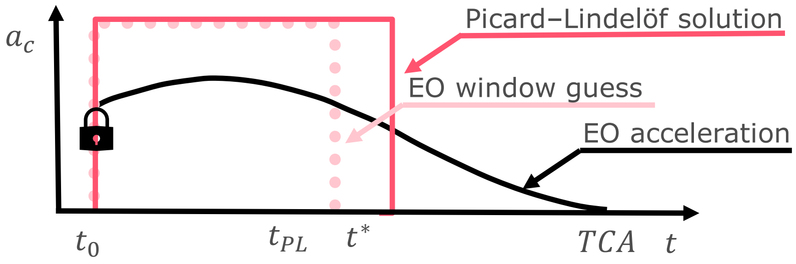
- **Single arc firing selection** with an **NLP** leveraging **analytical propagators** for tangentially thrusted (Bombardelli et al.) and ballistic arcs. The **EO window guesses** have the same area as the **EO acceleration** one (**same** Δv). Technique suitable for **Advance notification CAMS.**

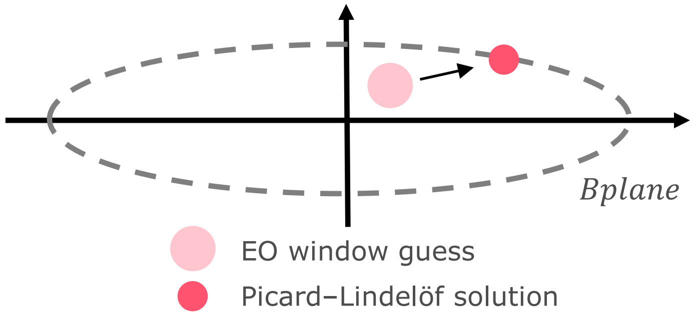




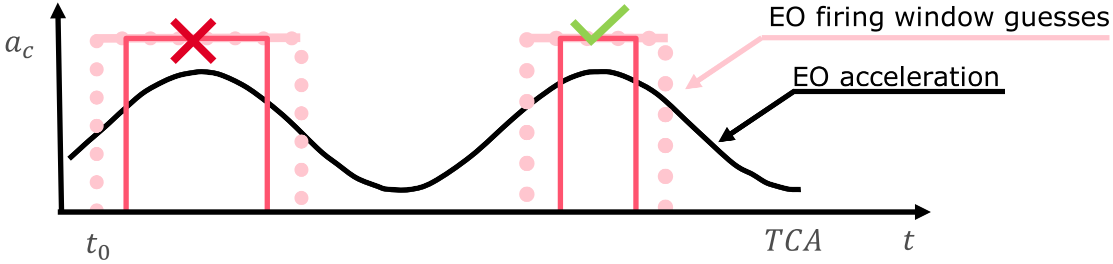
Task 3.3: Approach for collision avoidance
Fuel-Optimal Chan-based CAM

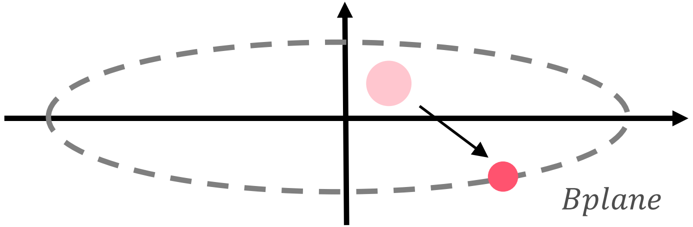
- With **DA-Picard Lindelöf** iterations, expand the **firing window guess** and then propagate **ballistically** to meet the constraint on PoC. The **EO window guess** has the same area as the **EO acceleration** one (**same** Δv). Technique suitable for **Just-in-time CAMS.**





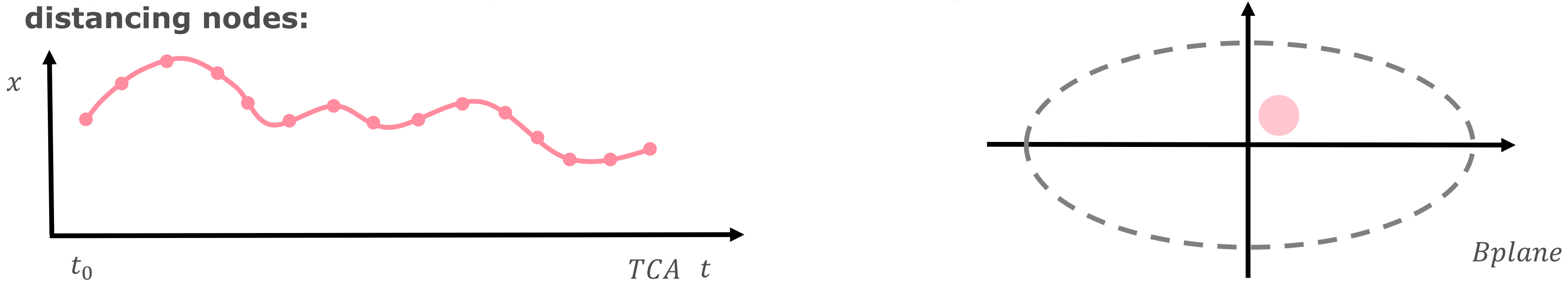
- **Single arc firing selection** with an **NLP** leveraging **analytical propagators** for tangentially thrusted (Bombardelli et al.) and ballistic arcs. The **EO window guesses** have the same area as the **EO acceleration** one (**same** Δv). Technique suitable for **Advance notification CAMS.**



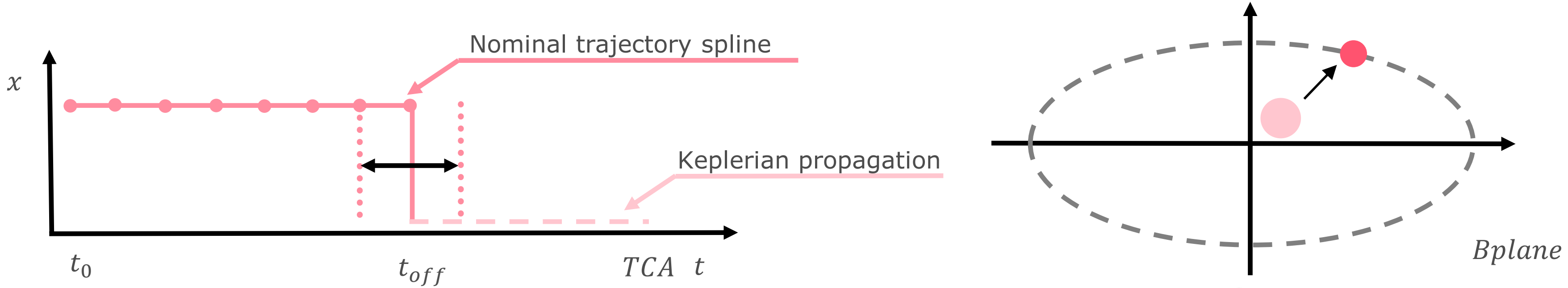


Task 3.3: Approach for collision avoidance
EOR Chan-based CAM

- Integrate the dynamics adopting a **predefined control strategy** described by an OPM with **equal distancing nodes:**

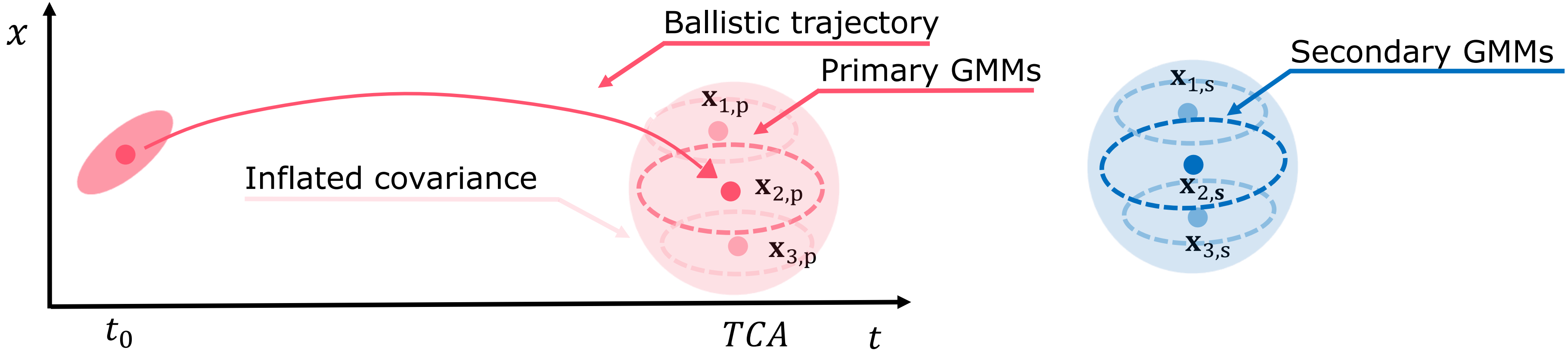


- The orbit-raising **switch-off time** is found through a **bisection-like algorithm** composed of a spline polynomial for the nominal trajectory state and an analytic Keplerian propagator:



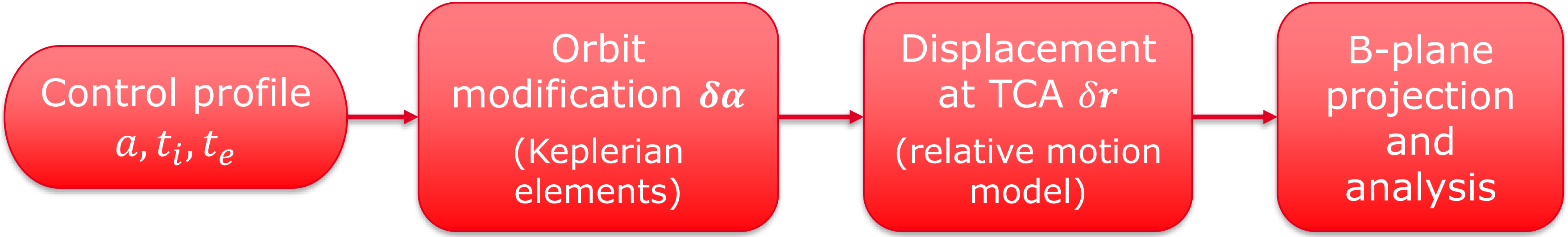
Task 3.3: Approach for collision avoidance
GMM Chan-based CAM

- Evaluate the effect of **non-linearities** for **covariance inflation due to thrust noise** with the FO Chan's CAM and generate a GMM for the primary and secondary



- Similar techniques have been implemented for **Chan's GMM CAM**, more specifically:
  - An **energy-Optimal** formulation for tangential and radial maneuvers (**semi-analytical**)
  - A **fuel-Optimal** formulation for tangential and radial maneuvers (**semi-analytical**)
  - An **EOR** formulation with shutdown time estimation (**semi-analytical**)
- All the techniques have a **slightly higher computational cost** due to the new PoC boundary function still attaining **same accuracy levels**.

Task 3.3: Approach for collision avoidance Single-averaged CAM models



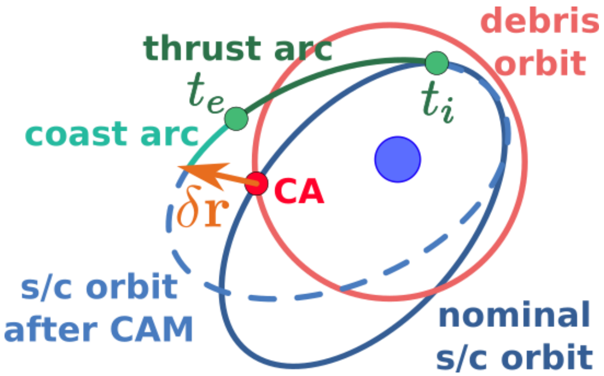
Control profile a, t_i, t_e

Orbit modification $\delta\alpha$ (Keplerian elements)

Displacement at TCA δr (relative motion model)

B-plane projection and analysis

- CAM modelled as **sequence of thrust and coast arcs**
- Constant thrust profile at each arc
- Orbit modification (in terms of Keplerian elements) for each arc based on **single-average analytical techniques**
- Quasi-optimal piecewise constant control profile derived from impulsive CAM model



Task 3.3: Approach for collision avoidance
Single-averaged CAM models

For small thrust acceleration $|\boldsymbol{a}| \ll 1$, contributions from each $\boldsymbol{a}$ component are treated linearly:

$$\Delta\boldsymbol{\alpha}\big|_{\Delta t_k} = \Delta\boldsymbol{\alpha}^t\big|_{\Delta t_k}\varepsilon_t + \Delta\boldsymbol{\alpha}^n\big|_{\Delta t_k}\varepsilon_n + \Delta\boldsymbol{\alpha}^n\big|_{\Delta t_k}\varepsilon_h + \mathcal{O}\left(\varepsilon_{t,n,h}^2\right) \qquad \varepsilon_{t,n} = \frac{a_{t,n,h}}{\mu/a_{ref}^2}$$

Fully analytical models in E for **tangential** [1,2] and **normal** [3] components, involving complete elliptic integrals of the reference orbit (evaluated just once) and trigonometric series:

Complete elliptic integrals (1st and 2nd kind) of e_{ref}^2

$$\boldsymbol{\alpha}^{t,n}(E;\varepsilon_{t,n}) = \boldsymbol{\alpha}_{ref} + \varepsilon_{t,n}\boxed{\boldsymbol{K}_{\alpha}^{t,n}}E + \varepsilon_{t,n}\boxed{\boldsymbol{\alpha}_{osc}^{t,n}(E)} + \mathcal{O}\left(\varepsilon_{t,n}^2\right)$$

$$\boldsymbol{\alpha}_{osc}^{t,n}(E) = \sum_{u.} e_{ref}^u \sum_{v}^{f(u)} M_{uv}^{\alpha}\sin vE$$

$$\Delta t\cdot n_{ref} = \underline{\left[E - e_{ref}\sin E\right]_{E_0}^{E}} + \varepsilon_t\left[\tau^t(E)\right]_{E_0}^{E} + \varepsilon_n\left[\tau^n(E)\right]_{E_0}^{E} + \mathcal{O}\left(\varepsilon_{t,n}^2\right)$$

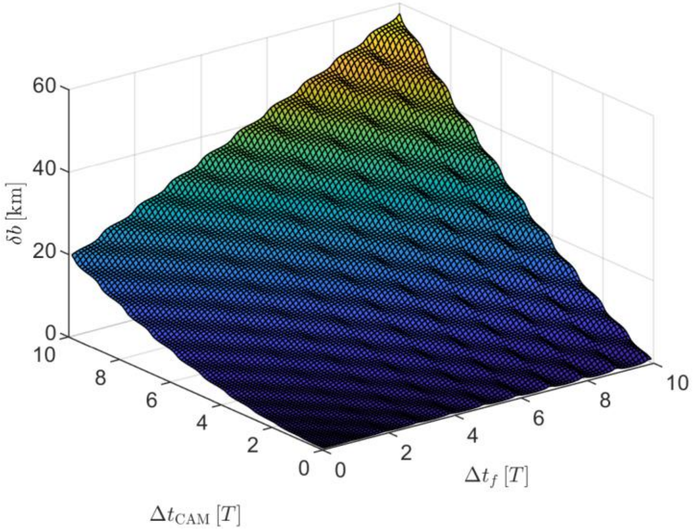
Time law $E \Leftrightarrow t$

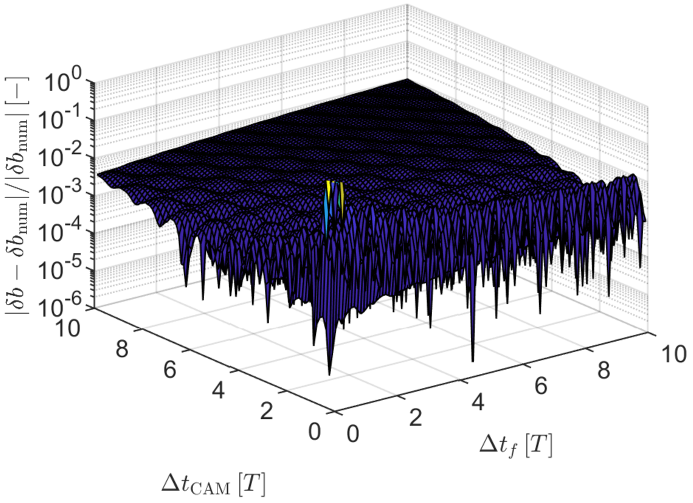
[1] J.L. Gonzalo, and C. Colombo, "Lightweight algorithms for collision avoidance applications," *ESA GNC 2021*, 22-25 June 2021
[2] J.L. Gonzalo, C. Colombo, and P. Di Lizia, "A semi-analytical approach to low-thrust collision avoidance manoeuvre design," *70th IAC*, 2019
[3] J.L. Gonzalo, C. Colombo and P. Di Lizia, "Single-averaged models for low-thrust collision avoidance under uncertainties," 73rd IAC, 2022.

Task 3.3: Approach for collision avoidance
Single-averaged CAM models

- Miss distance and error in GTO for single tangential thrust arc Δt_{CAM} + coast arc Δt_f

Nominal CA	$a\ [km]$	$e\ [-]$	$i\ [deg]$	$\Omega\ [deg]$	$\omega\ [deg]$	$M_0\ [deg]$
Spacecraft	24208.53	0.7282	26.498	318.984	179.962	336.063
Debris	13813.097	0.5208	27.043	318.452	180.839	335.662





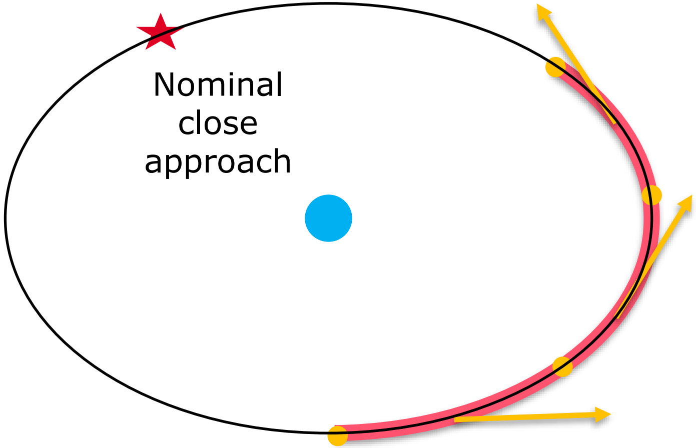
$$a_t = 10^{-7}\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

$$T = 10.41\ \mathrm{h}$$

Task 3.3: Approach for collision avoidance Single-averaged CAM models

- These models characterize orbit modification due to a low-thrust arc with given thrust profile
  - Useful for parametric analyses and fast orbit evaluations.
  - Do not provide directly directly an optimal CAM

- Quasi-optimal piecewise-constant control
  - Derived for each arc from impulsive model:
    - Linear model between impulsive $\delta\mathbf{v}$ and change in miss distance/PoC [1]. Optimal impulsive CAM design reduced to an eigenproblem.
    - CAM orientation on each arc given by dominant eigenvector.
    - Associated eigenvalue gives a measure of the local efficiency of the CAM. Useful to set relative weights between segments, define on/off sequences.

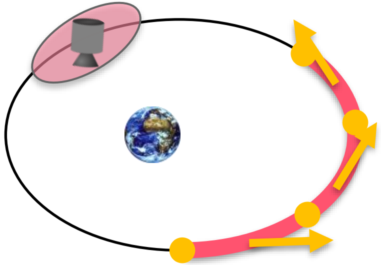


[1] J.L. Gonzalo, C. Colombo, and P. Di Lizia, "Analytical framework for space debris collision avoidance maneuver design," *Journal of Guidance, Control and Dynamics*, 44(3):469-487 2021.

Task 3.3: Approach for collision avoidance
Single-averaged CAM models

- Quasi-optimal control designed from impulsive model [1]

For single Gaussian, direct impact [1,2]



Depends on dynamics and combined covariance

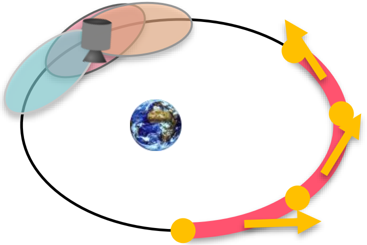
$$J = \underbrace{\delta\mathbf{v}^T \boxed{\boldsymbol{M}}\, \delta\mathbf{v}}_{\text{Related to Mahalanobis distance}} \Longrightarrow$$

$\max(J)$ reduces to eigenproblem:

$$(\gamma_1, \mathbf{ei}_1)$$

$$\delta\mathbf{v} = \delta v_{max}\, \mathbf{ei}_1$$

For GMM, non-direct impact [2]



$$J = \delta\mathbf{v}^T M^* \delta\mathbf{v} + B^* \Longrightarrow$$

- No longer eigenproblem
- Numerical solution

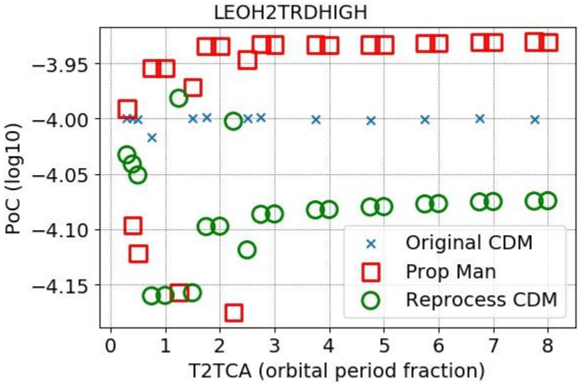
[1] J.L. Gonzalo, C. Colombo, and P. Di Lizia, "Analytical framework for space debris collision avoidance maneuver design," *JGCD*, 44(3), 2021.
[2] C. Bombardelli, J. Hernando-Ayuso, "Optimal Impulsive Collision Avoidance in low Earth orbit," *JGCD*, 38(2), 2015.

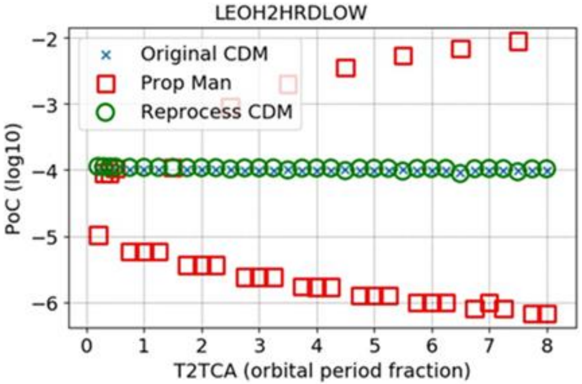
Task 3.3: Approach for collision avoidance Numerical methods for CAM

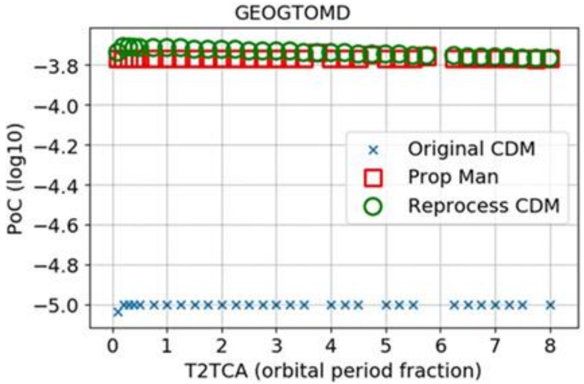
- **Analytical/semi-analytical** methods to serve as **initial guess** for higher fidelity **numerical methods**. Single thrusting arcs with acceleration level 0.1 mm/s2.
- Initial guess is **preliminarily re-evaluated:**
  - CDM **reprocessing** (recompute **PoC** Chan's → Akella's).
  - **Re-propagate OPM (with analytical CAM)** with higher-fidelity dynamics, reanalyse conjunction.
- Numerical CAM design (with and without constraints):
  - **Local** refinement
  - Design **from scratch**: Global optimiser

Task 3.3: Approach for collision avoidance
Numerical methods for CAM

- Preliminary **evaluation:**
  - On low LEO (LEOH2HRD**LOW**), Drag modelling plays a significant role.
  - **GEOGTOMD →** Conjunction geometry is similar to other analysed cases. However, covariance of the secondary has a large component **CN_N.** Thus, difference in PoC computation method (Chan vs Akella's) may be held accountable.





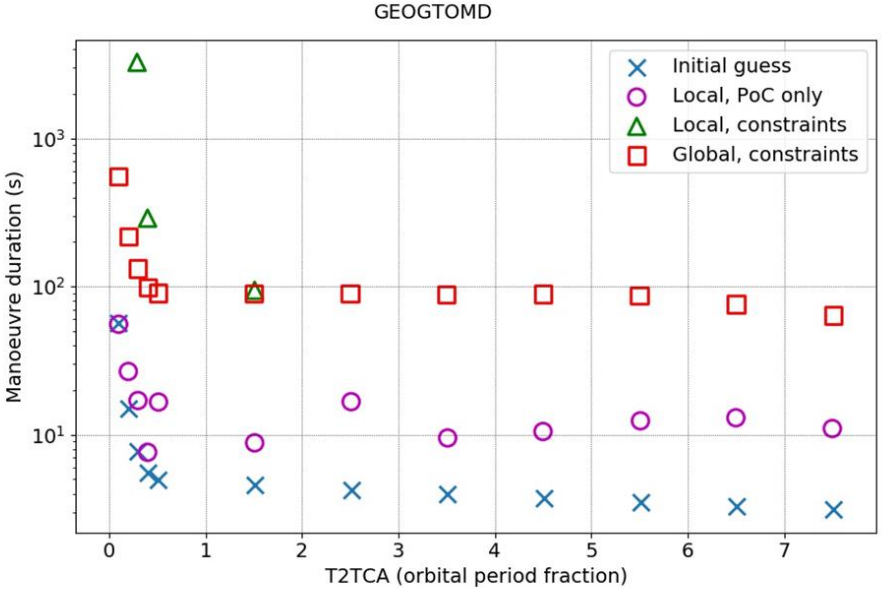


Task 3.3: Approach for collision avoidance
Numerical methods for CAM

CAM refinement: ballistic

For the case showing a mismatch in PoC with respect to the analytical environment:

- Local, PoC only requires a larger duration.
- Considering constraints toughens convergence



Task 3.3: Approach for collision avoidance
Numerical methods for CAM

- **CAM refinement: EOR**
  - **Analytical** methods → shut down until TCA
  - **Numerical** → shut down and restart.
  - Then:
    - Numerical → optimise orbital position, more efficient firing
  - SK scenarios → Intermittent firing!!

Conjunction	Analytical methods shutdown duration (hours)	Numerical methods shutdown duration (hours)
DISPOSAL	2.1	0.3
G2G1P	1.0	0.2
G2G1A	1.2	0.2
G2G3	1.4	1.1
L2L1	0.8	0.6
GEOSK	20.9	0.5
LEOSK	91.0	0.3

Task 3.4: Simulations. Operational concepts

- **Conjunction screening**

Concept ID	Operational Concept	Process Location	Applicable Scenario(s)	Uncertainty Reduction	Timeliness Improvement
MA4	Frequent calibration of thruster performances	G,H	RL1, RL2, RG1, RM1, LL1, LL2, RG2, ML1, MG1, MG2, ML3	M	N/A
MA5	Feedback control (GNSS measurements).	S	RL1, RL2, RG1, RM1, LL1, LL2, ML1, ML2, ML3, ML4, ML5, ML6	H	H
MA6	Feedback control (High-precision accelerometer)	S	RL1, RL2, RG1, RM1, LL1, LL2	H	N/A
OD1	The use of more precise data to reduce the initial uncertainties	G,H,S	ML2, ML4, ML5, ML6	L	N/A
OD2	Receive support from space surveillance sensor networks (e.g. telescopes, SLR, SST radar)	G,H	RG1, RM1, RG2, MG1, MG2, MM1	L	M

ID	Scenario Name
RL1	LEO-to-LEO EOR (Low)
RL2	LEO-to-LEO EOR (High)
RG1	GTO-to-GEO EOR
RM1	LEO-to-MEO EOR
LL1	LEO EOL (Low)
LL2	LEO EOL (High)
RG2	GEO Graveyard EOR
ML1	LEO Mega-constellation (Low)
ML2	LEO Mega-constellation (High)
MG1	GEO SK Full EP
MG2	GEO SK Hybrid EP
ML3	LEO Tube control
ML4	LEO Low dV
ML5	LEO Low acceleration
ML6	LEO Ground track control

NB: process locations are: G = ground; S = on board and H = hybrid. The uncertainty reduction, timeliness improvement and risk analysis values are: L = low; M = medium and H = high.

Task 3.4: Simulations. Operational concepts

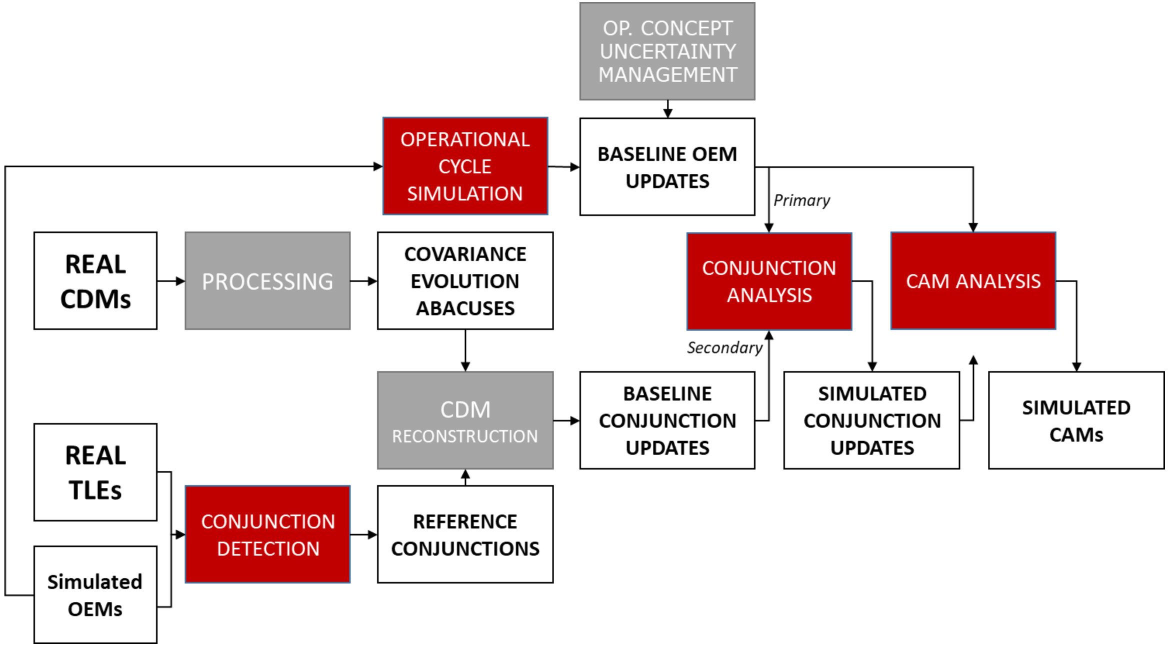
- **Collision avoidance**

Concept ID	Operational Concept	Process Location	Applicable Scenario(s)	CAM Delay	Mission Impact Reduction	Operational Workload Reduction	Robustness Increase
CD1	Late telecommand paths to postpone the decision time	G,H	RL1, RL2, RG1, RM1, LL1, LL2, RG2, ML1, ML2, MG1, MG2, ML3, ML4, ML5, ML6	H	L	L	L
OW1	CAM design on-board	S	RL1, RL2, RG1	H	L	H	M
MI5	Shut-down engine during EOR/EOL	G,H,S	RL1, RL2, RG1, RM1, LL1, LL2, RG2	L	H	M	M
MI6	Modify planned manoeuvres in EOR/EOL or SK	G,H	RL1, RL2, RG1, RM1, LL1, LL2, RG2, ML1, MG1, MG2, ML3	L	H	M	L
IR1	Model the uncertainty of the CAM	G,H,S	RL1, RL2, RG1, RM1, LL1, LL2, RG2, ML1, ML2, MG1, MG2, ML3, ML4, ML5, ML6	L	L	L	H
IR4	Consider multiple events in the CAM design	G,H,S	RL1, RL2, RG1, RM1, LL1, LL2, RG2, ML1, ML2, MG1, MG2, ML3, ML4, ML5, ML6	L	M	M	H
IR5	Use multiple metrics for the post-CAM thresholds	G,H,S	RL1, RL2, RG1, RM1, LL1, LL2, RG2, ML1, MG1, MG2, ML3	L	L	L	H

NB: process locations are: G = ground; S = on board and H = hybrid. The CAM delay, mission impact reductions, operational workload reduction and robustness values are: L = low; M = medium and H = high.

ID	Scenario Name
RL1	LEO-to-LEO EOR (Low)
RL2	LEO-to-LEO EOR (High)
RG1	GTO-to-GEO EOR
RM1	LEO-to-MEO EOR
LL1	LEO EOL (Low)
LL2	LEO EOL (High)
RG2	GEO Graveyard EOR
ML1	LEO Mega-constellation (Low)
ML2	LEO Mega-constellation (High)
MG1	GEO SK Full EP
MG2	GEO SK Hybrid EP
ML3	LEO Tube control
ML4	LEO Low dV
ML5	LEO Low acceleration
ML6	LEO Ground track control

Task 3.4: Simulations. Methodology



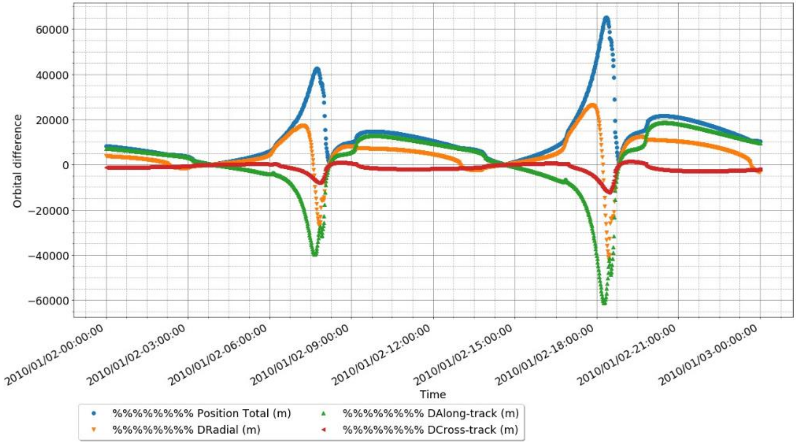
- **Output CDMs of simulations are analysed:**
  - Annual manoeuvring rate required to mitigate all conjunctions above a given **Accepted Collision Probability Level** (ACPL) and **Depth of Intrusion** (DOI)
  - Risk reduction and residual risk
  - Delta-V and successful execution/conjunction mitigation rate
- **CAM Design computed through optimisation using numerical methods**

Task 3.4: Simulations. Methodology

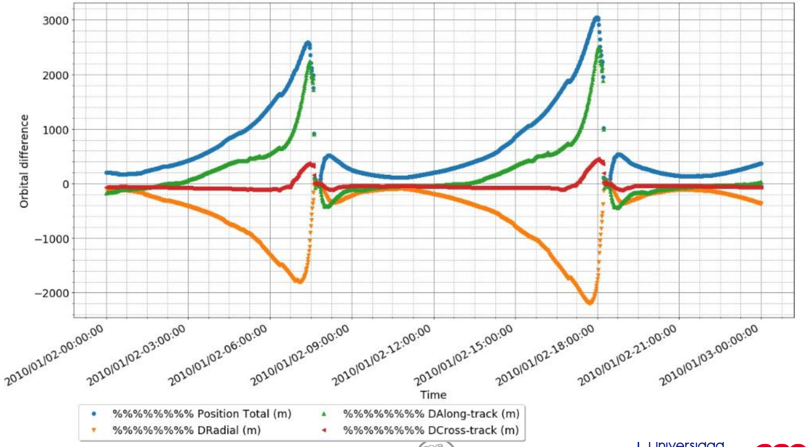
- **Primary OEM update:**

Scenario	Operational Concept	Decision Time to TCA (hrs)	Thrust Error Model		
			Modulus (%)	Pointing (deg)	Time Scale (Orbital Periods)
GTO-to-GEO Start EOR (RG1)	NOMINAL	36	1	0.5	1
	IMPROVED	7	1	0.5	0
GTO-to-GEO Insertion EOR (RG1)	NOMINAL	36	1	0.5	1
	IMPROVED	13	1	0.5	0

NOMINAL Op. Concept (RG1, Start) Sample Trajectory



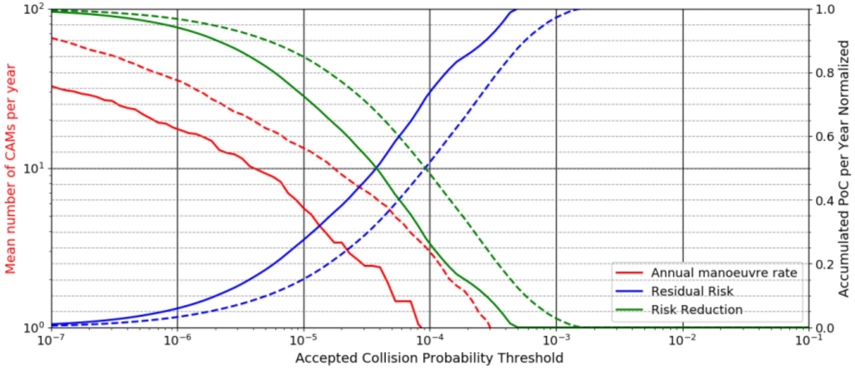
IMPROVED Op. Concept (RG1, Start) Sample Trajectory



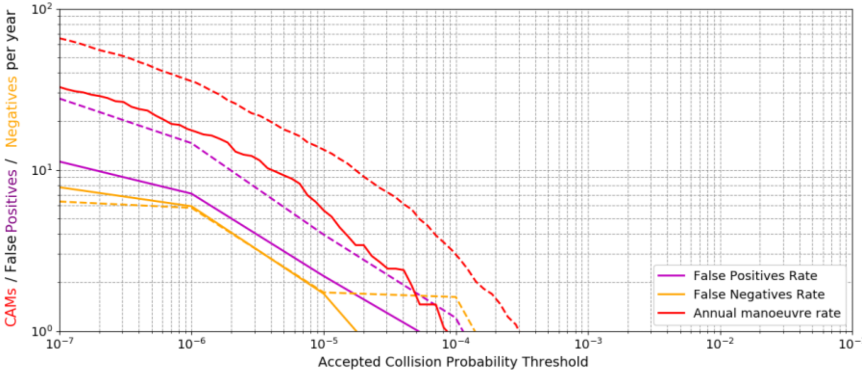
Task 3.4: Simulations. Results

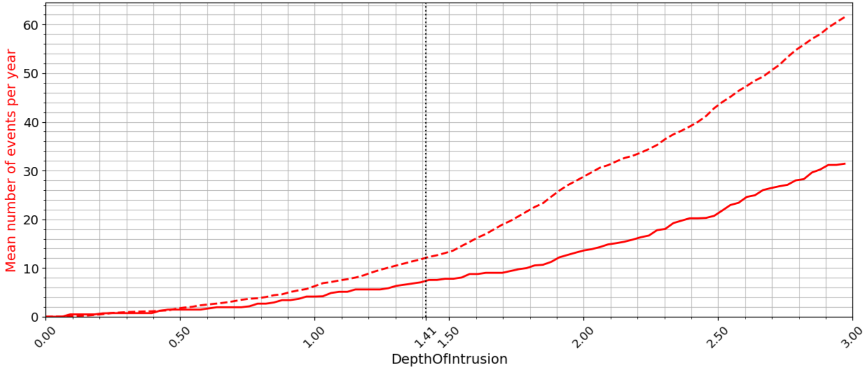
LEO-to-LEO EOR – Risk Assessment (GNSS measurements):

Dashed curves → Nominal op. concept



Solid curves → Improved op. concept





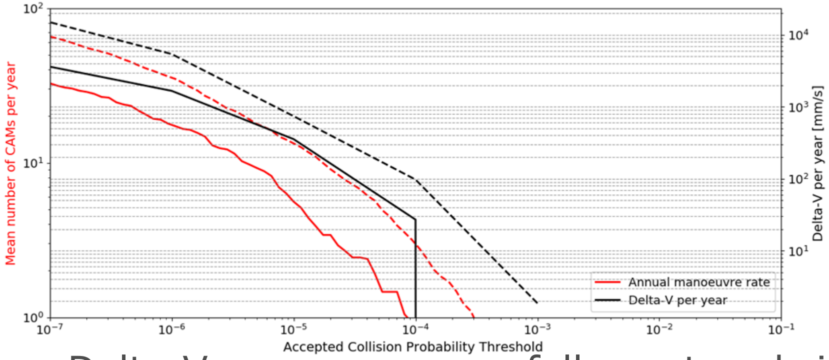
- Constant and controlled covariance + improved predictability → Improved risk assessment, fewer identified CAMs per year.
- Manoeuvring based on DoI criteria also benefits from improved Op. Concept.
- False positives represent a similar percentage of required CAMs.
- False negative rate unaffected.

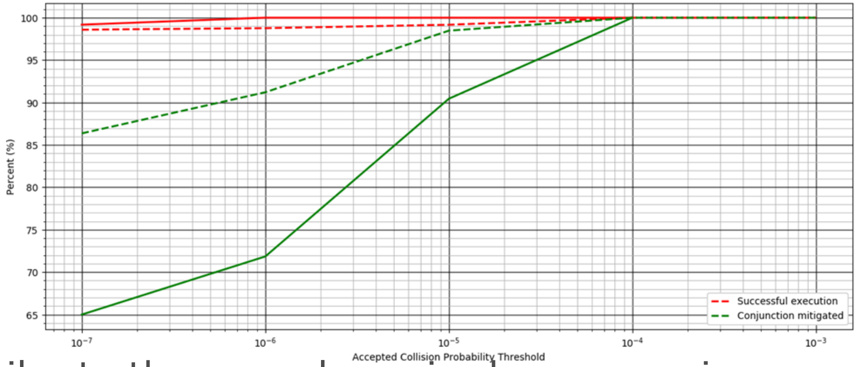
Task 3.4: Simulations. Results

- **LEO-to-LEO EOR – CAM with numerical methods(GNSS measurements):**

Dashed curves → Nominal op. concept

Solid curves → Improved op. concept



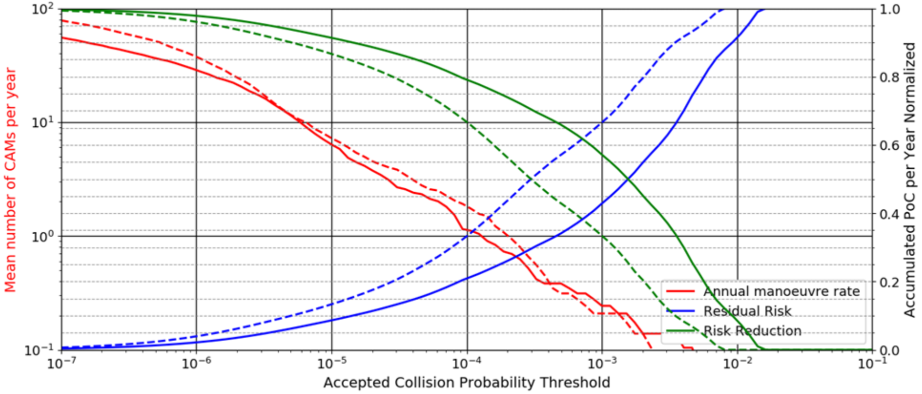


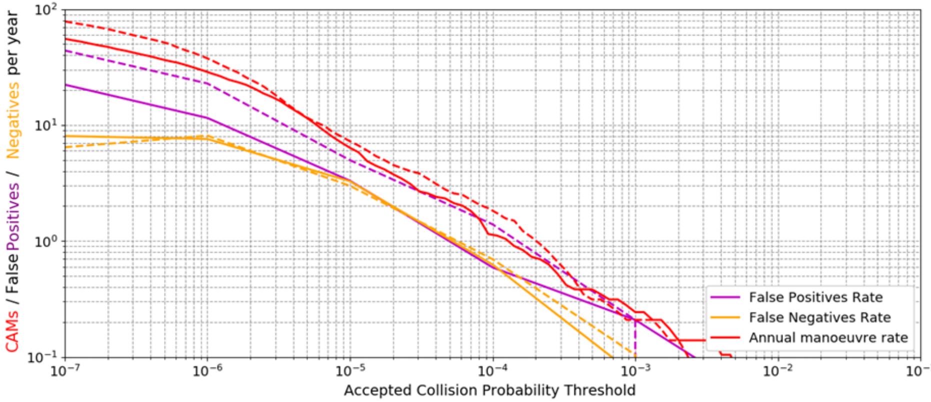
- Delta-V per year curves follow a trend similar to the annual required manoeuvring rate:
  - Improved covariance management (directly related to PoC evaluation) does not seem to have an impact in lowering required delta-V. Related to the observed not drastic reduction in DoI in the risk assessment.
- Software executes successfully >98% of occasions:
  - However, successful mitigation is seen to increase with increasing ACPL (due to a relaxed PoC threshold).
  - The tighter scheduling associated to the improved Op. Concept also lowers its mitigation rate compared to the Nominal case. Easily solvable.

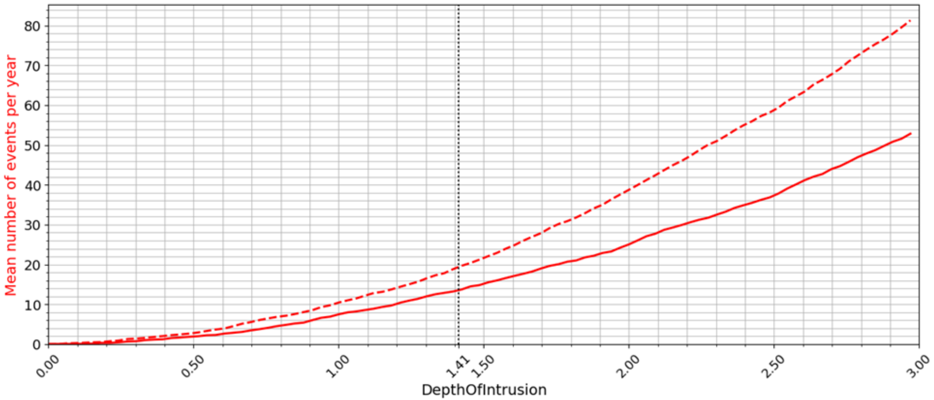
Task 3.4: Simulations. Results

■ LEO Large Constellation (SK) – Risk Assessment (On Board OD):

Dashed curves → Nominal op. concept **Solid curves → Improved op. concept**





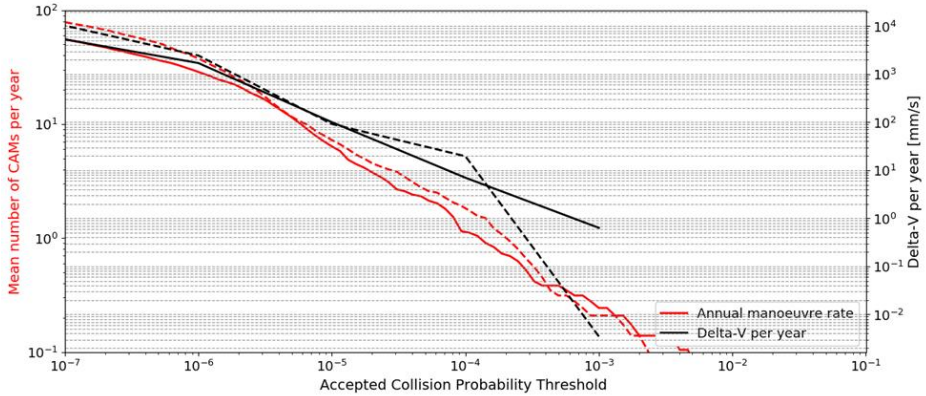


- Low delta-V and short duration manoeuvres:
  - Thruster related uncertainty is already low.
  - Improved Op. Concepts don’t display a large effect on this scenario.
  - Slightly improved covariance management reveals some extra conjunctions with ACPL > 1e-3.
- False positives/negatives represent a similar percentage of required CAMs in both cases.

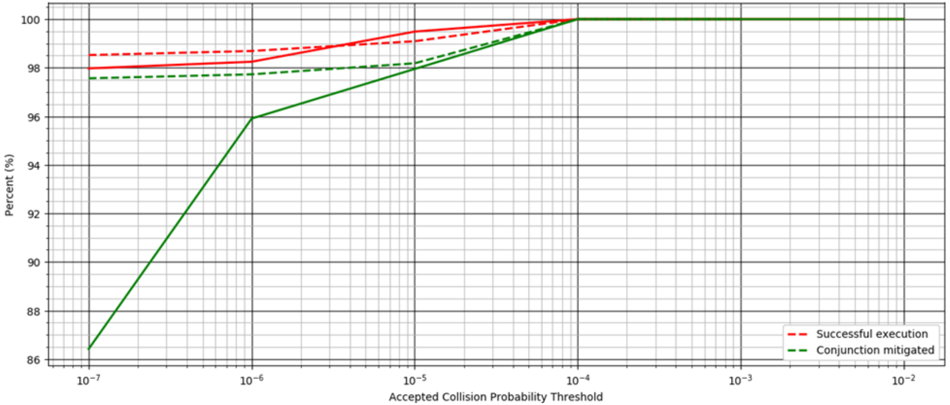
Task 3.4: Simulations. Results

LEO Large Constellation (SK) – CAM with numerical methods:

Dashed curves → Nominal op. concept



Solid curves → Improved op. concept

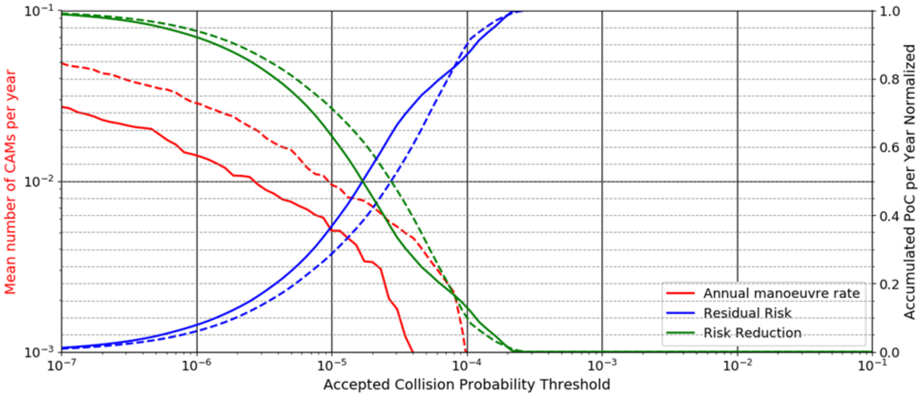


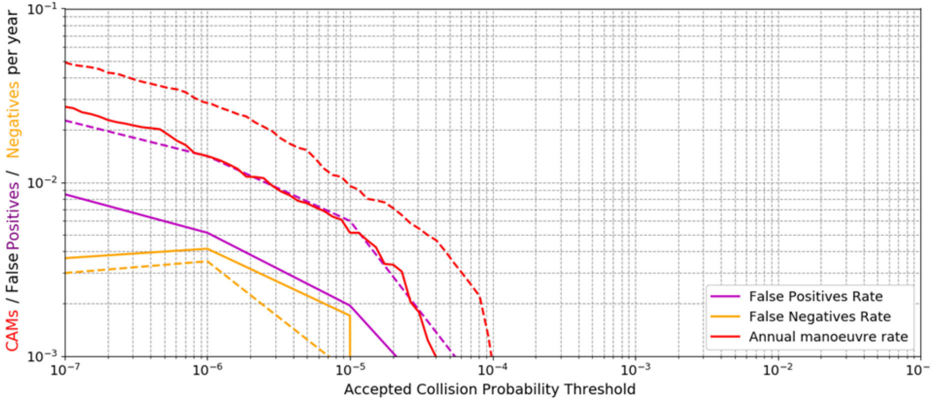
- Delta-V per year curve reveals a slightly greater delta-V to mitigate an ACPL of 1e-3, consistent with previous findings.
- Again, a tighter scheduling associated to the improved Op. Concept also lowers its mitigation rate compared to the Nominal case. Easily solvable.

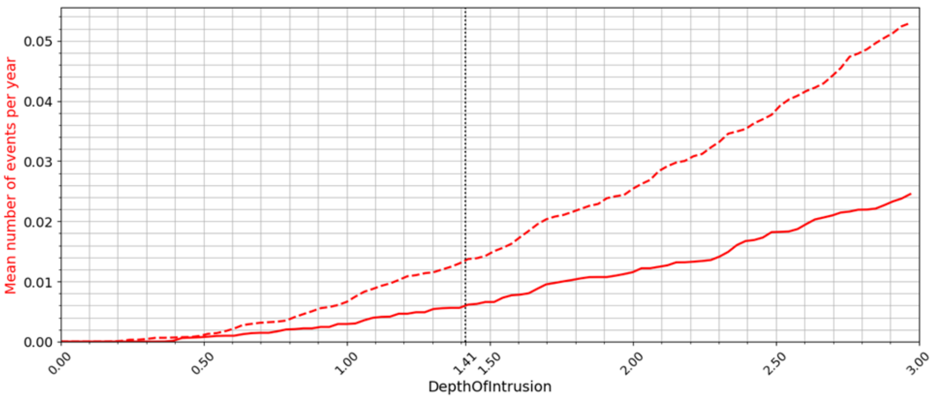
Task 3.4: Simulations. Results

- **GEO SK – Risk Assessment (accelerometer measurements):**

Dashed curves → Nominal op. concept **Solid curves → Improved op. concept**







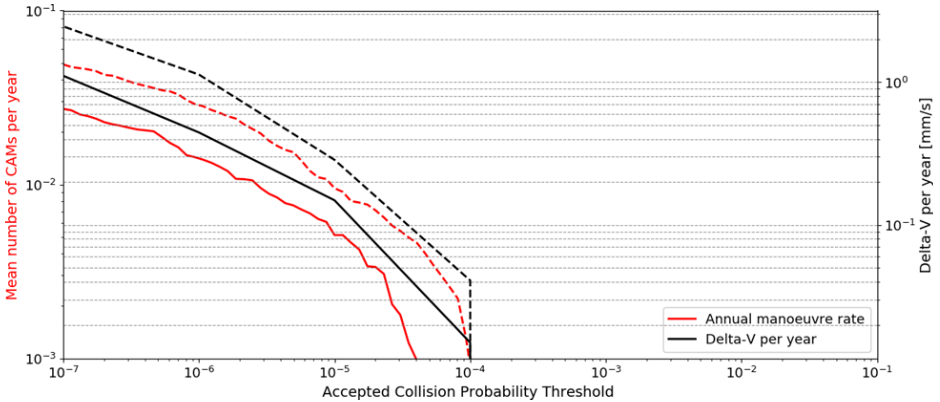
- Although SK scenario, manoeuvres have a larger duration relative to the orbital period.
  - Effect of improved Op. Concept is noticeable, reducing annual manoeuvring rate
- False negative rate slightly increases. Are the not identified conjunctions are meaningful (PoC evaluation)? Not a trivial definition of false negative/positives in a simulation environment.

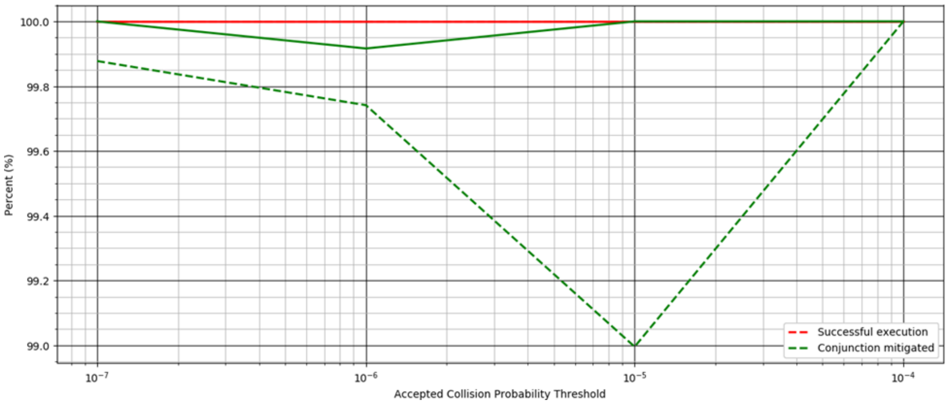
Task 3.4: Simulations. Results

GEO SK – CAM with numerical methods:

Dashed curves → Nominal op. concept

Solid curves → Improved op. concept





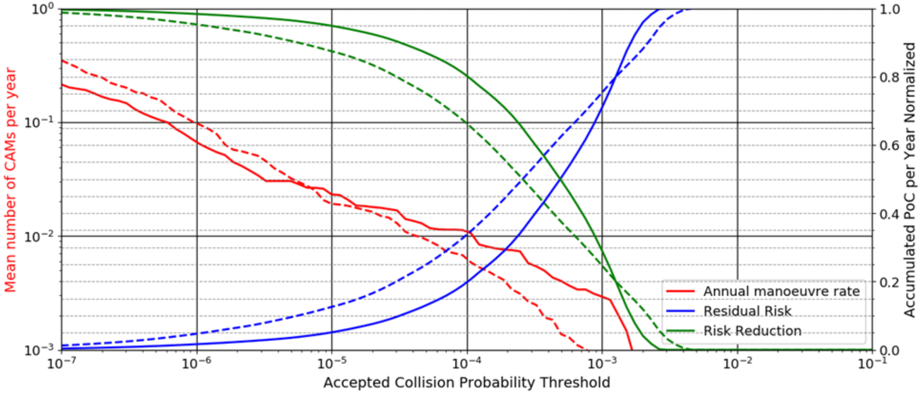
- Delta-V per year curve mimics that of manoeuvring rate.
- Weird behaviour of mitigation rate curve. Due to the numerical nature of the method. In any case, >99%.

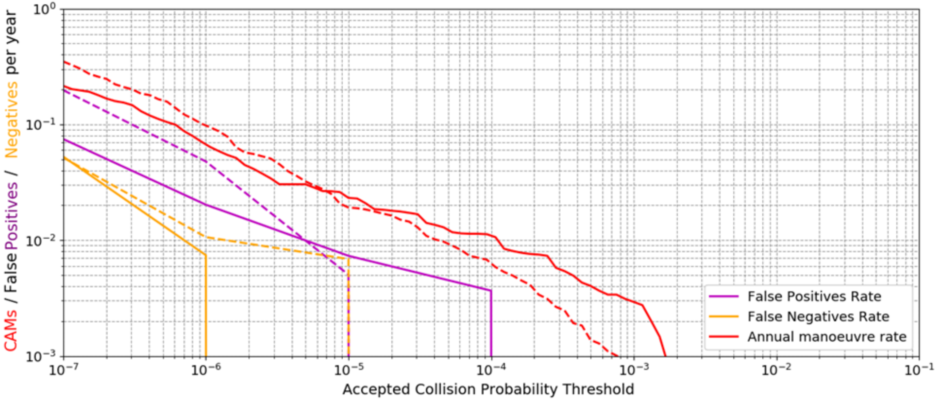
Task 3.4: Simulations. Results

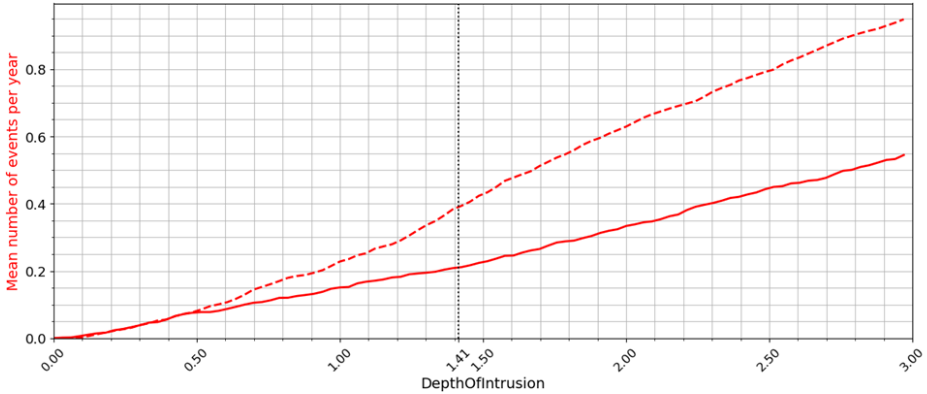
GEO GRAVEYARD EOR – Risk Assessment (acc. measurements):

Dashed curves → Nominal op. concept

Solid curves → Improved op. concept







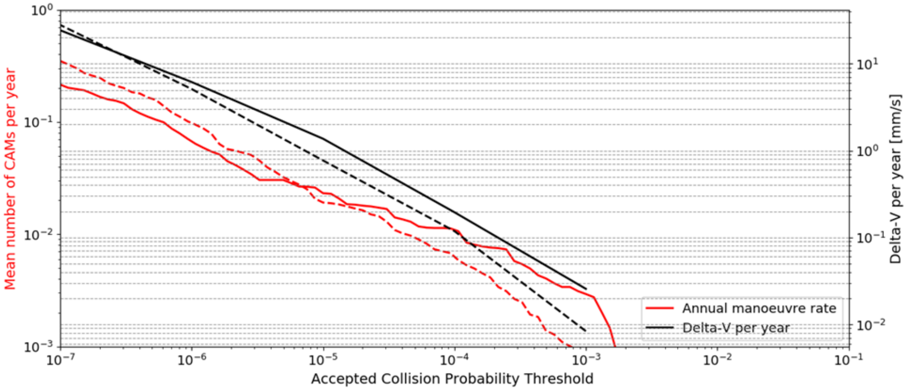
- Conjunctions are detected in the first quarter of the orbit:
  - Although the effect of the manoeuvre is large, its effect has not yet accumulated significantly when conjunctions are encountered.
- Main effect is a slight shift of conjunctions from lower to higher ACPLs (covariance, DoI).

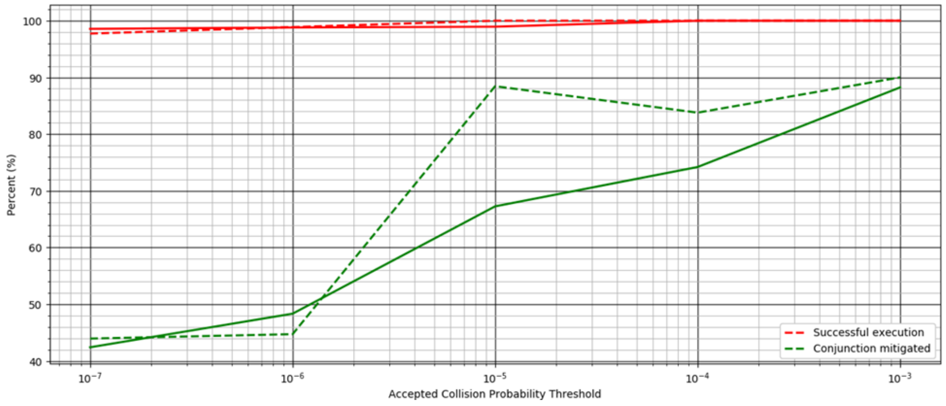
Task 3.4: Simulations. Results

GEO GRAVEYARD EOR – CAM with numerical methods:

Dashed curves → Nominal op. concept

Solid curves → Improved op. concept



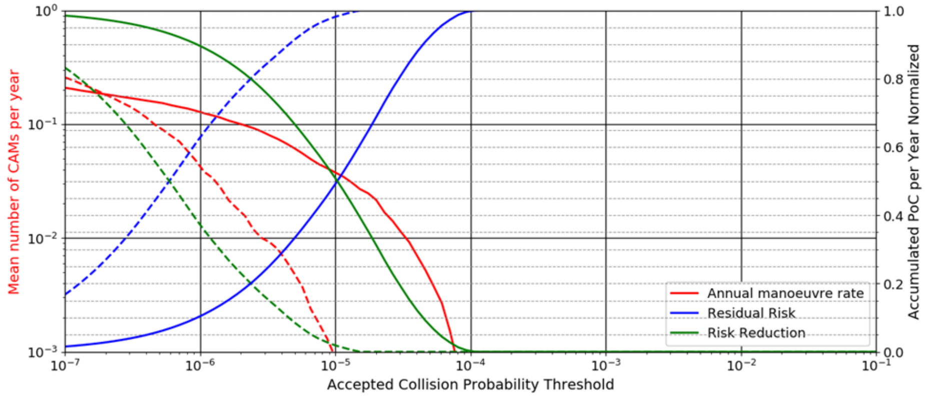


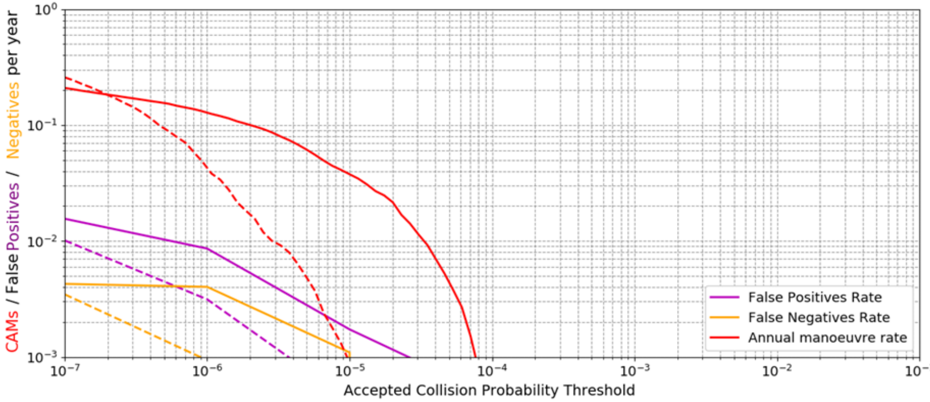
- Delta-V per year curve mimics that of manoeuvring rate.
- Mitigation rate is relatively poor, specially for low ACPLs:
  - Manoeuvres have a small lead time (conjunctions happening during first quarter of orbit).
  - In practical terms, transfer would be delayed to avoid conjunctions.

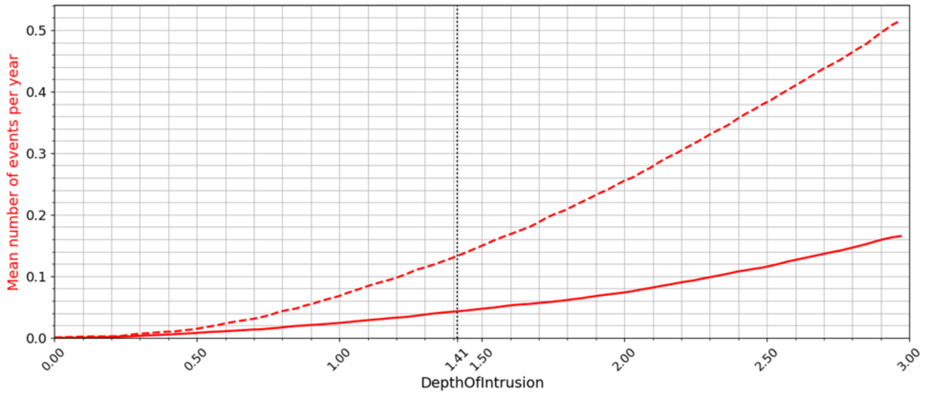
Task 3.4: Simulations. Results

GTO-to-GEO (GEO insertion) – Risk Assessment (acc. meas.):

Dashed curves → Nominal op. concept **Solid curves → Improved op. concept**







- Contrary to GEO GRAVEYARD, conjunctions are encountered at the end of the transfer → Uncertainty has accumulated.
- Main effect is to shift/reveal a larger set of conjunctions in the higher ACPL end, supported as well by DoI evolution.
- False positives/negatives follow accordingly.

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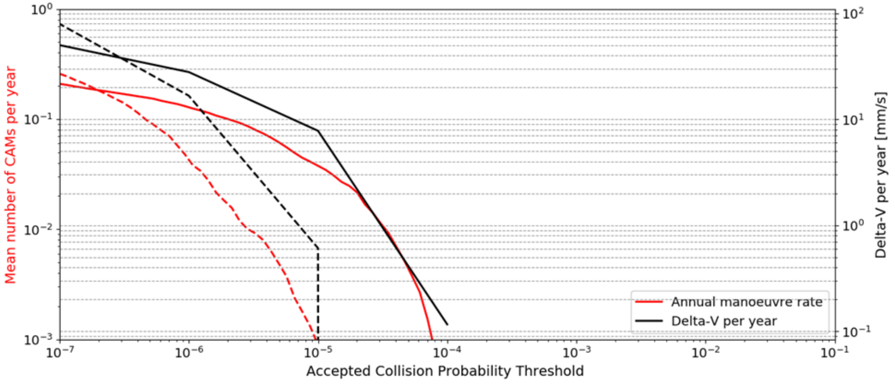
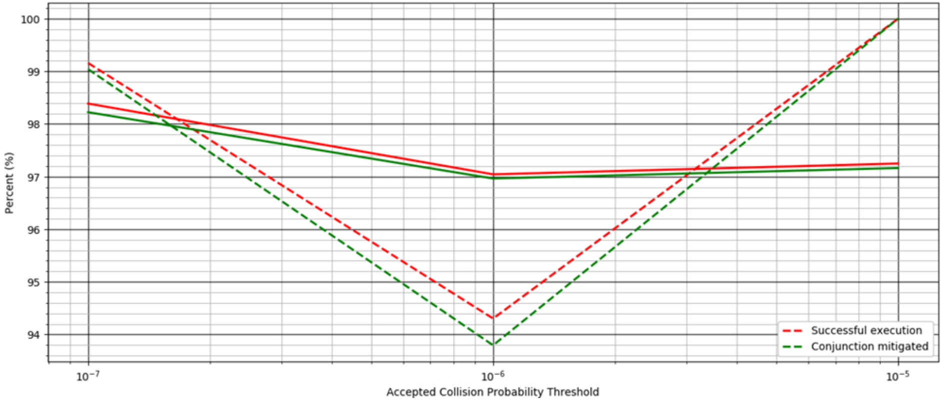
Task 3.4: Simulations. Results

- **GTO-to-GEO (GEO insertion) – CAM with numerical methods:**

Dashed curves → Nominal op. concept **Solid curves → Improved op. concept**



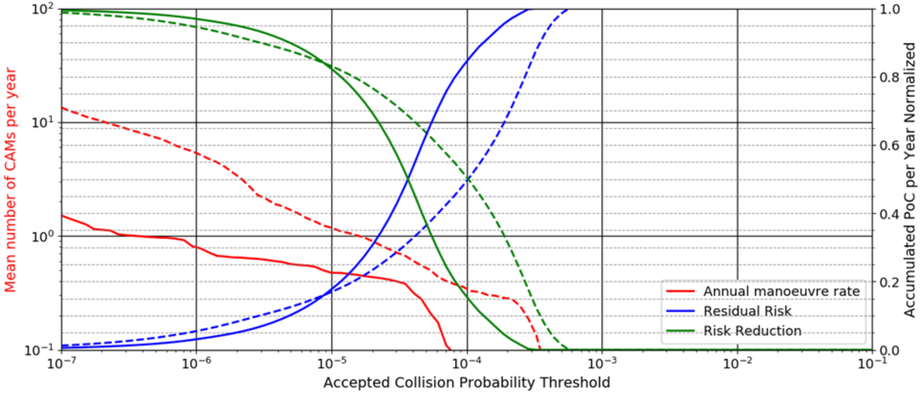


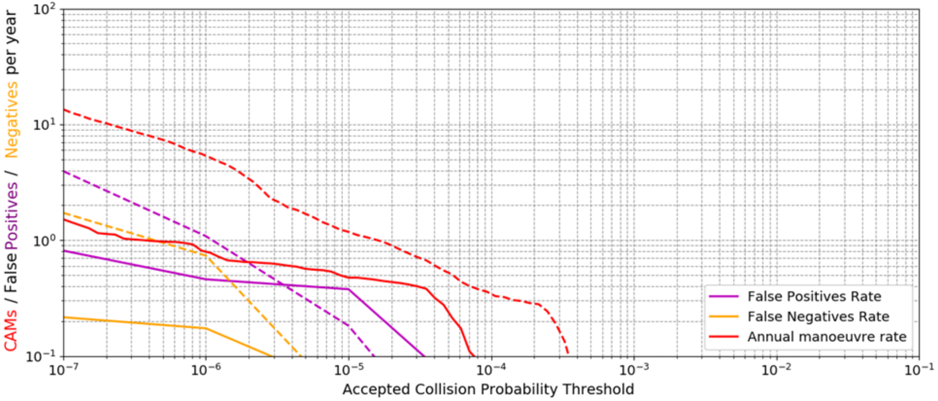
- Delta-V per year curve mimics that of manoeuvring rate.
- Mitigation rate is > 90%.

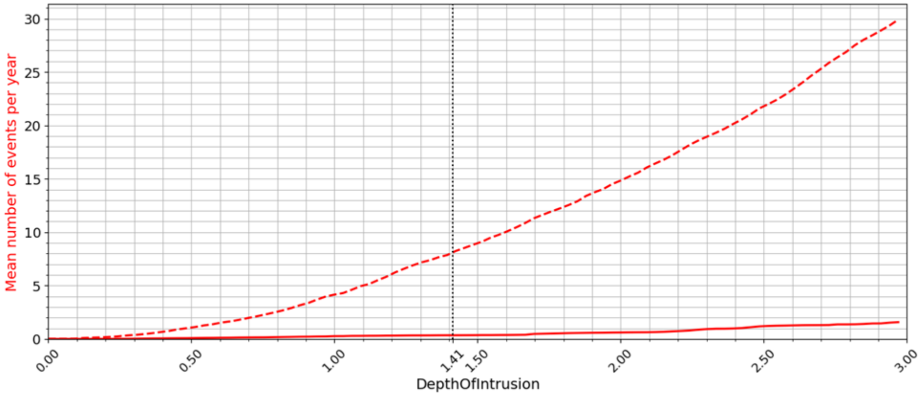
Task 3.4: Simulations. Results

GTO-to-GEO (beginning) – Risk Assessment (acc. + GNSS):

Dashed curves → Nominal op. concept **Solid curves → Improved op. concept**





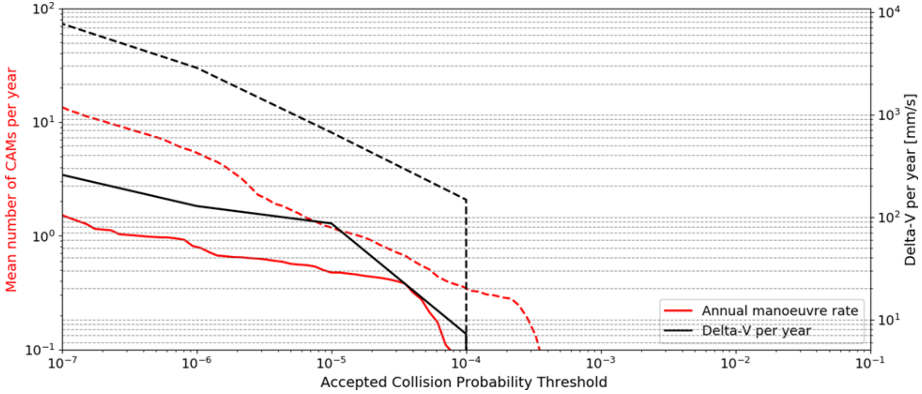


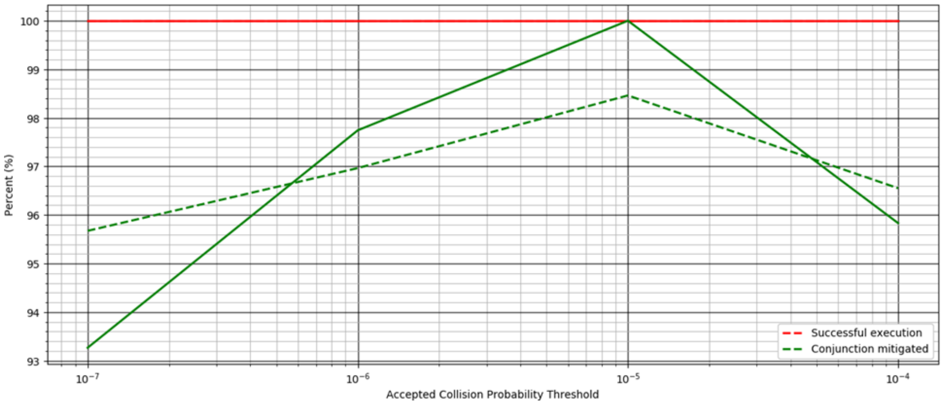
- Improved Op. Concept is a drastic improvement (100x10x10 m fixed TNW covariance).
- Predictability of trajectory and covariance management result in a better screening and risk assessment (annual manoeuvring, DoI…)
- False positives tend to annual manoeuvring rate (matching a real environment).

Task 3.4: Simulations. Results

- **GTO-to-GEO (beginning) – CAM with numerical methods:**

Dashed curves → Nominal op. concept **Solid curves → Improved op. concept**





- Delta-V per year curve mimics that of manoeuvring rate.
- Mitigation rate is > 90%, even for the tighter improved Op. Concept schedule.

Task 3.4: Simulations. Conclusions

- Operational concepts translate to an improvement in:
  - Primary’s trajectory predictability.
  - Primary’s covariance management.
- For scenarios involving GNSS coverage (LEO-to-LEO transfer, GTO-to-GEO early transfer), the improvement is significant across all ACPLs.
- For SK scenarios: found to be relevant for typical SK in GEO.
- Two particular scenarios: GEO insertion and GEO GRAVEYARD.
  - Improved Op. Concepts are seen to be more effective the longer the prediction horizon is, mitigating the uncertainty growth in the longer term.

Task 3.5: Sensitivity analysis

- **Outline:**
  - Test cases analysis for methods validation
  - Results on the large-scale simulations

Task 3.5: Sensitivity analysis – Test cases analysis
Sample test cases - Ballistic

- LEOH2HMD
- LEOH2HRD
- LEOH2TRD
- LEOTYPMD
- MEOGTOMD
- GEOGTOMD
- GEOTYPMD

$0.1\ mm/s^2$	10^{-4}	$15\ m$
$0.1\ mm/s^2$	10^{-5}	$15\ m$

Maneuvers are planned from 0.1 up to 8 orbits before TCA with 35 starting points to satisfy the **target PoC**

Task 3.5: Sensitivity analysis – Test cases analysis
Sample test cases - EOR

- acc-03-00028358
- DISPOSAL
- G2G1apogee
- G2G1perigee
- G2G3
- L2L1
- starlink-03

Given Control History

Thruster Shut down

10^{-5}

$15\ m$

Maneuvers are planned by selecting the thruster shutdown that complies with the **target PoC**

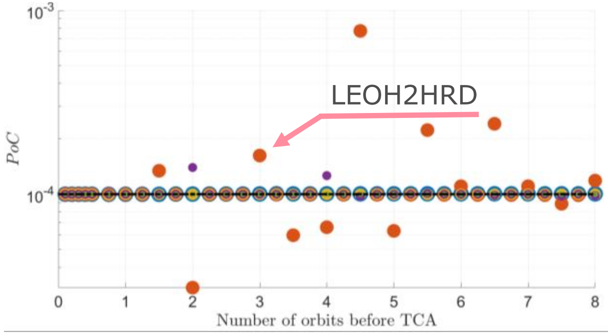
Task 3.5: Sensitivity analysis – Test cases analysis Energy-Optimal Chan-based CAM - Results

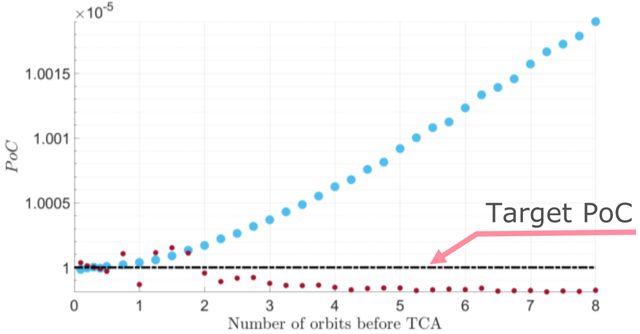
LEO + MEO

GEO

Remarks

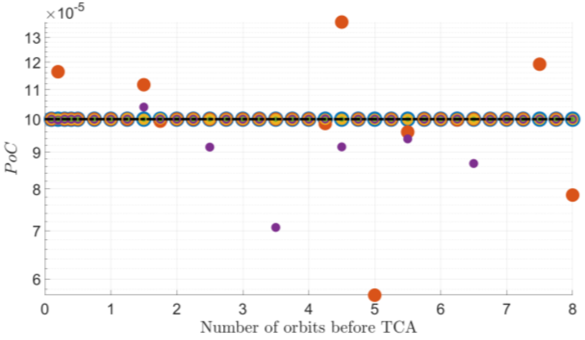
Tangential

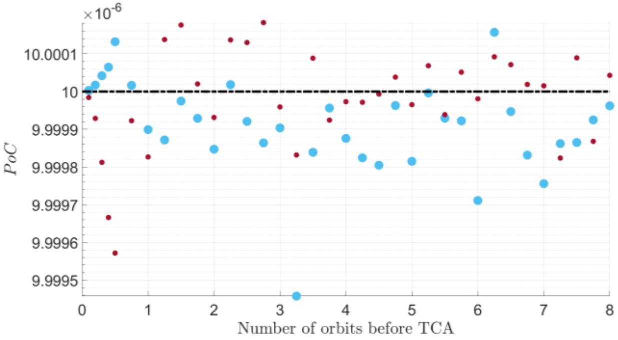

LEOH2HRD


Target PoC

- Good PoC targeting
- Computational cost is around $10^{-1}/10^{-2}$s depending on the **maneuvering point**

Radial





- ● LEOH2HMD
- ● LEOH2HRD
- ● LEOH2TRD
- ● LEOTYPMD
- ● MEOGTOMD
- ● GEOGTOMD
- ● GEOTYPMD

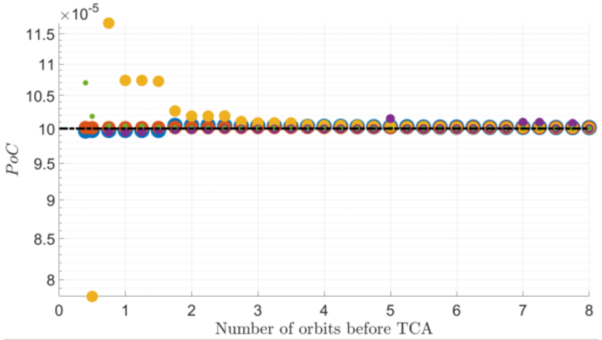
Task 3.5: Sensitivity analysis – Test cases analysis Fuel-Optimal Chan-based CAM - Results

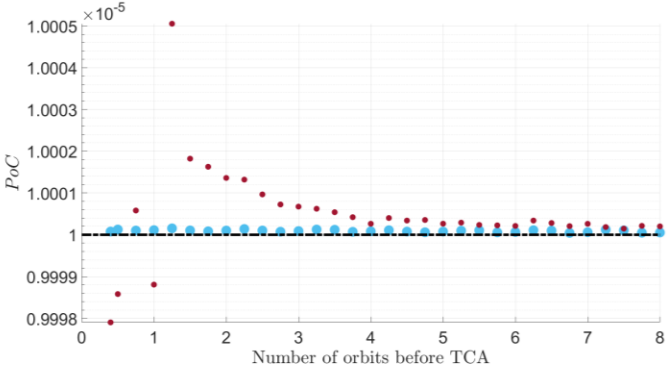
LEO + MEO

GEO

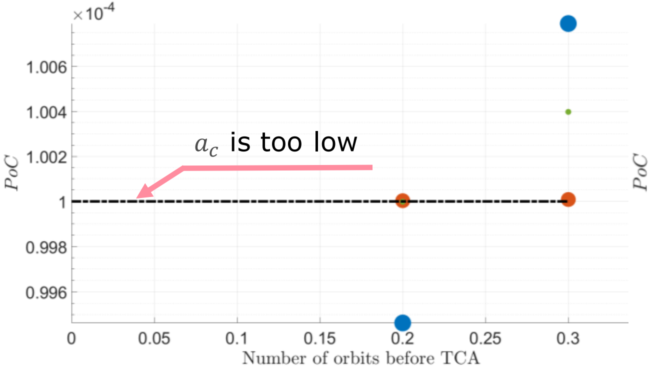
Remarks

Tangential

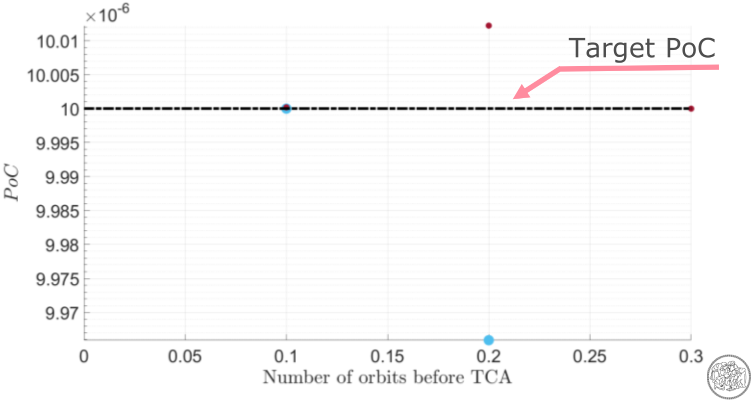




Radial



a_c is too low

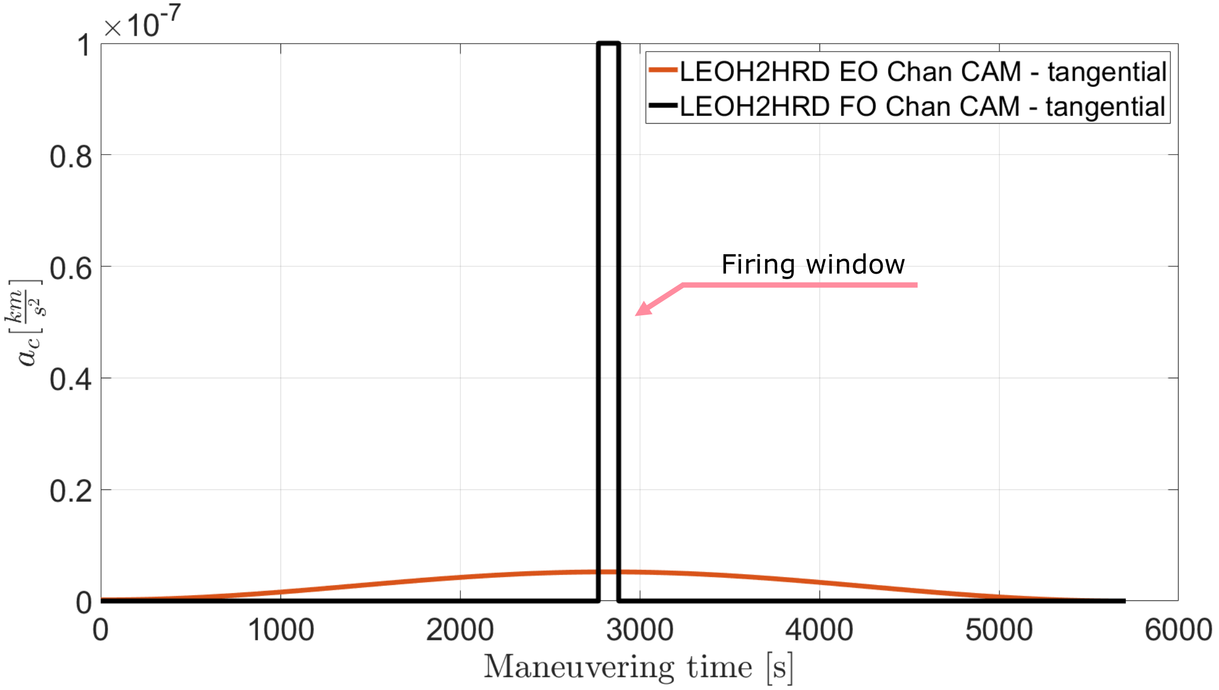


Target PoC

- Good **PoC targeting**
- The **radial** maneuver sometimes fails due to **short notice**
- Computational cost is 10^{-2}s for **tangential** and 10^{-1}s for **radial** **(on board implementation)**

- LEOH2HMD
- LEOH2HRD
- LEOH2TRD
- LEOTYPMD
- MEOGTOMD
- GEOGTOMD
- GEOTYPMD

Task 3.5: Sensitivity analysis – Test cases analysis Fuel-Optimal Chan-based CAM - Results

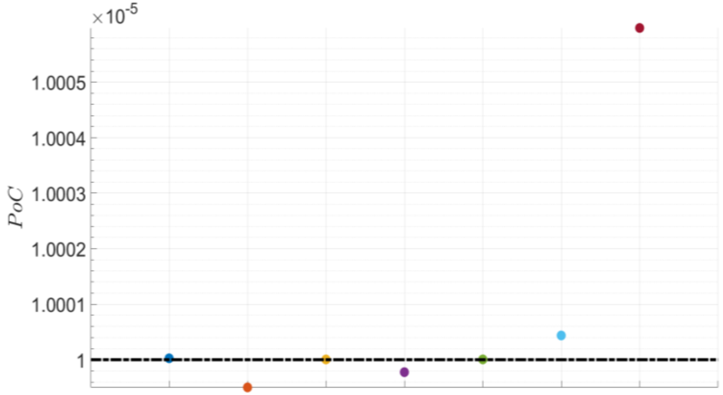


Remarks

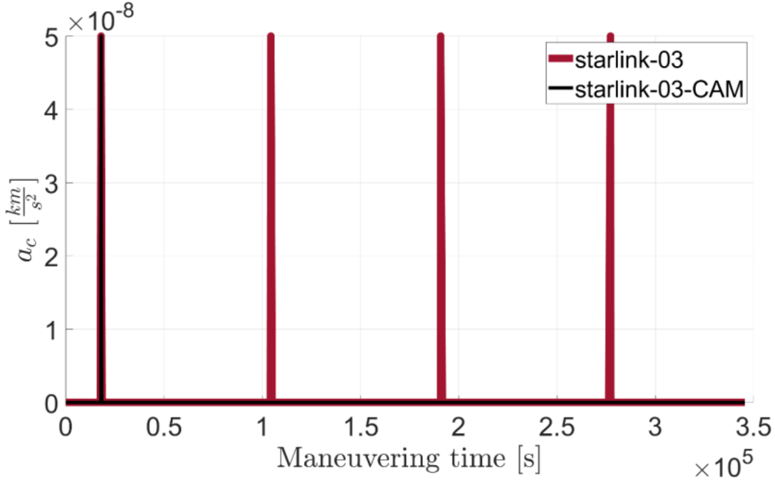
- No **discontinuity issues** generated by **smoothing based** approaches
- The firing window is **centered** on **the EO maximum.**
- It works with **shorter** and **longer** thrusting times
- **Independent tangential maneuver** computational cost wrt all **maneuvering times.**
- Closer to an **operational environment**

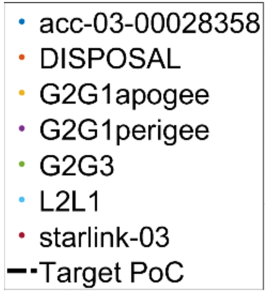
Task 3.5: Sensitivity analysis – Test cases analysis EOR Chan based CAM - Results

PoC



Thrust profile

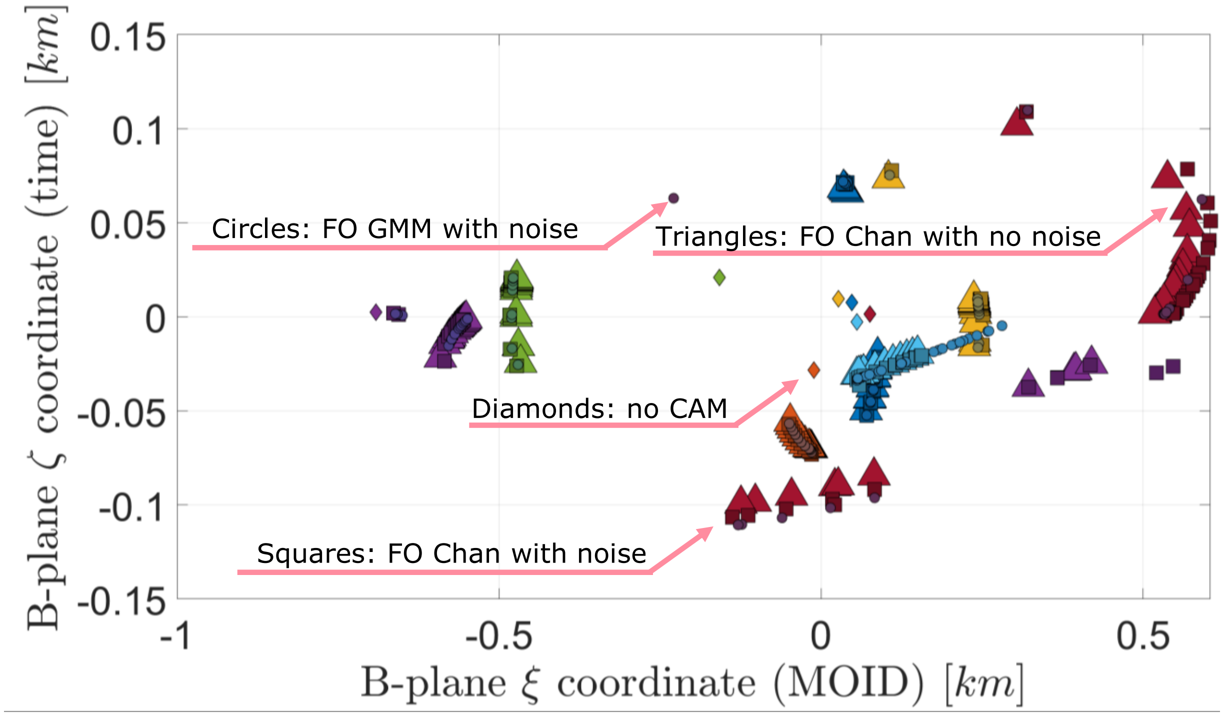




- Good PoC targeting (it depends on **node spacing** and **bisection tolerances**)
- Computational time around 10^{-1}s
- The method suffers from long shutdown periods (not applicable to **just-in-time CAMs**)
- Polimi is working on alternative strategies based on **convex optimization.**

Task 3.5: Sensitivity analysis – Test cases analysis GMM Chan-based CAM - results

B-plane



Circles: FO GMM with noise

Triangles: FO Chan with no noise

Diamonds: no CAM

Squares: FO Chan with noise

Remarks

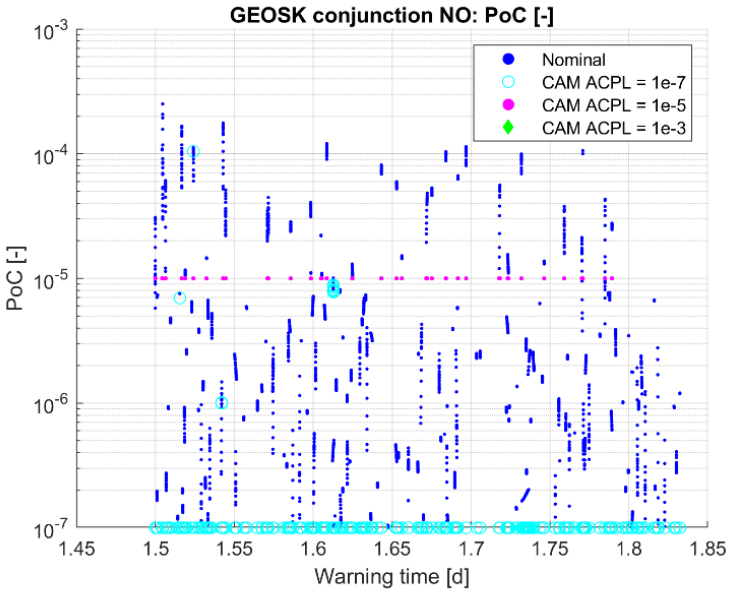
- Points **distribute on ellipses** in the Bplane for simple **Chan Cases**
- Noise causes to target **different ellipses** for the same maneuver
- Chan’s CAM with noise (**squares**) and with GMMs (**circles**) overlaps.
- GMMS can cope with **quasi-short** term scenarios

● LEOH2HMD
● LEOH2HRD
● LEOH2TRD
● LEOTYPMD
● MEOGTOMD
● GEOGTOMD
● GEOTYPMD

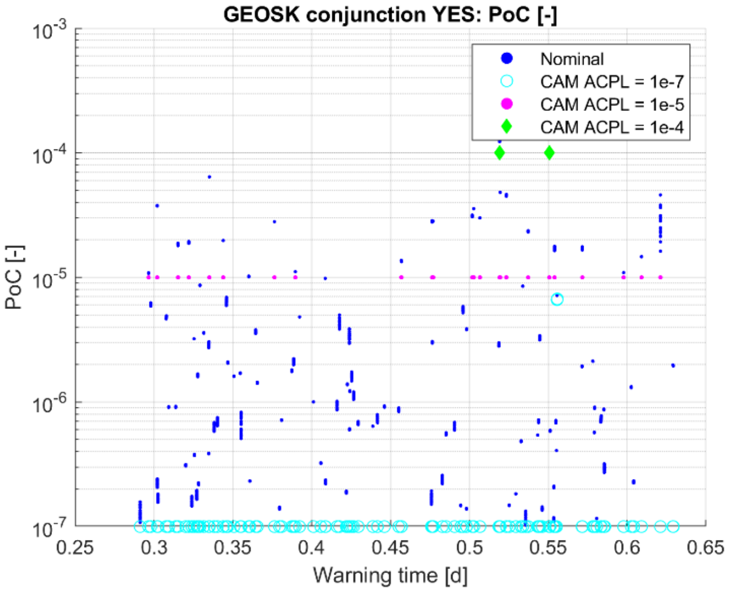
Task 3.5: Sensitivity analysis – Large scale sim.

Characterisation of the GEOSK ballistic CAM scenarios

PoC. Concept of operations: standard



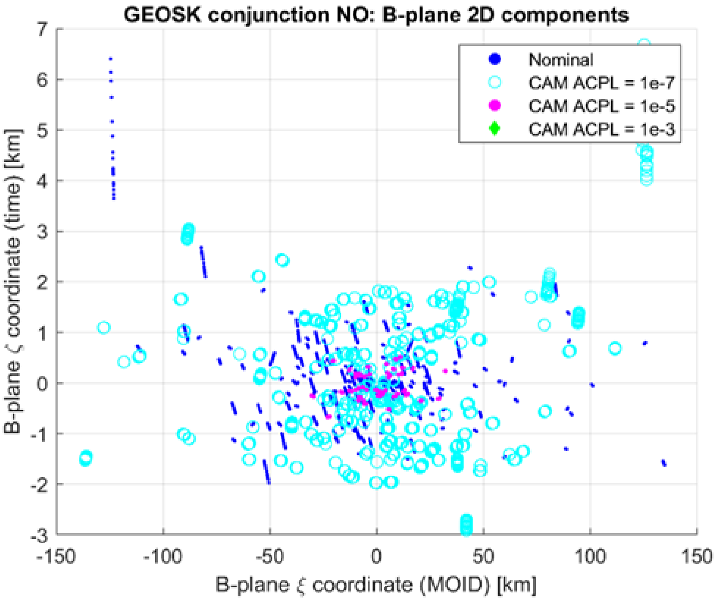
PoC. Concept of operations: new



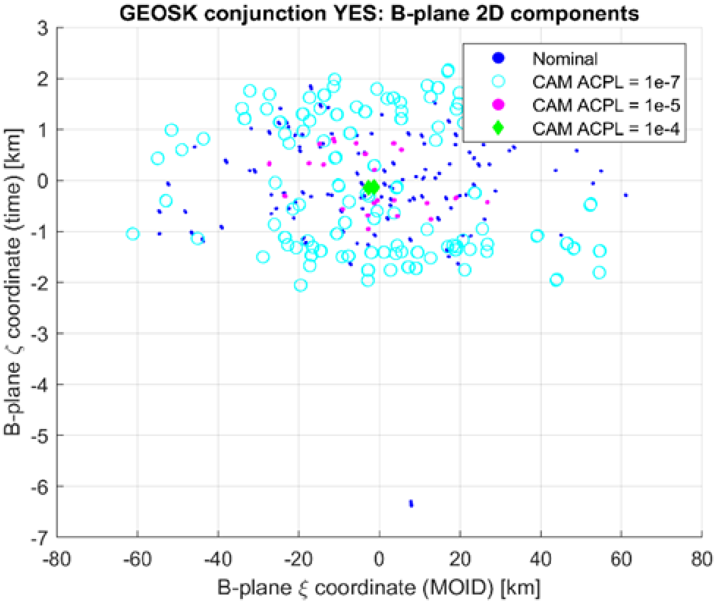
Task 3.5: Sensitivity analysis – Large scale sim.

Characterisation of the GEOSK ballistic CAM scenarios

B-plane representation. Concept of operations: standard



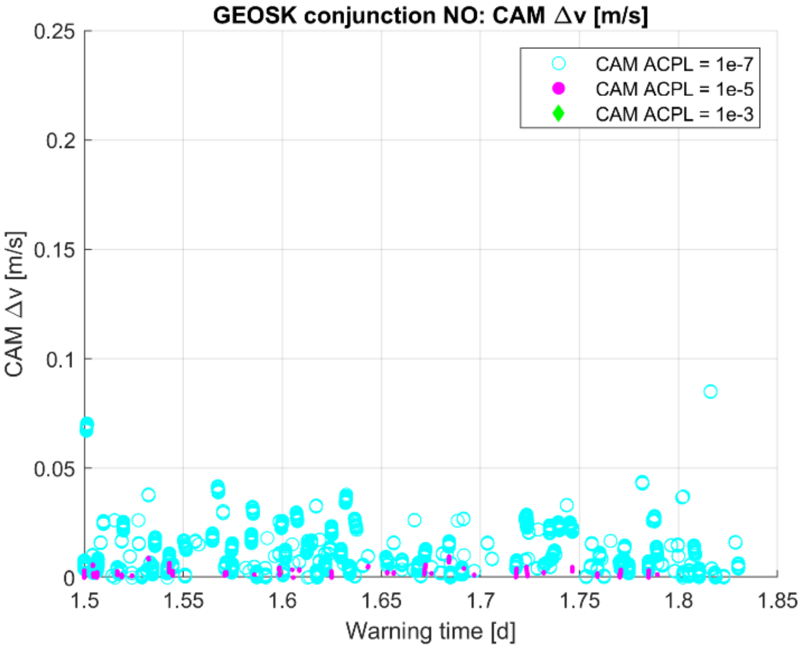
B-plane representation. Concept of operations: new



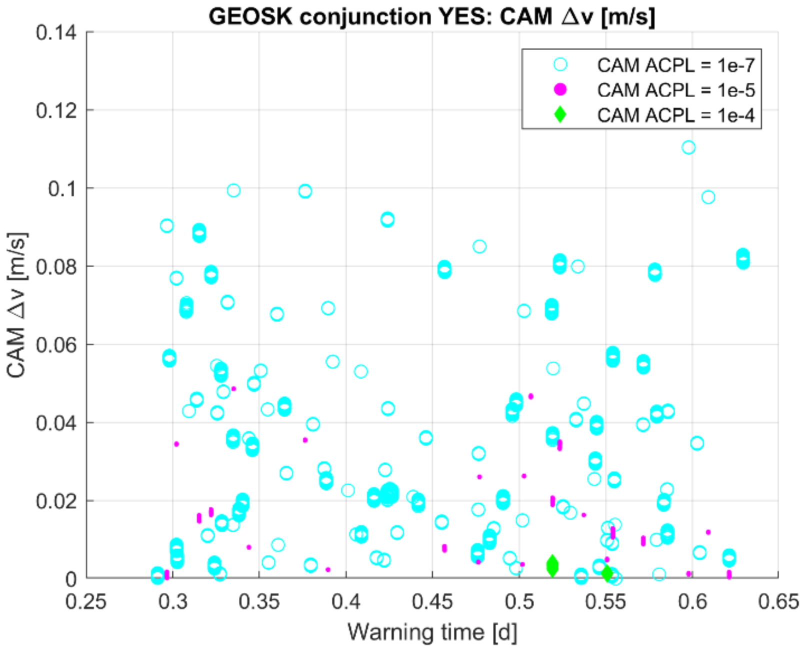
Task 3.5: Sensitivity analysis – Large scale sim.

Characterisation of the GEOSK ballistic CAM scenarios

CAM manoeuvre. Concept of operations: standard



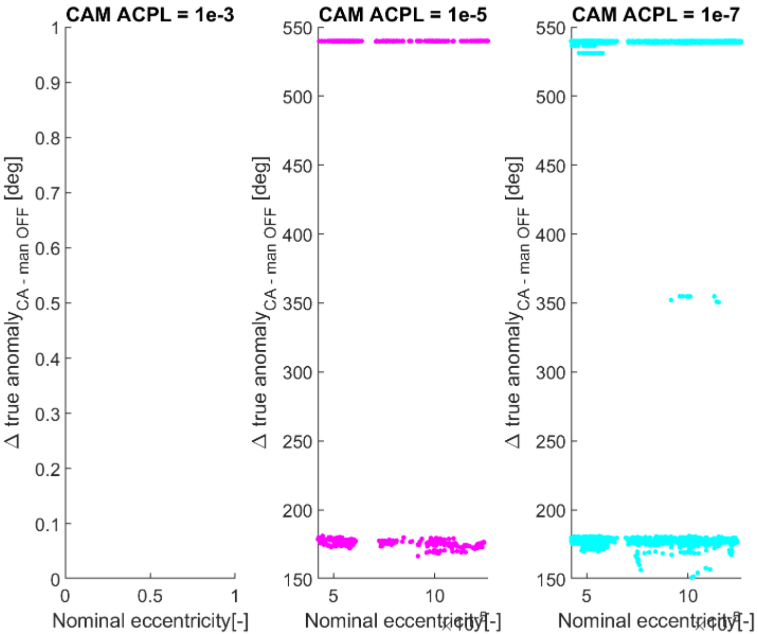
CAM manoeuvre. Concept of operations: new



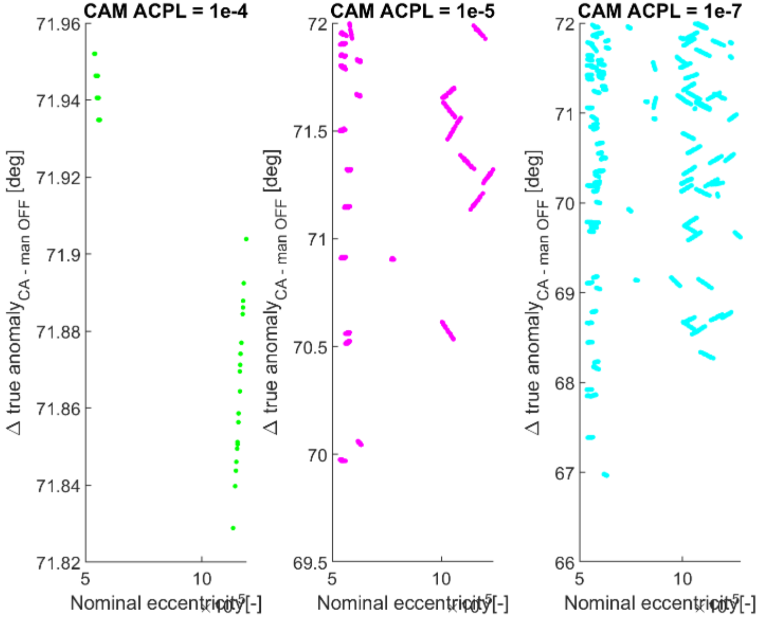
Task 3.5: Sensitivity analysis – Large scale sim.

Characterisation of the GEOSK ballistic CAM scenarios

Delta true anomaly between the engine is turned off and the CA. Concept of operations: standard



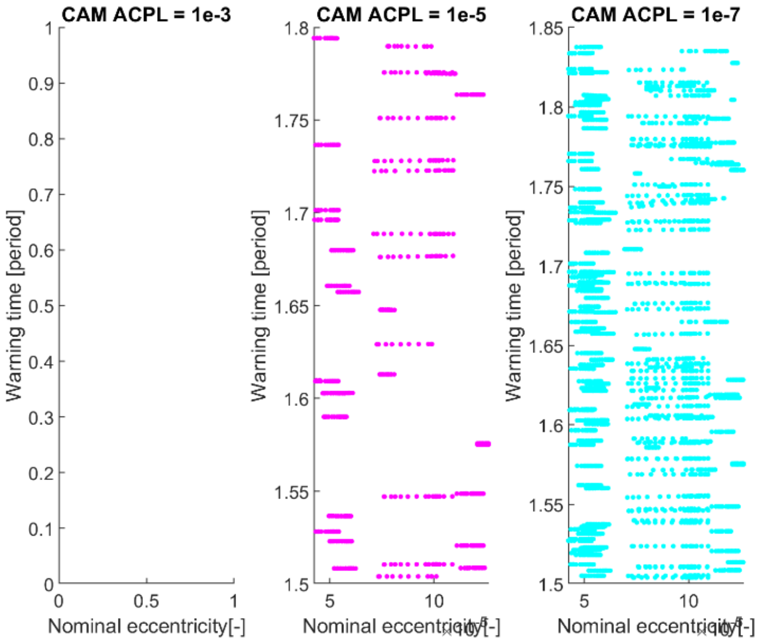
Delta true anomaly between the engine is turned off and the CA. Concept of operations: new



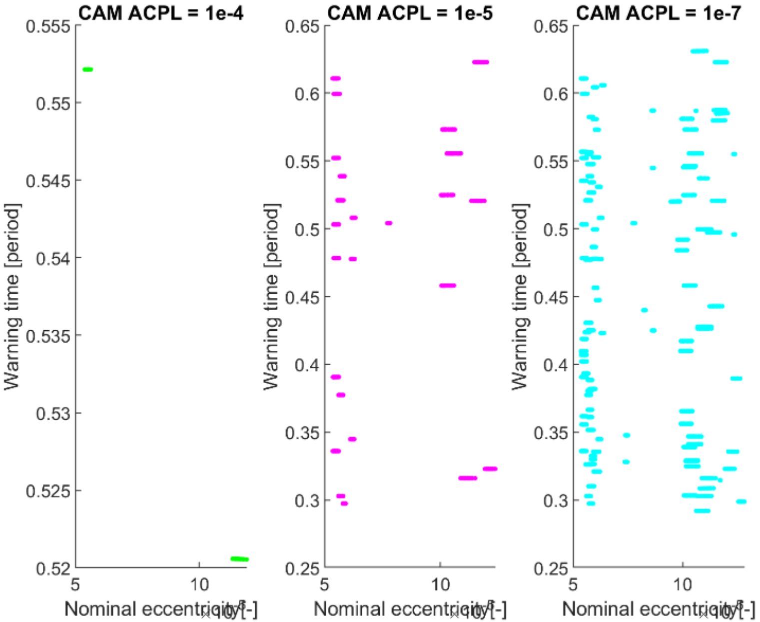
Task 3.5: Sensitivity analysis – Large scale sim.

Characterisation of the GEOSK ballistic CAM scenarios

Warning time. Concept of operations: standard



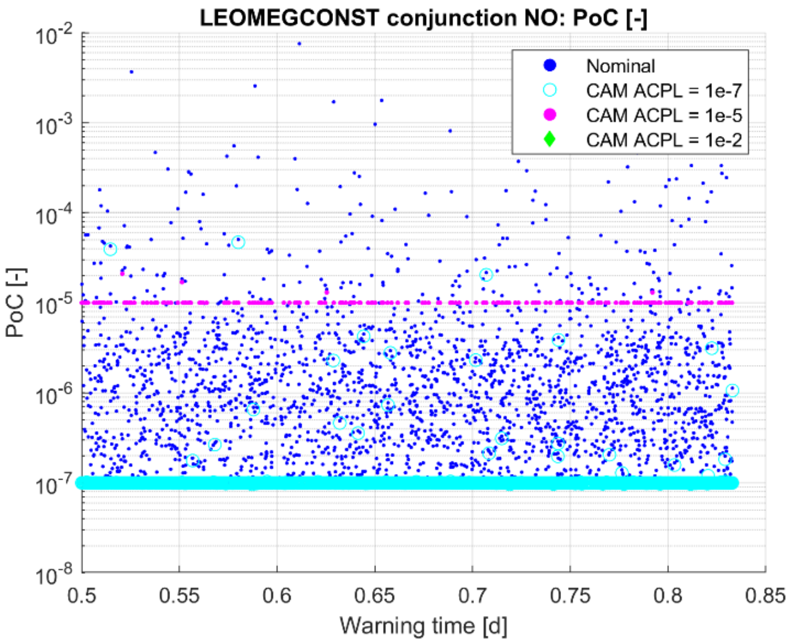
Warning time. Concept of operations: new



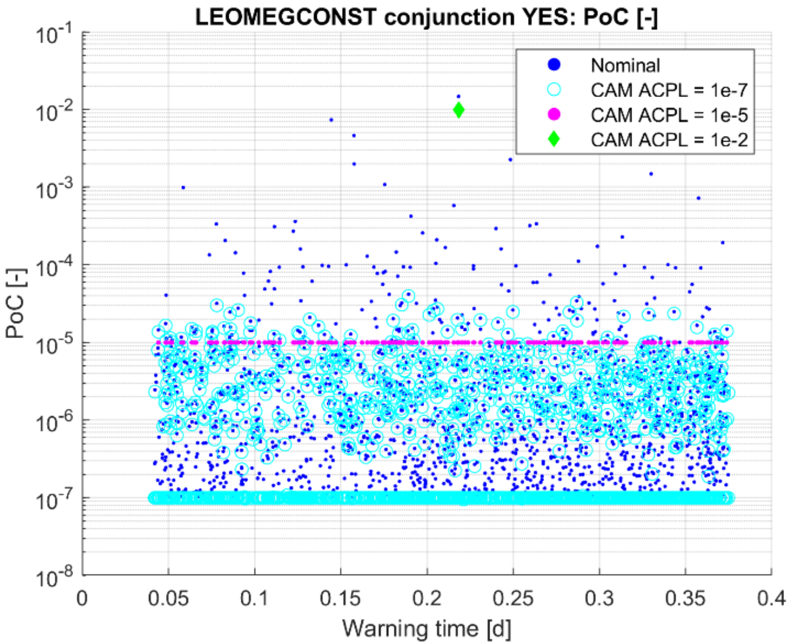
Task 3.5: Sensitivity analysis – Large scale sim.

Characterisation of the LEOMEGCONST ballistic CAM scenarios

PoC. Concept of operations: standard



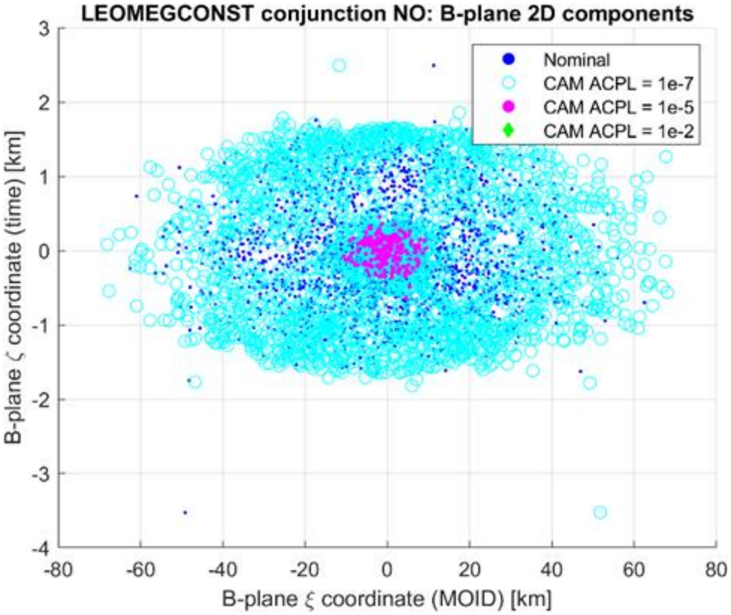
PoC. Concept of operations: new



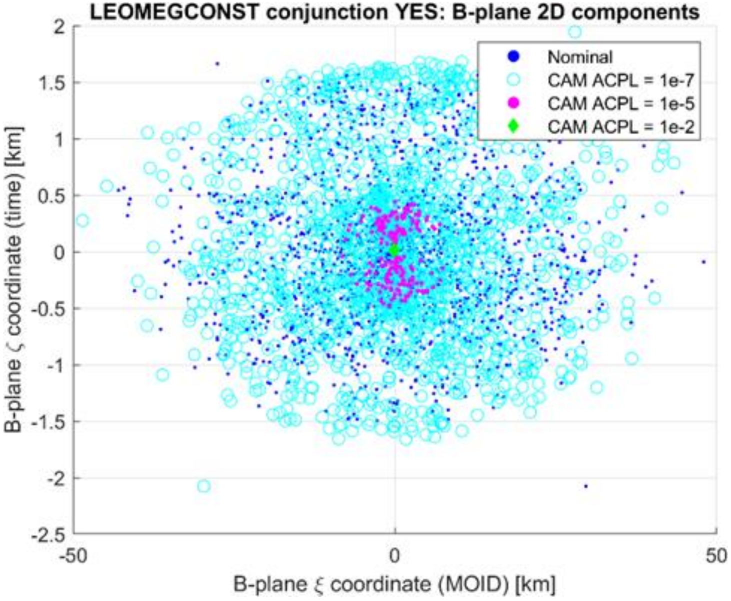
Task 3.5: Sensitivity analysis – Large scale sim.

Characterisation of the LEOMEGCONST ballistic CAM scenarios

B-plane representation. Concept of operations: standard



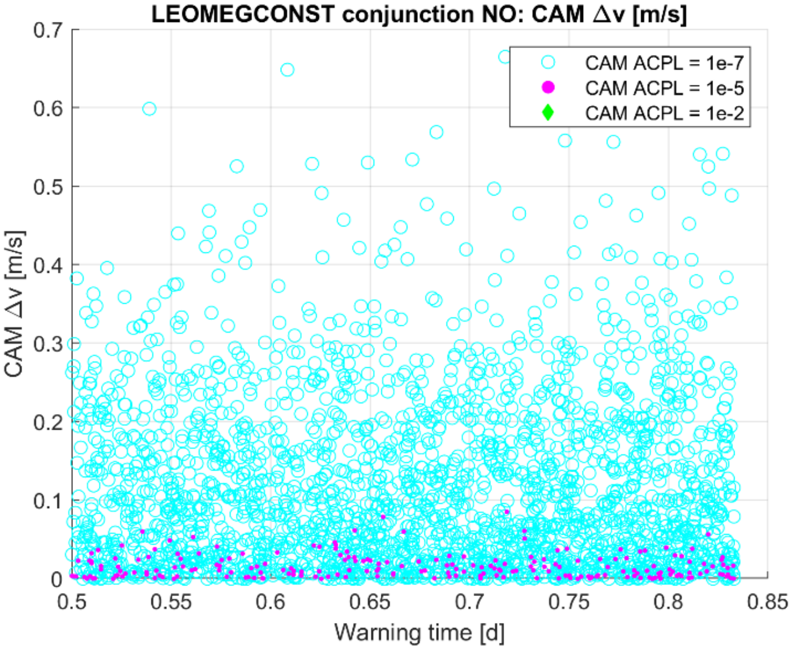
B-plane representation. Concept of operations: new



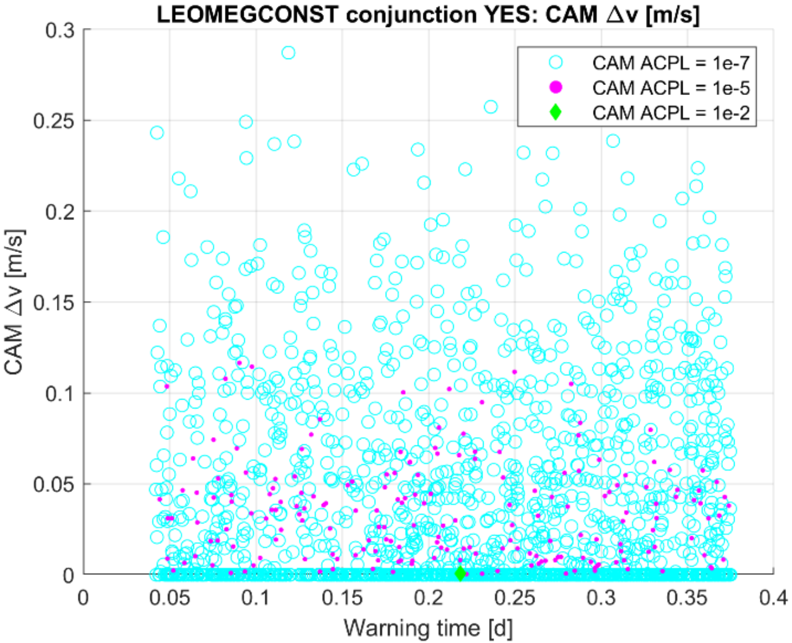
Task 3.5: Sensitivity analysis – Large scale sim.

Characterisation of the LEOMEGCONST ballistic CAM scenarios

CAM manoeuvre. Concept of operations: standard



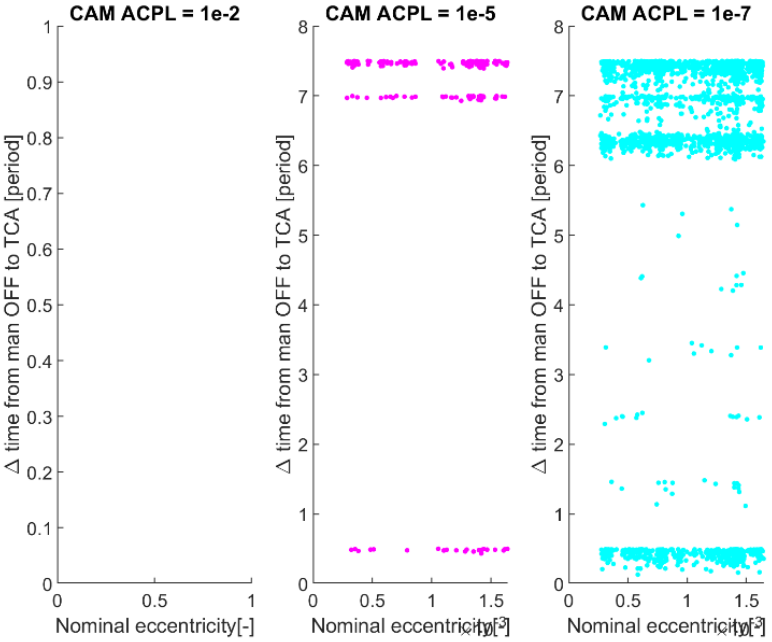
CAM manoeuvre. Concept of operations: new



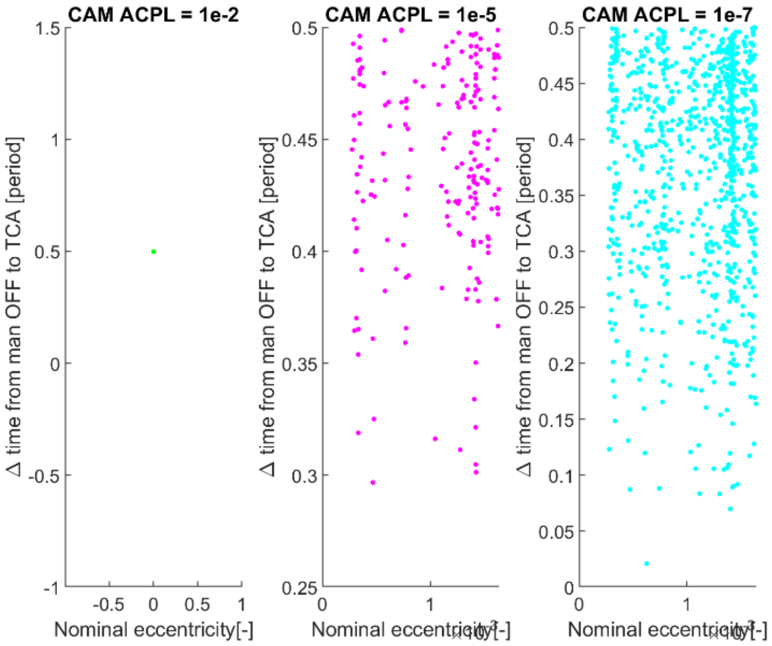
Task 3.5: Sensitivity analysis – Large scale sim.

Characterisation of the LEOMEGCONST ballistic CAM scenarios

Delta time from manoeuvre off until TCA.
Concept of operations: standard



Delta time from manoeuvre off until TCA.
Concept of operations: new



Task 3.5: Sensitivity analysis – Large scale sim.

Discussion

- In all the scenarios the method can achieve a pre-defined ACPL.
- For each of them two concepts of operations were considered, the STANDARD one (NO) and the new PROPOSED one (YES).
- In some cases, the lowest value of ACPL 1e-7 was not achieved due to numerical errors when targeting an exact value of the ACPL and to the very short warning time, especially in the new concept of operations approach. A future extension of the method will try to minimise the PoC when an exact ACPL cannot be achieved.

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Task 4: Update of the ESA DRAMA ARES tool

Task 4: Update of the ESA DRAMA ARES tool

- **Task 4.1**: Technical specification and documentation
- **Task 4.2**: Software
- **Task 4.3**: Software test plan
- **Task 4.4**: Successful software tests

Key aspects of the proposed solution

- **GMV:** ECSS compliant software development, tailored for analysis software
- **GMV:** Impact on ARES functionality: low-thrust delta-V, orbit evolution, etc.

Task 4.1: Technical specification and doc

- Documentation focused on software developments for low-thrust propulsion
- Documentation delivered:
  - **Software Requirements Specification (SRS)**
  - **Software Design Document (SDD)**
    - Software static and dynamic architecture
    - Interfaces
    - Software components high- and low-level design
    - Requirements to design traceability
  - **Software Design Justification (SDJ)**
    - Justification of algorithms: low-thrust CAM design, low-thrust transfer processing and low-thrust operational concepts
  - **Software Validation Specification (SVS)**
    - Definition of Test Designs
    - Description of Unit and System Test Cases, pass/fail criteria and covered requirements
    - Explanation of validation test procedures
    - Requirements to test cases traceability

Task 4.1: Technical specification and doc

- **Software Validation Results (SVR)**
  - Unit and System tests results
  - Non-regression tests in Continuous Integration/Continuous Delivery (CI/CD) pipeline of GitLab
- **Software User Manual (SUM)**
  - Explanation of the configuration of the new functionality:
    - Selection of ARES type of analysis
    - Selection of an electric thruster
    - Selection and configuration of the operational concepts as S/C covariance source
- **Software Reuse File (SRF)**
  - Summary of software changes
  - IPR analysis
  - Software dependencies
- **Software Release Note (SRN)**, software and documentation released
- **ARES Technical Note**
  - Updated with low-thrust CAM design, low-thrust transfer processing and low-thrust operational concepts

Task 4.2: Software

- Computation of collision statistics for a **pre-computed trajectory** (OEM) of a low-thrust transfer (e.g. EOR transfer):

  - **Read** input trajectory (OEM following CCSDS standards)
  - Trajectory sampled into several **reference orbits** according to the **orbital regime** (more divisions in regions with higher **debris density**)
  - **Residence time** associated to each reference orbit
  - **Recursive** call to DRAMA ARES existing functionality for the analysis of a single target orbit
  - Final collision statistics obtained as a weighted average of the results obtained for each reference orbit (according to the residence time in each of them)

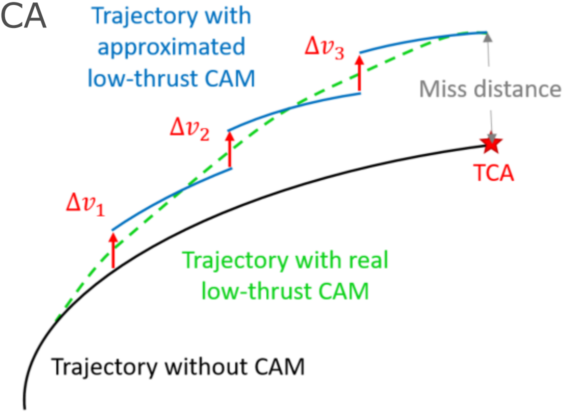
➢ Existing functionality (definition of a single target orbit with orbital parameters) also available for low thrust.

Task 4.2: Software

- Spacecraft's **covariance** at **TCA** obtained according to the **Operational Concept:**
  - Three operational concepts with associated look-up tables:
    - **GNSS based OD + feedback** control
    - **Autonomous** manoeuvre feedback control
    - **Uncontrolled** (nominal operational concept)
  - Covariance values derived from the analysis performed in Technical Report 3
  - Parameters:
    - **Orbital transfer** type: LEO disposal, LEO to LEO, LEO to MEO, GTO to GEO, GEO graveyard
    - **Thruster uncertainty**:
      - Low error level: 1% thrust magnitude error and 0.5 deg of pointing accuracy.
      - Moderate error level: 2% thrust magnitude error and 1 deg of pointing accuracy.
      - High error level: 5% thrust magnitude error and 2.5 deg of pointing accuracy.
    - **Time to event occurrence**. To take into account the time since the last orbit determination (0-7 days)

Task 4.2: Software

- Design of **low-thrust manoeuvres** for collision avoidance
  - Low-thrust **manoeuvre integration**:
    - Low-thrust manoeuvre arc divided into sub-arcs (impulsive burn in each of them)
    - Thruster acceleration obtained from user inputs
    - Secant method: compute the manoeuvre duration that provides the required miss distance
    - Initial guesses based on the impulsive delta-V
    - Keplerian propagation to account for the manoeuvre effect at TCA
  - **Manoeuvre direction**:
    - EOR/EOL shutdown: negative/positive along-track
    - Otherwise: same as the one used for the impulsive delta-V
  - Special cases:
    - Separation direction orthogonal to the B-plane
    - Recovery from differential drag manoeuvres
    - Objective: change the **semi-major axis** to reduce risk
  - **Orbital uncertainty** is not needed at TCA. PoC threshold translated beforehand to geometry thresholds for conjunction mitigation (ARES pre-existing approach)



Task 4.3: Software test plan

- Test environment: Debian 10.0.0 amd64
- Unit tests:
  - Existing Fortran modules extended
  - Can be executed manually with ./ares_unit_tests
  - Test Cases:
    - **Orbit transformations**: orbital changes (Keplerian propagation, reference frame transformations, SV to COE…).
    - **Low-thrust manoeuvre integration**: integration of a low-thrust manoeuvre (sub-arcs division).
    - **Delta-V low-thrust**: computation of the required delta-V (manoeuvre duration) to achieve the desired miss distance.
    - **Precomputed trajectory discretisation**: processing of a pre-computed trajectory (obtain reference orbits, identify the transfer type, compute the time span of the trajectory, compute the residence time associated to each reference orbit).
    - **Operational concept covariance preloaded table**: determination of the spacecraft’s covariance according to the operational concepts.
    - **Precomputed trajectory reading**: reading of an OEM following CCSDS standards.

Task 4.3: Software test plan

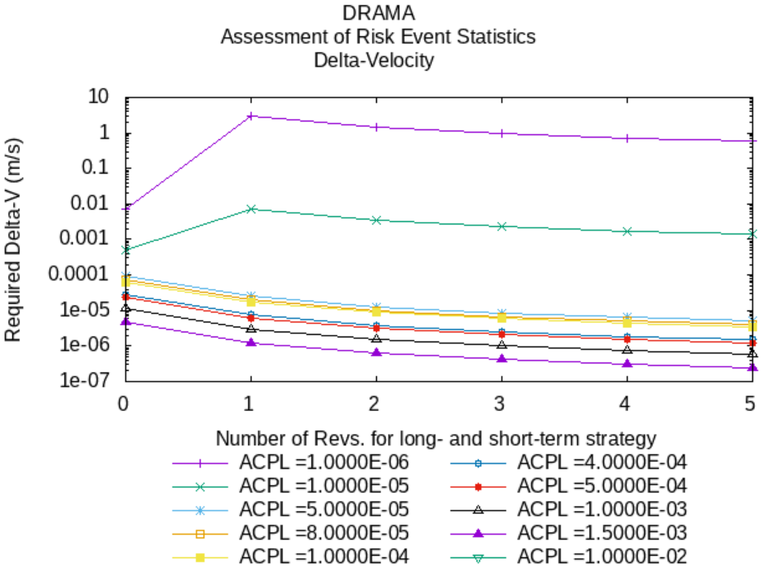
- System tests:
  - Existing system tests directory extended
  - Can be executed manually moving ares.cfg to the home directory
  - Test Cases:
    - **Preloaded trajectory low-thrust collision statistics**
      - Analyse trajectory with 2 ephemerides (same SV, close epoch)
      - Same partial results
      - Same final results as compared to analysis of a single target orbit
    - **Preloaded trajectory statistical weighted average**
      - Analyse trajectory. Save SV and epoch of reference orbits. Save final results
      - Reference orbits of the trajectory analysed as single orbits. Results aggregated externally
    - **Reference orbit low-thrust collision statistics**
      - Electric propulsion (with very high acceleration) vs. chemical propulsion
    - **Uncontrolled Operational concept**
    - **Autonomous manoeuvre Operational concept**
    - **GNSS-based OD + feedback control Operational concept**
      - Same results with covariance selected by ARES from look-up table and if covariance provided in input file

Task 4.4: Successful software tests

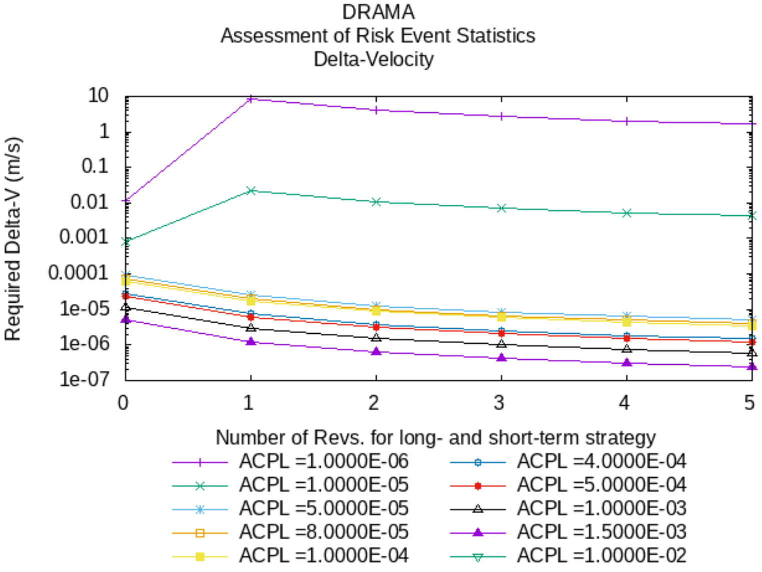
- Tests status: all unit and system tests successfully passed
- Tests running automatically in GitLab:
  - **Unit tests** running successfully in the CI/CD pipeline of GitLab for all distributions of Linux, Windows and MacOS
  - **System tests**:
    - Running successfully in the CI/CD pipeline of GitLab for Windows and some Linux distributions
    - Prepared as non-regression tests against validated references
    - System tests from DMF-03 project have also been included

Task 4.4: Comparison of results

Chemical propulsion system



Electric propulsion system, a=1m/s^2



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Other aspects

Participation in congresses and conferences

13 abstracts to conferences & congresses stemming from ELECTROCAM work:

- ASS 2022
- KEPASSA 2022
- IAC 2022
- AAS_Summer 2022
- AAS_34Edition 2023
- AEC 2023
- AIDAA 2023
- 2nd NEO-SST 2023
- EUCASS 2023
- IAC 2023
- SPACEOPS 2023
- CELMECVIII 2023
- SCITECH 2024

AOB

Thank you

27/07/2023